Optimization of Power Constrained Multi-Source Uplink Relay Networks

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***Abstract*— In this paper, we study the tradeoffs between basestation beamforming and distributed beamforming in the uplink. Here, multiple single-antenna sources access a multiple-antenna destination (basestation) through a network of single-antenna relays. In order to determine the solution, we decouple the problem into that of optimizing a linear decoder at the destination and that of optimizing relay beamforming weights. We find the optimal linear decoder at the destination and the corresponding feasible SINR feasible upperbound when the number of receive antennas at the destination is no less than the number of relays. We then apply a recently developed semidefinite relaxation technique that optimizes the relay weights. The proposed algorithm iteratively optimizes the weight matrix at the relays and the linear decoder at the destination and is able to reach a fixed point in only a few iterations. In addition, the tradeoffs between basestation and relay network complexity are investigated.**

I. Introduction

Due to the rapidly increasing demand for high data rate and reliable wireless communications, bandwidth efficient transmission schemes are critical. In recent years, user cooperation has been received increasing attention in the research community. By relaying each others' messages, mobile terminals can provide multiple versions of messages to the final destination arriving via different paths. These are termed cooperative diversity techniques [1] [2] and have been shown to significantly improve network performance, in large part, by mitigating detrimental effects of signal fading. Various signalling protocols have been proposed to achieve spatial diversity through user cooperation [1], [3]. The most popular schemes are amplify-and-forward (AF), decode-and-forward (DF), and coded cooperation [4]. Recently, AF has been combined with space-time coding strategies for relay networks, creating a new research area called distributed space-time coding [5] [6].

This paper, however, focusses on achieving overall network power efficiency in the physical layer. In [7], a distributed beamforming system with a single transmitter, receiver, and multiple relay nodes are studied, where second order statistics of the channel are employed to design optimal beam weights at the relays. In [8], single-antenna source-destination pairs that communicate through a relay network is studied, and the relay weight optimization is formulated in terms semidefinite programming (SDP) and solved through semidefinite relaxation. In [9], a distributed beamforming scheme with two relay nodes is proposed which has the advantage of limited feedback and improved diversity.

In [10], the corresponding downlink distributed beamforming problem is investigated, and the optimum precoder is derived. We note that the uplink distributed beamforming problem considered in this paper is not the dual of the problem in [10], as a result of the relay network.

In the literature, the multiple access channel (e.g., uplink) has been studied in depth. In [11], the joint optimization of a transmitter and receiver for a multi-user multiple-input multiple output (MIMO) multi-access channel with sum mean-squared error (MSE) as the objective is studied. In [12], a distributed beamforming strategy is developed for the case where relaying nodes cooperate to form a beam towards the receiver under individual relay power constraints. In [12], the amplitude and the phase of the transmitted signals are adjusted such that they constructively add at the receiver. In [13], the rate maximization of a parallel relay network with noise correlation is studied.

While prior research assumes a single antenna at the source, relay and destination, in the following, we address a scenario where the destination has multiple antennas, corresponding to the situation when multiple users access a base station through a network of relays. In our proposed scheme, we derive the optimal decoder for a fixed set of relay weights. The relay weight optimization in [7] and [8] is extended to a multiple antenna cooperative system. We also propose an iterative algorithm to iteratively optimize the decoder and relay weights.

More specifically, consider a wireless network that consists of M single-antenna sources, R single-antenna relays and an N-antenna destination basestation. The sources access the destination through a cooperative relay network. The system operates in a half-duplex mode. In the first time slot, the sources transmit data to the relay network. In the second time slot the relays amplify and forward the received signal to the destination. First, we separately consider (i) the optimization of the linear decoder for fixed relay beamforming weights and (ii) the optimization of relay weights for fixed decoder. Then, we propose an iterative algorithm to minimize the sum power at the relays with SINR constraints for the whole

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system. Only full channel state information from relays to the destination is required. Second order statistics from sources to relays are also assumed to be known at the destination. The beam weights are calculated at the destination and fed back to the relays.

The remainder of the paper is organized as follows: In Section II, we present the system model. A linear decoder optimization technique assuming known relay weights is developed in Section III, followed by a relay weight optimization procedure in Section IV. Numerical results are provided in Section V.

