

Review

# Froude and the contribution of naval architecture to our understanding of bipedal locomotion<sup>☆</sup>

Christopher L. Vaughan<sup>a,b,\*</sup>, Mark J. O'Malley<sup>b</sup>

<sup>a</sup> MRC/UCT Medical Imaging Research Unit, Department of Human Biology, Faculty of Health Sciences, University of Cape Town, Observatory, Western Cape 7925, South Africa

<sup>b</sup> Department of Electronic and Electrical Engineering, University College Dublin, Belfield, Dublin 4, Ireland

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## Abstract

It is fascinating to think that the ideas of two 19th century naval architects could offer useful insights for 21st century scientists contemplating the exploration of our planetary system or monitoring the long-term effects of a neurosurgical procedure on gait. The Froude number, defined as  $Fr = v^2/gL$ , where  $v$  is velocity,  $g$  is gravitational acceleration and  $L$  is a characteristic linear dimension (such as leg length), has found widespread application in the biomechanics of bipedal locomotion. This review of two parameters,  $Fr$  and dimensionless velocity  $\beta = (Fr)^{1/2}$ , that have served as the criterion for dynamic similarity, has been arranged in two parts: (I) historical development, including the contributions by William Froude and his son Edmund, two ship designers who lived more than 130 years ago, the classic insights of D'Arcy Wentworth Thompson who, in his magnum opus *On Growth and Form*, espoused the connection between mathematics and biology, and the pioneering efforts of Robert McNeill Alexander, who popularised the application of  $Fr$  to animal locomotion; and (II) selected applications, including a comparison of walking for people of different heights, exploring the effects of different gravitational fields on human locomotion, establishing the impact of pathology and the benefits of treatment, and understanding the walking patterns of bipedal robots. Although not all applications of  $Fr$  to locomotion have been covered, the review offers an important historical context for all researchers of bipedal gait, and extends the idea of dimensionless scaling of gait parameters.

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## 1. Introduction

Bipedalism is the fundamental evolutionary adaptation that sets hominids—and therefore humans—apart from other primates [1]. A toddling infant, taking its first few halting steps on the living room carpet, uses essentially the same walking pattern as a 2 m tall adult striding down the road [2]. However, it is not only humans who utilise bipedalism as their primary mode of terrestrial locomotion; prehistoric dinosaurs, some of which had a height of over 5 m, used a running gait [3], while the hopping crow stands less than 0.3 m tall [4]. This leads to the obvious question: how do we compare the gait patterns of all these different animals? The

answer lies in the field of physical similarity and dimensional analysis [5]. It was the naval architect William Froude who, 130 years ago, introduced a non-dimensional parameter that served as the criterion for dynamic similarity when comparing boats of different hull lengths [6]. This parameter, now known as the Froude number or  $Fr$ , is equal to  $v^2/gL$  where  $v$  is velocity,  $g$  is gravitational acceleration, and  $L$  is a characteristic length (in nautical engineering, the hull length). This review of the contribution of naval architecture—the Froude number—to our understanding of bipedal locomotion has been compiled in two parts, numbered I and II, and seven sections, numbered 2.1–3.4.

## 2. Part I: historical development

### 2.1. William and Edmund Froude

William Froude was born in Devon, England in 1810 and studied classics and mathematics at Oxford University (cf.

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\* Corresponding author. Tel.: +27-21-406-6235;

fax: +27-21-448-7226.

E-mail address: [kvaughan@cormack.uct.ac.za](mailto:kvaughan@cormack.uct.ac.za) (C.L. Vaughan).

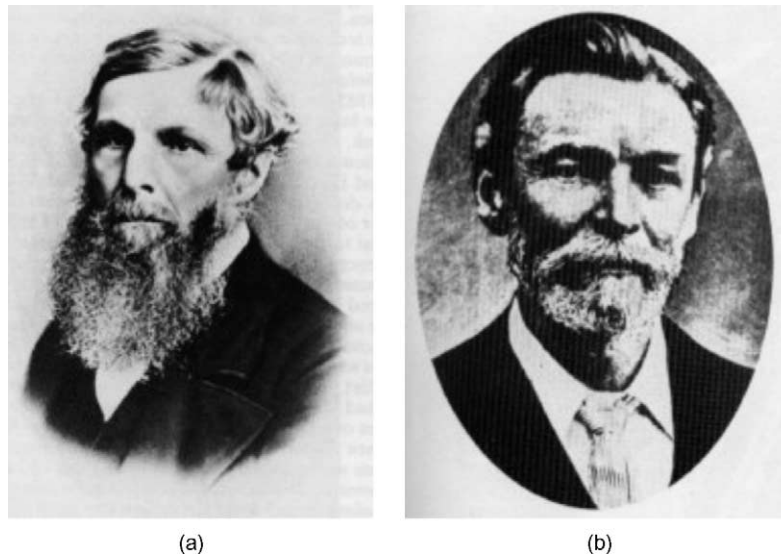


Fig. 1. (a) William Froude (1810–1879) and (b) his third son Robert Edmund Froude (1846–1924). Reproduced with the permission of Russell [7].

Fig. 1a). Following graduation he worked in the field of civil engineering, assisting Isambard Kingdom Brunel with the building of the Bristol and Exeter Railway in 1838. By 1846, he had retired from full time employment to run the family estates but it was also an opportunity to turn his attention to the sea and ships that had always exercised a special fascination for him [7]. His work was influenced by a particularly expensive mistake [8]. Froude had been consulted by his old boss Brunel and the mistake involved the design of a huge iron-clad ocean liner, the *Great Eastern*, which was the largest ship in the world at that time. Even though the designers had included paddle wheels and a screw propeller, as well as auxiliary sails (Fig. 2a), the sheer size of the ship meant that it had insufficient power. Its speed was so slow that there was no way the ship could earn enough to pay for the cost of its fuel [8], a significant shortcoming, since the *Great Eastern* laid the undersea telegraph cable between Co. Kerry, Ireland and Newfoundland, Canada in 1869, linking the continents of Europe and North America for the first time. Although Froude was involved with the engineering of the *Great Eastern* in only a minor capacity, he clearly appreciated how poorly naval architects in the mid-19th century understood wave resistance and the effects of size.

Froude's own approach to science, and indeed religion, is summed up in a phrase that he often quoted: "Our sacred duty [is] to doubt each and every proposition put to us including our own" [9]. He turned his attention to experimentation in the River Dart on two scale models, called *Raven* and *Swan*, in which he demonstrated that there was no ideal form and that performance varied with speed. With this evidence he persuaded the government to fund the building in the early 1870s of a towing tank, almost 100 m long, across the road from his house. With the assistance of his third son Robert Edmund (Fig. 1b), born in 1846, he was able to tow his models at a known speed through still water us-

ing a steam-powered winch that pulled the carriage along a track suspended over the tank (Fig. 2b). The drag force acting on the models was monitored by a custom-designed dynamometer [10].