II. System Model

We consider a multi-access channel where multiple terminals communicate through a relay network to a destination as shown in Fig. 1. We assume a multi-relay network with R single-antenna relays, and M distributed sources, each with single antenna. In order to communicate with the destination, each source transmits its data $s_i, 1 \leq i \leq M$, to the relay network, which then delivers all data streams to the destination. The channel from the M sources to the relay $r, 1 \leq r \leq R$, is represented as

$$x_r = \sum_{i=1}^{M} f_{r,i} s_i + \nu_r \tag{1}$$

where $f_{r,i}$ is the channel from the ith source to the rth relay and ν_r is the noise at the rth relay. Each source uses its maximum power P_i, i.e. $E|s_i|^2 = P_i$ for $i = 1, 2, \ldots, M$. Using vector notation, we can rewrite (1) as

$$\mathbf{x} = \sum_{i=1}^{M} \mathbf{f}_i s_i + \boldsymbol{\nu} \tag{2}$$

where

$$\mathbf{x} = [x_1\ x_2\ \ldots\ x_R]^T$$
$$\boldsymbol{\nu} = [\nu_1\ \nu_2\ \ldots\ \nu_R]^T$$
$$\mathbf{f}_i = [f_{1,i}\ f_{2,i}\ \ldots\ f_{R,i}]^T.$$

The ith relay multiplies its received signal by a complex weight coefficient w_i^*. The vector of signals $\mathbf{t}$ transmitted from the relays is

$$\mathbf{t} = \mathbf{W}^H \mathbf{x} \tag{3}$$

where $\mathbf{W} = \operatorname{diag}(w_1, w_2, \ldots, w_R)$. The received signal vector at the N-antenna destination destination is expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{t} + \mathbf{n} \tag{4}$$

where $\mathbf{H}$ is the channel matrix from the relays to the destination. Denote the linear decoder as $\mathbf{G}$, σ_ν^2 as the noise power at each of the relays and $E(\mathbf{n}\mathbf{n}^H) = \sigma_n^2 \mathbf{I}$. Then the estimated signal vector denoted as $\hat{\mathbf{s}}$ is

$$\begin{aligned} \hat{\mathbf{s}} &= \mathbf{G}\mathbf{y} \\ &= \mathbf{G}\left(\mathbf{H}\mathbf{W}^H\left(\sum_{i=1}^{M}\mathbf{f}_i s_i + \boldsymbol{\nu}\right) + \mathbf{n}\right). \end{aligned} \tag{5}$$

The estimated signal for the kth source is

$$\hat{\mathbf{s}}_k = \underbrace{\mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \mathbf{f}_k \mathbf{s}_k}_{Desired\ signal} + \underbrace{\mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \sum_{j=1, j\neq k}^{M} \mathbf{f}_j \mathbf{s}_j}_{Interference} + \underbrace{\mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \boldsymbol{\nu} + \mathbf{g}_k^T \mathbf{n}}_{Colored\ noise} \tag{6}$$

where $\mathbf{g}_k^T$ is the kth row of $\mathbf{G}$.

III. Linear decoder optimization assuming known relay weights

An expression for the signal-to-interference and noise ratio (SINR) for the kth source,

$$SINR_k = \frac{P_s^k}{P_i^k + P_n^k}. \tag{7}$$

where P_s^k,P_i^k and P_n^k denote the desired signal power, interference power and noise power of the kth source, respectively. In (7), the desired signal power is

$$\begin{aligned} P_s^k &= \mathbf{g}_k^T \mathbf{H}\mathbf{W}^H E\{\mathbf{f}_k \mathbf{f}_k^H\} \mathbf{W}\mathbf{H}^H \mathbf{g}_k^* E\{|s_k|^2\} \\ &= P_k \mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \mathbf{R}_f^k \mathbf{W}\mathbf{H}^H \mathbf{g}_k^* \end{aligned} \tag{8}$$

where $\mathbf{R}_f^k$ is the covariance matrix of the channel from the sources to relays for the kth source. We assume independence of the different channel paths from sources to relays and from relays to destination.