What Froude observed was that large and small models of geometrically similar hulls produced different wave patterns when towed at the same speed (Fig. 2c). However, if the larger hull was pulled at greater speeds, there was a speed at which the wave patterns were nearly identical. This occurred when the ratio of the velocity squared to the hull length was the same for both large and small hulls. He had thus demonstrated that geometrically similar hulls would also be dynamically similar, in terms of wave resistance, when this ratio—now known as the Froude number—was constant. In his own words [6]:

That "Law of Comparison" is that if the speeds of the ships are proportional to the square roots of their dimensions, their resistances at those speeds will be as the cubes of their dimensions.

From 1873 until his death in 1879 in Simonstown, a naval base near Cape Town, South Africa (Fig. 3), William Froude published many papers in journals such as *Nature* [11] and in other scholarly publications [6,10,12]. After the passing of his father, Edmund continued the family tradition of research on hydrodynamics, publishing his own findings on wave-making resistance of ships, and introducing the analogy of a simple pendulum [13]. He was also well known for having recommended the suitable dimensions of screw propellers [14]. By the time of Edmund's own death in 1924, the father and son had left an indelible legacy—between them they published 34 papers in the *Transactions of the Institution of Naval Architects*—with every ship in the world today owing its performance to their insights and steadfast endeavour.

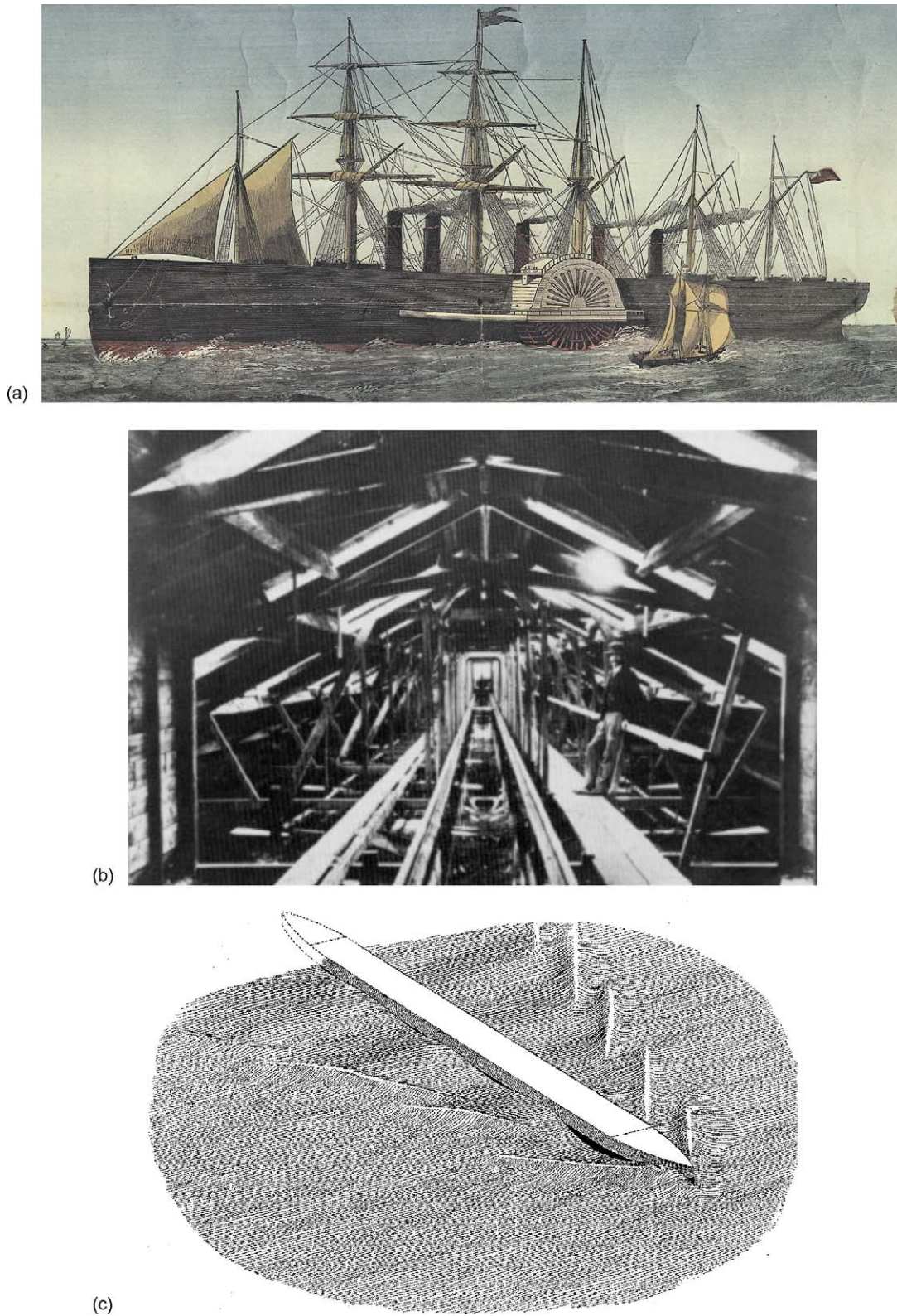


Fig. 2. (a) The *Great Eastern* was a ship which convinced William Froude that naval architects of the mid-19th century did not understand wave resistance and the effects of size. Reproduced with the permission of the Maritime Museum, Valentia Island, Co. Kerry, Ireland. (b) The towing tank built by William Froude in the early 1870s where he conducted hydrodynamic experiments on scale models. Reproduced with the permission of Russell [7]. (c) William Froude conducted experiments on the resistance of model boats of different lengths, allowing him to study diverging bow waves [12]. Reproduced with permission of the Royal Institution of Naval Architects.



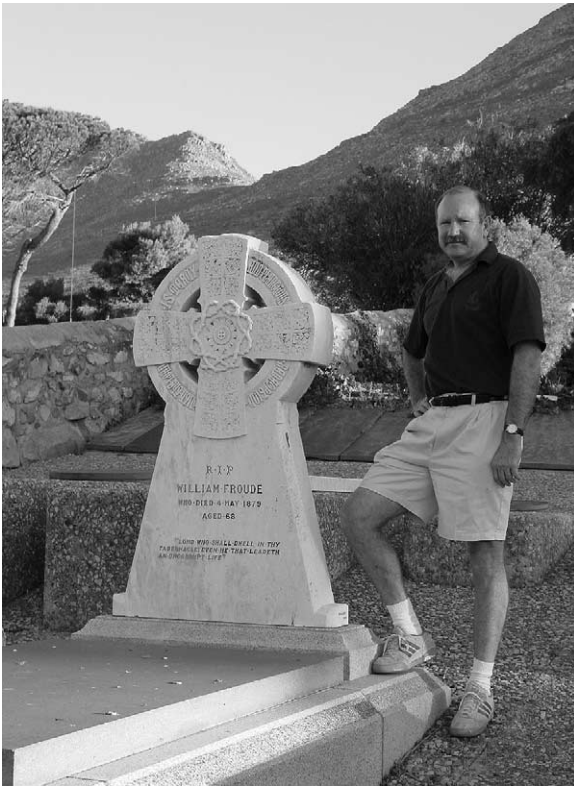


Fig. 3. William Froude's gravestone in Simonstown, a naval base near Cape Town, with the first author (CLV) standing nearby.

## 2.2. D'Arcy Wentworth Thompson

While the Froudes had concentrated on the movement of ships, it was the polymath D'Arcy Wentworth Thompson who first recognised the connection between the Froude number and animal locomotion, although his derivation of Froude's Law was based on skin friction and did not acknowledge the interaction between gravity and inertia. Born in Edinburgh, Scotland in 1860, he won prizes for the Classics, Greek, mathematics and modern languages in his final year of high school [15]. After beginning his medical studies at Edinburgh University, he switched to science at Cambridge University, earning a BA in zoology in 1883. The following year Thompson was appointed Professor at Dundee (later incorporated within the University of St. Andrews) and occupied this chair until his death in 1948, a remarkable record of 64 years (Fig. 4). Thompson possessed a unique set of skills: he was a Greek scholar, a biologist and a mathematician. Although he was a prolific writer, publishing almost 300 scientific articles and books [16], he is best known for his famous book *On Growth and Form*, first published in 1917 [17]. His primary thesis was that all living creatures could only be properly understood in terms of pure mathematics. These arguments were advanced in beautiful prose that was a pleasure to read with poetry in the sentences [18].

Thompson's understanding of the relationship between scale, Froude and bipedal locomotion are best revealed in

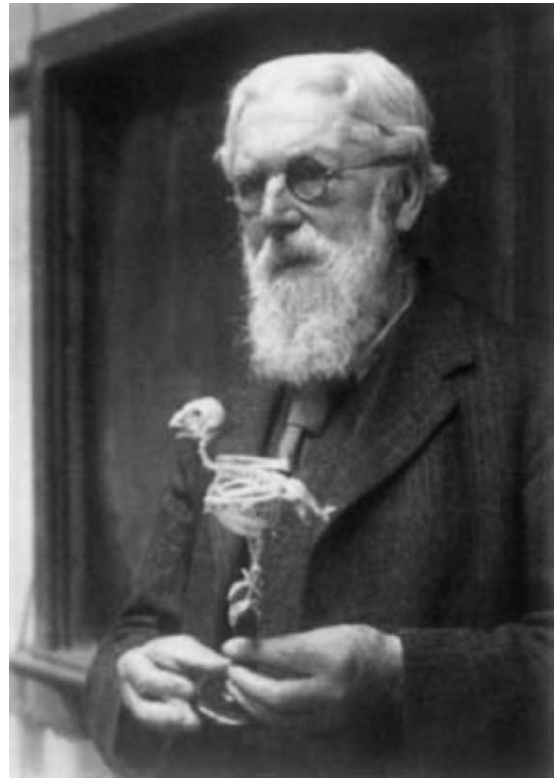


Fig. 4. D'Arcy Wentworth Thompson (1860–1948), author of *On Growth and Form* [17]. Reproduced with the permission of O'Connor and Robertson [15].

three quotations that have been extracted from his magnum opus *On Growth and Form* [17]. On page 17, he introduces the concept of dynamic similarity:

For *scale* has a very marked effect upon physical phenomena, and the effect of scale constitutes what is known as the principle of similitude, or of dynamical similarity.

This statement forms the basis for a discussion on the strength of a muscle and the resistance of a bone to crushing stress, both of which vary with their cross-sections. In considering the movement of terrestrial animals, living under the direct action of gravity, he argues that there is a limit to the size of an animal, with an elephant approaching the limit. Then on page 23 Thompson introduces the analogy between two dynamically similar animals and two steamships and their propulsion:

In two similar and closely related animals, as also in two steam engines, the law is bound to hold that the rate of working must tend to vary with the square of the linear dimensions, according to Froude's Law of steamship comparison.

He explores the movement of birds and fish through a fluid medium and shows that by applying Froude's Law their velocities squared are proportional to a linear dimension (e.g. the animal's length). Finally, on page 30, he compares bipedal walking patterns by means of a pendulum model:

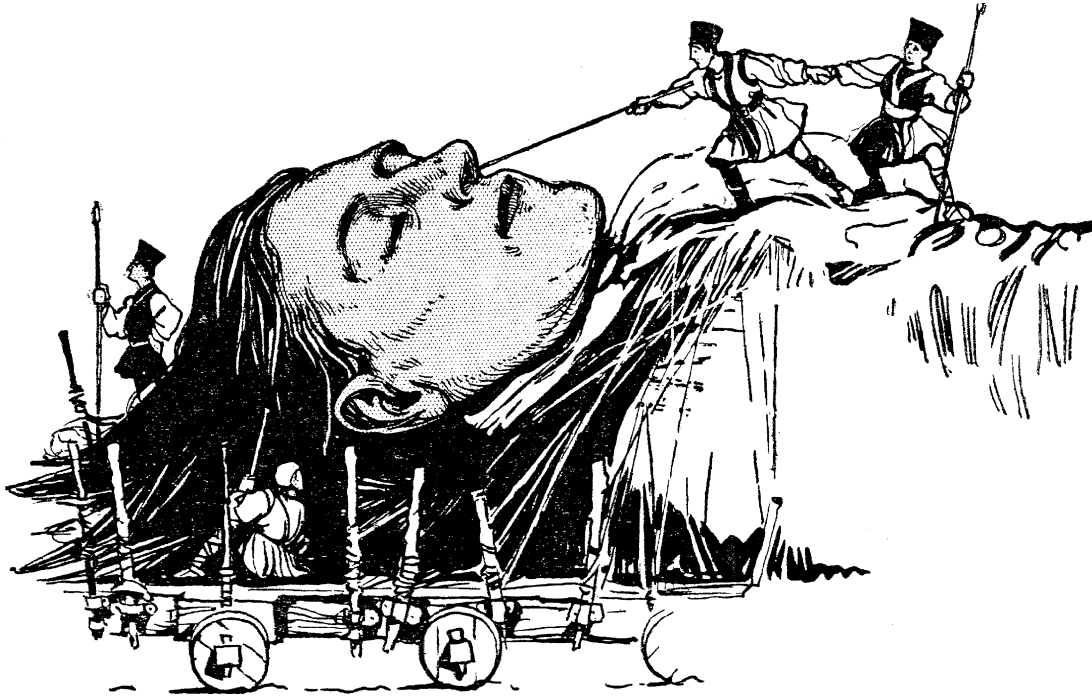


Fig. 5. The tiny inhabitants of Lilliput await the revival of Captain Lemuel Gulliver, in a scene from Jonathan Swift's book, *Gulliver's Travels*, which was first published in 1726. The different-sized characters in this classic inspired Thompson [17] to apply Froude's Law to compare their stride lengths.

Now let two individuals walk in a similar fashion with a similar angle of swing. The arc through which the leg swings will vary as the length of the leg, but the time of the swing will vary as the square root of the pendulum length. Therefore the velocity will also vary as the square root of the length of the leg.