The interference power at the kth source is

$$\begin{aligned} P_i^k &= E\left\{\left(\mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \sum_{j=1,j\neq k}^{M} \mathbf{f}_j \mathbf{s}_j\right)\left(\mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \sum_{j=1,j\neq k}^{M} \mathbf{f}_j \mathbf{s}_j\right)^H\right\} \\ &= \mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \left(\sum_{j=1,j\neq k}^{M} P_j \mathbf{R}_f^j\right) \mathbf{W}\mathbf{H}^H \mathbf{g}_k^*. \end{aligned} \tag{9}$$

The noise, which is colored, has power given by

$$P_n^k = \sigma_\nu^2 \mathbf{g}_k^T \mathbf{H}\mathbf{W}^H \mathbf{W}\mathbf{H}^H \mathbf{g}_k^* + \sigma_n^2 \mathbf{g}_k^T \mathbf{g}_k^*. \tag{10}$$

If the relay weights are fixed, the SINR for the kth source is

$$\text{SINR}_k = \frac{\mathbf{g}_k^T \mathbf{A} \mathbf{g}_k^*}{\mathbf{g}_k^T \mathbf{B} \mathbf{g}_k^*}. \tag{11}$$

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where $\mathbf{A} = P_k \mathbf{H}\mathbf{W}^H \mathbf{R}_f^k \mathbf{W}\mathbf{H}^H$ and $\mathbf{B} = \mathbf{H}\mathbf{W}^H (\sum_{j=1, j\neq k}^{M} P_j \mathbf{R}_f^j)\mathbf{W}\mathbf{H}^H + \sigma_\nu^2 \mathbf{H}\mathbf{W}^H\mathbf{W}\mathbf{H}^H + \sigma_n^2 \mathbf{I}$. It can be shown that the maximum value of $SINR_k$ is given by the maximum generalized eigenvalue, denoted as $\lambda_{max}(\mathbf{A}, \mathbf{B})$, and the optimal $\mathbf{g}_k^*$ is the corresponding principal eigenvector denoted as

$$\mathbf{g}_k^* = \wp\{\mathbf{B}^{-1}\mathbf{A}\}. \tag{12}$$

IV. Relay weight optimization

A. Minimization of sum power at relays

We next consider the problem of relay weight optimization for a fixed decoder. We rewrite (6) as

$$\hat{\mathbf{s}}_i = \underbrace{\mathbf{w}^H \mathrm{diag}(\mathbf{g}_k^T \mathbf{H})\mathbf{f}_k \mathbf{s}_k}_{Desired\ \ Signal} + \underbrace{\mathbf{w}^H \mathrm{diag}(\mathbf{g}_k^T \mathbf{H}) \sum_{j=1, j\neq k}^{M} \mathbf{f}_j \mathbf{s}_j}_{Interference} + \underbrace{\mathbf{w}^H \mathrm{diag}(\mathbf{g}_k^T \mathbf{H})\boldsymbol{\nu} + \mathbf{g}_k^T \mathbf{n}}_{Colored\ \ Noise} \tag{13}$$

where $\mathbf{w}$ is a column vector formed by the diagonal elements of $\mathbf{W}$ such that $\mathbf{w}_i = \mathbf{W}_{i,i}$.

Here we consider the problem to minimize the sum transmission power of the relay network for a fixed linear decoder at the destination while the sources' quality of service (QoS) are kept above pre-defined thresholds. In the following, we use SINR as a measure of QoS. We therefore aim to solve:

$$\begin{aligned} &\min_{\mathbf{w}} \ \ P_R \\ s.t.\ \ &SINR_k \geq \gamma_k, \quad for \ \ k = 1, 2, \ldots, M \end{aligned} \tag{14}$$

where s.t. stands for "subject to", P_R is the sum transmit power at the relays given as

$$\begin{aligned} P_R &= E\{\mathbf{t}^H \mathbf{t}\} \\ &= \mathrm{Tr}\{\mathbf{W}^H E\{\mathbf{x}\mathbf{x}^H\}\mathbf{W}\} \\ &= \mathbf{w}^H \mathbf{D}\mathbf{w} \end{aligned} \tag{15}$$

and where $\mathbf{D} \triangleq \mathrm{diag}\left([\mathbf{R}_x]_{1,1}, [\mathbf{R}_x]_{2,2}, \ldots, [\mathbf{R}_x]_{R,R}\right)$. The matrix $\mathbf{R}_x$ is expressed as

$$\mathbf{R}_x = \sum_{j=1}^{M} P_j \mathbf{R}_f^j + \sigma_\nu^2 \mathbf{I}. \tag{16}$$