In a later abridged edition of *On Growth and Form*, which John Tyler Bonner had the temerity to edit [18], an illustration of a simple pendulum was included, and the step lengths of characters from Jonathan Swift's book *Gulliver's Travels* (Fig. 5) were compared. The inhabitants of Lilliput and Brobdingnag, with heights of 0.15 and 20 m, respectively, would have had step lengths of 0.06 m and 8.4 m on the basis of geometric similarity. By extending Thompson's argument, and applying Froude's Law and dynamic similarity to the inhabitants, we see that they would have had walking velocities of 0.29 and 3.3 m/s, respectively. These values can be compared to Captain Lemuel Gulliver's height of 1.8 m, a step length of 0.76 m and walking velocity of 1.2 m/s.

### 2.3. Robert McNeill Alexander

Despite the widespread impact of Thompson's book [17], the legacy of the Froudes and their contribution to the understanding of scale and propulsion in biology lay dormant for decades. The one person who changed that and so popularised the application of the Froude number to animal locomotion was Robert McNeill Alexander, Professor of Zoology at the University of Leeds. Through the publication of his books on biomechanics [19–22], his book chapters

that arose from special conferences on animal locomotion [23–26], and most importantly his articles in high impact journals [3,4,27–36], Alexander has ensured that the Froude number can now take its rightful place as an important parameter for us to employ when studying bipedal gait.

It all started in 1976 with a study to estimate the speeds of dinosaurs [27]. Because there were reasonably well-documented dinosaur tracks from which stride lengths could be measured (Fig. 6), Alexander used observations of living animals, including humans, and applied these to dinosaurs. He argued that the movements of animals of geometrically similar form but of different sizes would be dynamically similar when they moved with the same Froude number  $Fr = v^2/gL$ , where  $L$  was the height of the hip joint above the ground. He also posited that geometrically similar movements required equal values of the relative stride length, the dimensionless ratio  $\lambda/L$ , where  $\lambda$  was the stride length (Fig. 6). Then, based on data from mammals as diverse as jirds (a type of gerbil), men and horses, he plotted  $Fr$  as a function of  $\lambda/L$  on logarithmic coordinates and established the empirical relationship

$$\frac{\lambda}{L} = 2.3(v^2/gL)^{0.3} \quad (1)$$

And this equation could then be rewritten as

$$v = 0.25 g^{0.5} \lambda^{1.67} L^{-1.17} \quad (2)$$

Since  $g$  was known ( $9.8 \text{ m/s}^2$ ), while  $\lambda$  could be measured directly from the dinosaur footprints and  $L$  estimated from intact dinosaur skeletons,  $v$  could be readily calculated. The estimated speeds were rather low, between 1.0 and 3.6 m/s,

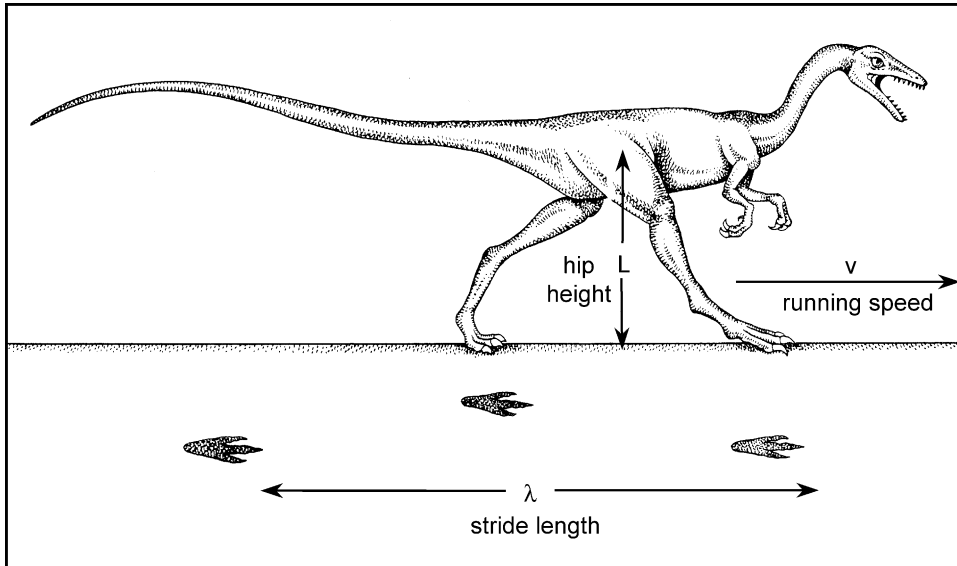


Fig. 6. Alexander [27] showed that it was possible to estimate the running speed  $v$  of a dinosaur, with hip height  $L$  and a stride length of  $\lambda$  (calculated from the fossil record and preserved footprints respectively), using the Froude number data for contemporary animals. Adapted with permission from reference [3].

but it was difficult to know if the tracks were made when the dinosaurs were walking or running [27]. Others used Alexander's approach [37], and a new track site showed that some dinosaurs probably achieved speeds of up to 11 m/s [38]. While Alexander continued to refine the relationship between relative stride length and  $Fr$  (Fig. 7a), the release of

the movie *Jurassic Park* in the early 1990s led to further debate about the maximum speed at which a bipedal dinosaur could run [36,39]. Using arguments based on Froude numbers [3,27] as well as bone strength [3,40], Alexander concluded [36] that *Tyrannosaurus rex* was probably not very good at chasing Jeeps!

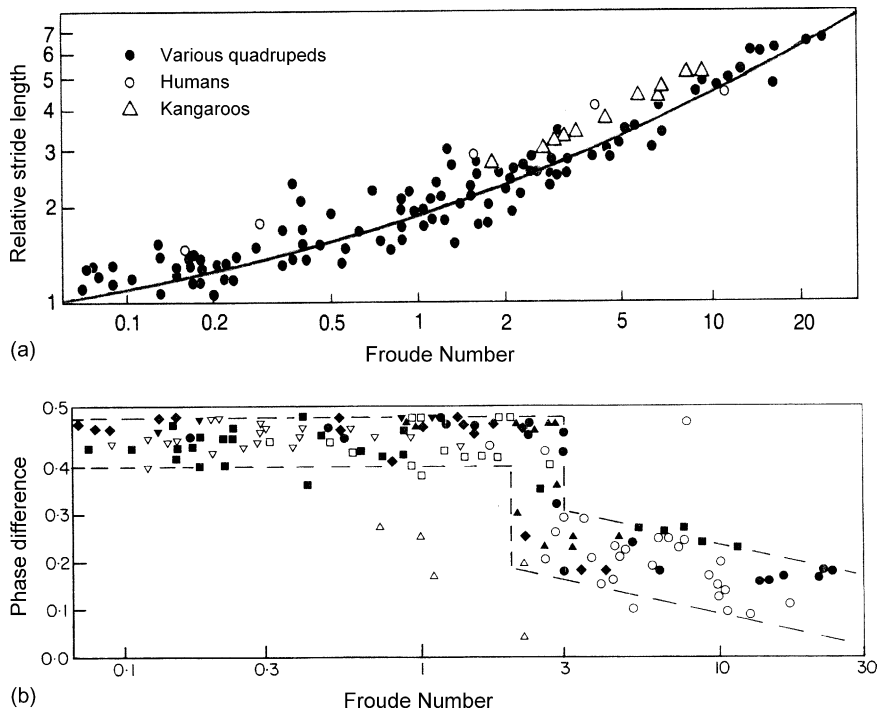


Fig. 7. (a) Relationship between relative stride length  $\lambda/L$  and the Froude number  $v^2/gL$  for bipeds (kangaroos and humans) and quadrupeds [3], with the curve through the data represented in Eq. (1) (note the logarithmic scale on each axis). (b) Phase difference between the forefeet versus the Froude number for quadrupedal mammals, with a logarithmic scale on the horizontal axis. These graphs have been adapted with permission from references [3,31] respectively.

One of Alexander's most highly cited articles was his dynamic similarity hypothesis [31]. He noted that the galloping movements of cats and rhinoceroses are remarkably similar even though the animals are so different [22], and postulated five dynamic similarity criteria: (1) each leg has the same phase relationship; (2) corresponding feet have equal duty factors (% of cycle in ground contact); (3) relative (i.e. dimensionless) stride lengths are equal; (4) forces on corresponding feet are equal multiples of body weight; and (5) power outputs are proportional to body weight times speed. He hypothesised, and provided the necessary experimental evidence to demonstrate, that animals meet these five criteria when they travel at speeds that translate to equal values of  $Fr$  [31]. Evidence in support of criterion 3 has been presented in Fig. 7a, while the data for criterion 2 may be seen in Fig. 7b. At  $Fr$  values below 2, the phase differences lie between 0.4 and 0.5, and the animals utilise symmetrical gaits such as walking, trotting and pacing. There is an abrupt transition at  $Fr$  values between 2 and 3, and above 3 the animals use asymmetrical gaits such as cantering and galloping. Although Alexander developed the dynamic similarity hypothesis for quadrupedal animals [31], it may also be applied to bipedal gait [33].

### 3. Part II: selected applications

#### 3.1. Effects of size

As indicated above in Section 2.1, D'Arcy Thompson used the Froude number to compare the walking speeds of different sized characters in *Gulliver's Travels*. This is clearly one of the major benefits of the Froude number, with the primary application being in the study of children's gait [1,2,32,41–44]. Alexander [32] showed that when dimensionless stride length was plotted as a function of dimensionless speed  $\beta$  where

$$\beta = v/(gL)^{1/2} = (Fr)^{1/2} \quad (3)$$

then data for children aged over 4 years were the same as adults. He used this empirical relationship to predict the walking speeds for two small hominids (with estimated heights of 1.19 and 1.39 m) who left their footprints in volcanic ash at Laetoli in East Africa 3.7 million years ago [1]. Alexander estimated the speeds to be 0.64 and 0.75 m/s, respectively, which corresponds to modern humans walking in small towns [32]. Minetti and his colleagues have studied two other groups that have short statures: Pygmies from West Africa [45,46]; and pituitary dwarfs suffering from growth hormone deficiency [46,47]. This latter group will be described later in Section 3.3 when the effects of pathology and treatment are considered.

Minetti et al. [45] simultaneously measured oxygen consumption and kinematics for Pygmy adults (height  $1.53 \pm 0.04$  m) and Caucasian adults (height  $1.77 \pm 0.04$  m) walking and running on a treadmill. They showed that for walk-

ing, the metabolic power (oxygen consumption per kilogram per minute) was the same for both groups when expressed as a function of  $Fr$ . For running, however, they discovered that the Pygmies had a lower metabolic cost, suggesting that the two groups probably did not run in a dynamically similar fashion. Saibene and Minetti [46] combined the walking data for children aged 1–12 years [48] with that of the Pygmy adults and plotted the recovery of mechanical energy (expressed as a percentage) as a function of  $Fr$ . They demonstrated that, despite the size differences in the subjects, all the data could be fitted by a single curve with a peak energy recovery value of 65% at the same Froude number ( $Fr = 0.25$ ), which represents the optimal walking speed for all humans.

When the fundamental gait parameters (step length, step frequency, single limb stance time, and step width) are rendered dimensionless according to the method advocated by Hof [49], these parameters change during the first 6 years of a child's life [1,41,42]. Thereafter, they are invariant, with the values for 7 year olds, teenagers and adults being the same [41]. This finding has been referred to by Vaughan [1] as a *risk aversion hypothesis*: when a child takes its first few halting steps, its biomechanical strategy is to minimise the risk of falling. Vaughan et al. [50] have argued that when step length and step frequency are scaled non-dimensionally, they account for increases in a child's physical size (i.e. biomechanical changes) and any residual changes in the fundamental locomotor parameters reveal ontogenetic development. They posited that dimensionless velocity  $\beta$  (Eq. (3)), which is the product of dimensionless step length and frequency, serves as a measure of neural development. All three parameters increased from the age of 18 months and reached maturity (i.e. adult values) between 50 and 90 months (Fig. 8a). Based on a study of 200 children, the findings of Vaughan et al. [50] lend support to a theory that posits a neuromaturation growth curve:

$$\beta(t) = 0.45(1 - e^{-0.05t}) \quad (4)$$

where  $t$  is the child's age in months, 0.45 is the adult value for  $\beta$ , and 0.05 is the growth coefficient (Fig. 8b).

#### 3.2. Effects of gravity

Five years before Neil Armstrong took his first historic steps on the moon in 1969, scientists were already intrigued by the problems that could face humans walking in a sub-gravity environment [51]. The Apollo missions of the early 1970s then provided the impetus for further exploration of the mechanics of locomotion when the gravitational acceleration has a magnitude that is either smaller or greater than that on Earth. Since gravity appears explicitly in the Froude number equation, where  $Fr$  is equal to  $v^2/gL$ , it lends itself very well to the purpose of testing the dynamic similarity hypothesis for different values of  $g$  [46,52–57]. If we compare the same subject (i.e. the leg length  $L$  is con-