The $SINR$ constraints for the kth source can be expressed as

$$\frac{\mathbf{w}^H \mathrm{diag}(\mathbf{g}_k^T \mathbf{H})\left(P_k \mathbf{R}_f^k\right)\mathrm{diag}(\mathbf{g}_k^T \mathbf{H})^H \mathbf{w}}{\mathbf{w}^H \mathbf{E}\mathbf{w} + \sigma_n^2 \mathbf{g}_k^T \mathbf{g}_k^*} \geq \gamma_k \tag{17}$$

where

$$\mathbf{E} = \mathrm{diag}(\mathbf{g}_k^T \mathbf{H}) \left(\sum_{j=1, j\neq k}^{M} P_j \mathbf{R}_f^j + \sigma_\nu^2 \mathbf{I} \right) \mathrm{diag}(\mathbf{g}_k^T \mathbf{H})^H. \tag{18}$$

Using (15) and (17) we can rewrite (14) as

$$\begin{aligned} &\min_{\mathbf{w}} \ \ \mathbf{w}^H \mathbf{D}\mathbf{w} \\ s.t.\ \ &\mathbf{w}^H \mathbf{U}_k \mathbf{w} \geq \gamma_k \sigma_n^2 \mathbf{g}_k^T \mathbf{g}_k^* \\ &for \ \ \ k = 1, 2, \ldots, M \end{aligned} \tag{19}$$

where

$$\mathbf{U}_k = \mathrm{diag}(\mathbf{g}_k^T \mathbf{H}) \left(P_k \mathbf{R}_f^k - \gamma_k \sum_{j=1, j\neq k}^{M} P_j \mathbf{R}_f^j - \gamma_k \sigma_\nu^2 \mathbf{I} \right) \mathrm{diag}(\mathbf{g}_k^T \mathbf{H})^H. \tag{20}$$

It is important to observe that there are constraints in problem (19) that are not convex. Therefore convex optimization cannot be applied and (19) may not be easily solvable. Using the methods developed in [7] and [8], we apply a semidefinite relaxation approach to solve a relaxed version of (19) which is summarized briefly: Denoting $\mathbf{Z} = \mathbf{w}\mathbf{w}^H$, (19) can be rewritten as

$$\begin{aligned} &\min_{\mathbf{Z}} \mathrm{Tr}(\mathbf{Z}\mathbf{D}) \\ s.t.\ \ &\mathrm{Tr}(\mathbf{Z}\mathbf{U}_k) \geq \gamma_k \sigma_n^2 \mathbf{g}_k^T \mathbf{g}_k^* \quad for \quad k = 1, \ldots, M \\ &\mathbf{Z} \succeq 0, \ and \ \ \ \mathrm{rank}(\mathbf{Z}) = 1. \end{aligned} \tag{21}$$

where $\succeq 0$ denotes positive semi-definite. Applying semidefinite relaxation we remove the non-convex constraints in problem (21), i.e, the rank one constraint, resulting in

$$\begin{aligned} &\min_{\mathbf{Z}} \mathrm{Tr}(\mathbf{Z}\mathbf{D}) \\ s.t.\ \ &\mathrm{Tr}(\mathbf{Z}\mathbf{U}_k) \geq \gamma_k \sigma_n^2 \mathbf{g}_k^T \mathbf{g}_k^* \quad for \quad k = 1, \ldots, M \\ &\mathbf{Z} \succeq 0. \end{aligned} \tag{22}$$

The above optimization problem can be efficiently solved using optimization software, e.g., SeDuMi [14] by introducing slack variables $\beta_k, k = 1, \ldots, M$ to transform (22) into standard SDP form as follows:

$$\begin{aligned} &\min_{\mathbf{Z} \in \mathbf{C}^{R\times R}} \ \ \mathrm{vec}(\mathbf{D})^T \mathrm{vec}(\mathbf{Z}) \\ s.t.\ \ &\mathrm{vec}(\mathbf{U}_k)\mathrm{vec}(\mathbf{Z}) - \beta_k = \gamma_k \sigma_n^2 \mathbf{g}_k^T \mathbf{g}_k^* \\ &\beta_k \geq 0 \quad for \ \ k = 1, \ldots, M \\ &\mathbf{Z} \succeq 0. \end{aligned} \tag{23}$$