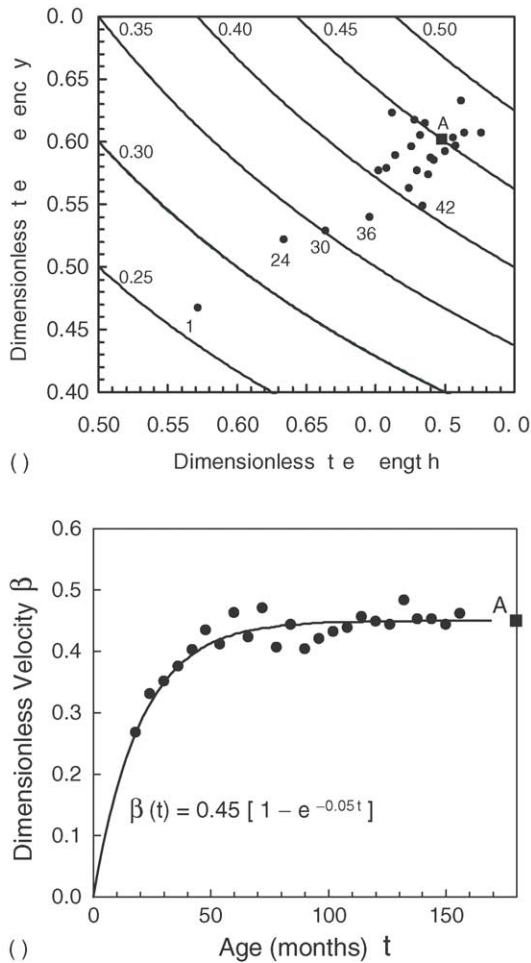


Fig. 8. The dimensionless temporal-distance parameters for 200 young children were clustered into 6-month epochs, beginning at age 18 months, and the average value for each epoch was calculated [50]. (a) Dimensionless step frequency is plotted as a function of dimensionless step length. The numbers next to the dots indicate the age in months, the isocurves are plots of dimensionless velocity  $\beta$ , and the small square A is the mean value for 15 adults. (b) Dimensionless velocity  $\beta$  is plotted as a function of the child's age in months. At 18 months of age,  $\beta$  is equal to 0.27 and this parameter steadily increases up to about 60 months when it reaches the adult value of 0.45. These data have been modelled by a neuromaturation growth curve where the dimensionless velocity at maturity  $\beta_{\infty} = 0.45$ , and the growth coefficient  $k = 0.05$  per month. Reproduced with the permission of Springer-Verlag, publishers of *Experimental Brain Research* [50].

stant) walking on different planets, and assume the Froude numbers are equal, then

$$v_{\text{planet}} = v_{\text{earth}} \left[ \frac{g_{\text{planet}}}{g_{\text{earth}}} \right]^{1/2} \quad (5)$$

There are two basic methods for simulating hypogravity. The first technique involves the subject either walking or running on a treadmill while his torso is suspended (e.g. on a bicycle saddle) via a system of cables and pulleys [51,58]. An almost constant vertical force, in the opposite direction to the gravity vector, thus unloads the subject and enables the researchers to simulate values of the ratio  $g_{\text{planet}}/g_{\text{earth}}$

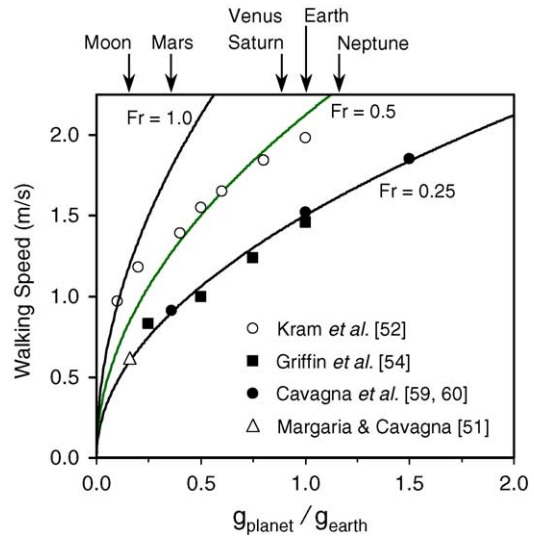


Fig. 9. Walking speed plotted as a function of gravity, which is expressed as a fraction of the Earth's gravity, for a human with leg length  $L = 0.92$  m [46,56,57]. The three theoretical curves (Eq. (5)) represent optimal walking speed ( $Fr = 0.25$ ), walk-to-run transition speed ( $Fr = 0.5$ ), and the physical limit of walking ( $Fr = 1.0$ ). The experimental data of Cavagna and co-workers [51,59,60] and Kram and colleagues [52,54] have been superimposed on these curves. Modified from the publications by Minetti and co-workers [46,56,57].

between 0.1 and 1.0 [52–55]. The one drawback of this method is that the four limbs are not strictly unweighted [46]. The second technique, which can simulate both hypo- and hyper-gravity, is based on aeroplanes flying along a parabolic trajectory [59,60]. While all the body segments experience the same gravitational acceleration, this method has the drawback that the time available to perform the locomotor experiments is limited to between 20 and 30 s [46].

Minetti and co-workers [46,56,57] have utilised Eq. (5) and plotted theoretical curves of walking speed as a function of the gravity ratio (Fig. 9). These three curves represent the prediction of dynamic similarity where leg length  $L$  equals 0.92 m and three separate Froude numbers are plotted:  $Fr = 0.25$  (optimal walking speed);  $Fr = 0.5$  (the walk-to-run transition speed); and  $Fr = 1.0$  (physical limit of walking, after which subject becomes airborne). Superimposed on these curves are the experimental data of Cavagna and co-workers [51,59,60] and Kram and co-workers [52,54]. As can be seen in Fig. 9, there is generally good agreement between the dynamic similarity theory and experiment [46,57].

On Earth, an average man will have an optimal speed of walking of 1.5 m/s and a walk-to-run transition speed of 2.0 m/s. On the Moon, which has a gravity about 0.16 times that of Earth, the corresponding speeds would be about 0.40 times that on Earth (i.e. the square root of 0.16 from Eq. (5)), or 0.6 and 0.8 m/s, respectively (cf. Fig. 9). These values were predicted by Margaria and Cavagna 40 years ago [51], although not using the Froude number approach [57]. The difficulty that astronauts had in trying to walk at terrestrial



speeds on the Moon was evident from the footage of skipping gaits seen on television and debriefings of the Apollo missions [61]. Cavagna et al. [59,60] simulated the gravitational field on Mars (0.40 times the Earth) and hyper-gravity of 1.5 times the Earth (between Neptune's value of 1.13 times Earth's gravity and Jupiter's value of 2.4) and again their optimal speeds are in good agreement with theory, the filled circles in Fig. 9 (note: Jupiter is not shown because it is beyond the illustrated scale for the horizontal axis). Interestingly, as the ratio  $g_{\text{planet}}/g_{\text{earth}}$  increases beyond 1.0, so the distance between the curves for optimal speed ( $Fr = 0.25$ ) and the walk-to-run transition ( $Fr = 0.5$ ) increases, suggesting that an astronaut exploring Jupiter would have a greater range of possible walking speeds available to him [57,60].

### 3.3. Impact of pathology and benefits of treatment

One of the obvious clinical manifestations in adult patients with childhood-onset growth hormone deficiency (GHD) is shortness of stature, in addition to reduced maximal isometric muscle strength and muscle size [46]. Saibene and Minetti [46] compared GHD patients (average height  $1.45 \pm 0.07$  m) with normal age-matched controls (average height  $1.76 \pm 0.04$  m) walking and running on a treadmill. They showed that when metabolic cost and energy recovery were plotted as a function of  $Fr$  for walking, the two groups exhibited responses that were dynamically similar, demonstrating a substantial "normality" in this group of childhood-onset GHD patients [46].

An important challenge facing researchers who perform longitudinal studies on paediatric populations with neurological and orthopaedic disorders (e.g. cerebral palsy, congenital hip dysplasia) is to control or account for growth and maturation [62]. This is a vitally important topic when long-term follow-up studies are conducted to try and understand the natural history and outcome of treatment [63]. As highlighted in Section 3.1 of this paper, dimensionless velocity  $\beta$ , which is the square root of  $Fr$  (Eq. (3)), provides a convenient parameter for comparing the gait of different-sized children [1,50].

An alternative normalisation approach was introduced by O'Malley et al. [64,65], in which a statistical technique based on leg length was applied to a group of 68 normal children and 88 children with the spastic diplegic form of cerebral palsy. Using just two gait parameters (also known as features)—stride length and cadence—a fuzzy clustering technique was used to produce five cluster centres [65]. These data have been re-analysed using non-dimensional scaling [50], rather than statistical detrending [64], and Fig. 10 illustrates the normal cluster (V1) as well as four other clusters (V2 to V5) that characterise the children with cerebral palsy. Note that every child will have a membership in each cluster, with the magnitude depending on the Euclidian distance from the cluster centre [65], and the smaller the dimensionless velocity  $\beta$ , the more disabled the child [62]. Note too that clusters V2 to V5 not only correspond

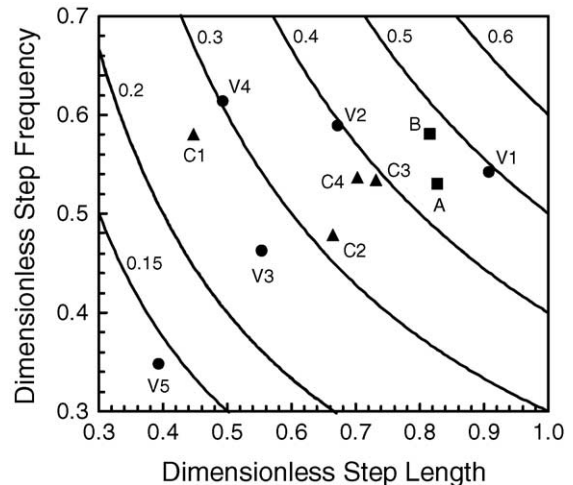


Fig. 10. Five dimensionless cluster centres V1 through V5 (represented by solid dots), constant dimensionless velocity isocurves (cf. Fig. 8a), two normal test subjects A and B, and a child with cerebral palsy C before neurosurgery at 8 years of age (C1), and then at 1 year (C2), 3 years (C3) and 10 years (C4) after surgery [62,65].

with a particular velocity but this dimensionless velocity can be achieved by different combinations of dimensionless step length and frequency (cf. Figs. 8a and 10).

The Froude number has utility not only to assess the impact of pathologies such as cerebral palsy and GHD, but also to track the benefits of treatment. In the case of selective dorsal rhizotomy, a neurosurgical technique designed to reduce spasticity in children with cerebral palsy, there was a pressing need to understand the long-term functional implications of this radical and somewhat controversial procedure [62]. Subramanian et al. [66] performed gait analysis 10 years after surgery for 11 patients who had also been evaluated pre-operatively and at 1 and 3 years post-operatively. Most of the children were less than 12 years of age when they underwent the rhizotomy (the youngest in fact was aged 2 years) so that they had grown considerably in stature during the intervening decade. While the joint kinematics could be easily compared [66], Vaughan et al. [62] have scaled the temporal-distance parameters—step length, step frequency and velocity—to render them dimensionless (Fig. 11). In the case of dimensionless velocity  $\beta$  and dimensionless step length, there was a steady increase up to 3 years after surgery with a slight decrease thereafter. Perhaps the most interesting parameter was the dimensionless step frequency that was unchanged by the surgery and was always significantly less than normal.

As seen in Fig. 10, the real power of the fuzzy clustering approach is that it allows the progress of individual children to be monitored [62,65]. The data for two neurologically intact subjects A and B, who have different ages and genders (a 13-year-old male and a 19-year-old female, respectively) and were not part of the original data set of 68 normal children, both fell close to the control group cluster at V1. Subject C had spastic diplegia and was part of the Cape Town

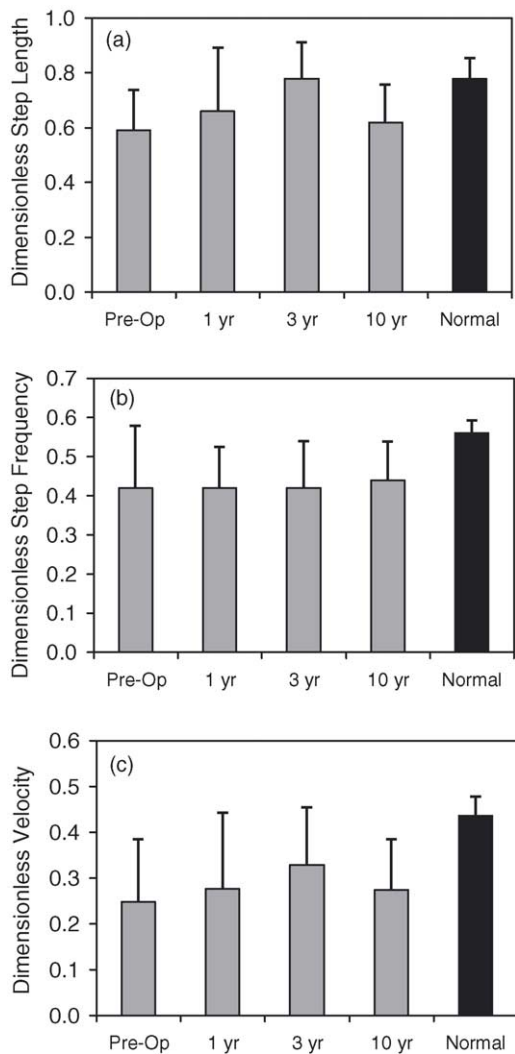


Fig. 11. Dimensionless temporal-distance parameters: (a) step length; (b) step frequency; and (c) velocity, for 11 subjects before rhizotomy, and at 1, 3 and 10 years after surgery compared with normal controls [66]. These figures have been adapted from Vaughan et al. [62].

rhizotomy study [62,66] but was not one of the 88 cerebral palsy children from the Virginia study [65]. She was 8 years of age prior to surgery (C1) and was then studied at 1 year (C2), 3 years (C3) and 10 years (C4) post-surgery. Before surgery subject C1 was very close to cluster V5, the short step length and high cadence strategy, with a dimensionless velocity equal to 0.26 ( $Fr = 0.068$ ). Thereafter, she moved progressively closer to normal, a position which was maintained a decade after her original surgery, by which time she was a young woman of 18.

### 3.4. Bipedal robots

In the late 19th century, George Fallis invented a bipedal walking toy, for which the central claim of his patent stated “This invention consists of a toy which is designed to simulate the human frame and which is a combined pendulum and rocker construction, whereby when placed upon

an inclined plane it will be caused by the force of its own gravity to automatically step out and walk down the said plane” [67]. What Fallis had described was a *passive* bipedal robot (i.e. it lacked *active* power) and, because its centre of gravity was always within its base of support, the gait was *static* (in *dynamic* walking the centre of gravity falls outside the support base during the transition from one foot to the other). These contrasting approaches to the design of bipedal robots—passive versus active and static versus dynamic—has led to an interesting debate [1] and, as highlighted by Vaughan and Verrijzer [68], the Froude number can shed some light on this subject.

Active bipedal robots were first described theoretically over three decades ago [69–71], with a group at Waseda University in Japan building the first successful active walker [72]. It employed a static gait pattern, with active dynamic walking only being achieved in the late 1980s [73,74]. McGeer [75,76] demonstrated that a passive walker based on the Fallis design could achieve dynamic gait despite the lack of any feedback control. His pioneering work has led to more recent efforts to explore the potential of passive dynamic gait to yield biomechanical insights [77–79]. Collins et al. [79] have built a three-dimensional passive robot (mass = 4.8 kg, height = 0.85 m) that walks down a 5 m ramp with a slope of  $3^\circ$  (Fig. 12a). Furusho and Sano [74] built one of the first active dynamic robots (mass = 25 kg, height = 0.97 m). The most recent developments in implementing active dynamic gait in bipedal robots have been made by two Japanese companies: Honda’s ASIMO (mass = 43 kg, height = 1.2 m) [80] and Sony’s SDR-3X (mass = 5 kg, height = 0.5 m) [81] are both anthropomorphic robots with  $24^\circ$  of freedom (Fig. 12b). Although the passive robots are clearly more energy efficient than their actively powered counterparts, their dynamic similarity with human walking is much less certain [68].

McGeer [76] provided the necessary data for his 2D passive walker ( $v = 0.56$  m/s,  $L = 0.80$  m,  $\beta = 0.20$ ) while Collins et al. [79] reported similar values for their 3D walker ( $v = 0.51$  m/s,  $L = 0.82$  m,  $\beta = 0.18$ ). Furusho and Sano [74] also provided the relevant data for their active dynamic robot ( $v = 0.18$  m/s,  $L = 0.6$  m,  $\beta = 0.07$ ). Based on the technical specifications published on the World Wide Web sites for the Honda [80] and Sony [81] robots, we have established the corresponding data for ASIMO ( $v = 0.44$  m/s,  $L = 0.67$  m,  $\beta = 0.17$ ) and SDR-3X ( $v = 0.25$  m/s,  $L = 0.28$  m,  $\beta = 0.15$ ). The dimensionless velocity  $\beta$  has been plotted as a function of leg length  $L$  for these five robots in Fig. 13. While it is clear that the passive walkers have dimensionless velocities—and therefore Froude numbers—that are slightly larger than the two actively powered robots manufactured by Honda and Sony, all these values must be seen in perspective. A comparison of the  $\beta$  values for normal children and adults (Fig. 8a) with the  $\beta$  values for these active and passive robots (Fig. 13) demonstrates that their gait is not dynamically similar to that of adult humans whom they have been designed to mimic. In fact, from a developmen-

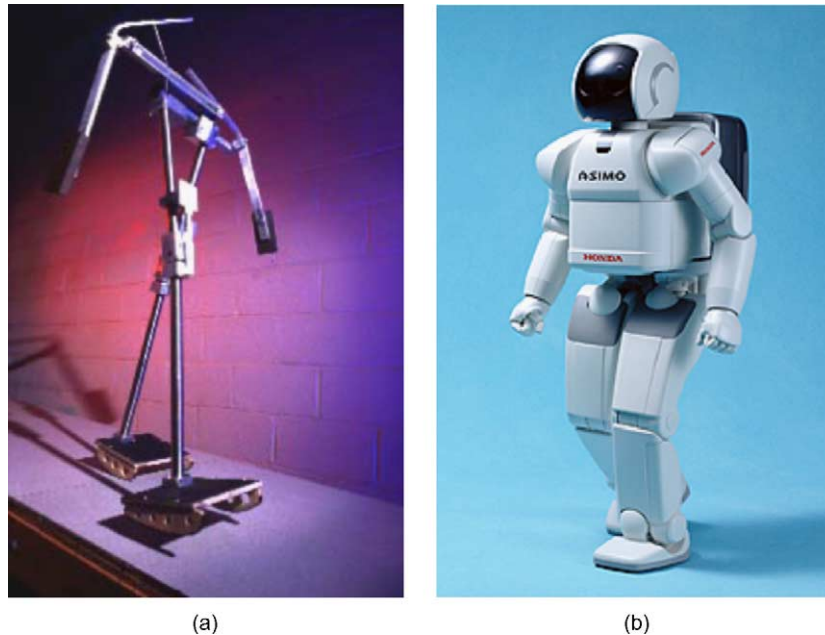


Fig. 12. (a) A passive dynamic walker designed by Collins et al. [79] from Cornell University. Reproduced with permission of Steve Collins. (b) An anthropomorphic bipedal robot called ASIMO with  $24^\circ$  of freedom that demonstrates active dynamic gait, and was manufactured by Honda [80]. Reproduced with permission of the company.

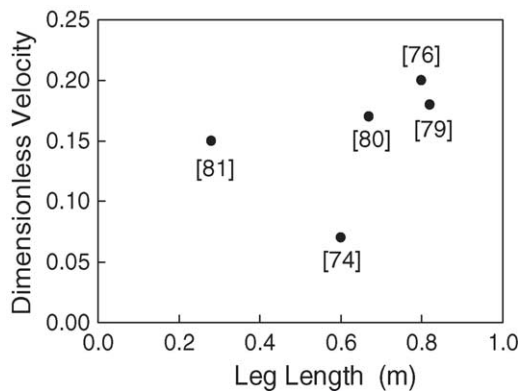


Fig. 13. Dimensionless velocity plotted as a function of leg length for five bipedal robots: Furusho and Sano [74]; McGeer [76]; Collins et al. [79]; ASIMO from Honda [80]; and SDR-3X from Sony [81].

tal point of view, the gait of the most sophisticated bipedal robots is only just beginning to approach that of toddling infants with both having values of  $Fr = 0.04$  and  $\beta = 0.20$  [68]. Interestingly, the monopedal robot developed 20 years ago by Raibert et al. [82] was able to hop at speeds comparable to animals of similar size.

#### 4. Concluding remarks

We have provided just a few important examples showing how the Froude number has been applied to bipedal locomotion. Some other applications that have not been considered include: walking and brachiating in apes [83,84]; running in humans, with an emphasis on understanding

concepts such as stiffness [85,86] and the walk-to-run transition [52,55,87]; and a comparison of the similarities and differences between bipedal and quadrupedal gaits [33,46]. Our review has nevertheless covered a wide spectrum of ideas, from the pioneering work of two naval architects to the advancements in robotic science.

The Froude number, the dimensionless ratio  $Fr = v^2/gL$ , seems a deceptively simple concept. However, as seen in Sections 2 and 3 of this paper,  $Fr$  has been used for a wide range of purposes: to design a properly powered ship; to compare the walking speeds of the characters in *Gulliver's Travels*; to estimate the running speeds of long-extinct dinosaurs and to formulate a dynamic similarity hypothesis for locomotion; to understand the neuromaturation that occurs in the ontogeny of infant walking; to explore the effects of pathologies such as cerebral palsy and growth hormone deficiency and the benefits of a treatment such as rhizotomy; and to demonstrate that bipedal robots walk with a gait pattern that is more dynamically similar to a toddling infant than a striding adult. As Minetti has observed [57], it is fascinating to ponder the idea that the work of two 19th century naval architects could offer biomechanical insights for scientists in the 21st century.

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