In the operations research literature [15] it has been shown that there always exists a rank-one solution to the relaxed problem (22) as long as $M < 3$. In our problem, recall that M represents the number of sources. From extensive computer simulations for cases of $M > 3$, we have observed that in the vast majority of cases, there also exists a rank one solution. In cases when the solution is not rank one, we use a modified version of the randomization method in [16] [17] [18] to search for a solution as follows:

Randomization method:

1) Assuming $\mathbf{W}_{opt}$ to be thesolution obtained by solving (22), calculate the eigen-decomposition of $\mathbf{W}_{opt}$ =

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$\mathbf{U\Sigma U}^H$ and choose $\mathbf{w}_l = \mathbf{D}^{-1/2}\mathbf{U\Sigma}^{1/2}\mathbf{e}_l$, where the elements of $\mathbf{e}_l$ are independent random variables, uniformly distributed on the unit circle in the complex plane: i.e., $[\mathbf{e}_l]_i = e^{j\theta_{l,i}}$, where $\theta_{l,i}$ are independent and uniformly distributed on $[0, 2\pi)$. It can be shown that $\mathbf{w}_l^H\mathbf{D}\mathbf{w}_l = trace(\mathbf{W}_{opt})$, i.e., the relay individual and sum power is fixed irrespective of the particular realization of $\mathbf{e}_l$. (See Appendix I for details.)

2) A collection of $\mathbf{w}_l$ is then generated and each realization is scaled to satisfy the constraints. Only realizations that satisfy all the constraints are considered as candidates.
3) Among all candidates of $\mathbf{w}_l$ in Step 2, the one with minimum $\mathbf{w}_l^H\mathbf{D}\mathbf{w}_l$ is chosen as the suboptimum solution.

B. FEASIBILITY

Before going through the effort of solving the optimization problem above, it is possible to check the feasibility in advance. For a given set of SINR criteria, the feasibility of (14) can be tested for the following case:

Lemma 1: If the number of receive antennas at the destination destination is larger than or equal to the number of relays, the asymptotic upper bound of the achievable SINR at the kth relay is the maximum generalized eigenvalue $\lambda_{max}(P_k\mathbf{R}_f^k, \sum_{j=1,j\neq k}^M P_j\mathbf{R}_f^j + \sigma_\nu^2\mathbf{I})$.

Proof: Details have been omitted due to space limitations.

C. JOINT DETERMINATION OF LINEAR DECODER AND RELAY WEIGHTS

To jointly optimize the linear decoder and relay weights, the following alternation algorithm is proposed:

1. Initialize the relay beamforming vector as $\mathbf{w} = c * vec(\mathbf{v})$, where c is a large value relative to σ_n^2 and $\mathbf{v}_i = e^{j\theta_i}$, θ_i is a random variable uniformly distributed in [0,2π].

2. Calculate the SINR upperbound for each source according to Lemma 1 and check the feasibility of the constraints. If not feasible then modify the SINR constraints according so that the problem is feasible.

3. Apply (12) to find the optimal decoder vector for each source with fixed relay weights.

4. With the decoder obtained from Step 3, apply (22) to minimize the relay sum power. (Note: if the solution is not rank one, then use the randomization method in Section IV.A)

5. Alternate between Step 3 and Step 4 until the relay sum power reaches a fixed point.

As the sum power of the relays are lower-bounded and the constraints to be satisfied in each step are equivalent, the total power will reduce for each step. It is therefore easy to show that the algorithm will converge to a fixed point, but not necessarily to the globally optimum point.

V. SIMULATION RESULTS

We assume that the second order statistics of the channel coefficients (rather than their instantaneous values) from the M sources to the K relays are available to the destination and that the channel coefficients from the relays to the destination are known at the destination and that the beamforming weights for relays are to be determined. This destination then broadcasts the beamforming weights to the relays. The channel coefficients $\mathbf{H}$ and $\mathbf{f}_k$ are assumed to be mutually independent where $\mathbf{H}$ represents the distributed channel from the relays to the destination and $\mathbf{f}_k$ represents the distributed channel from the kth source to the relays. We also consider imperfect CSI for the link from the sources to the relays as in [7]. Assume that $\mathbf{f}_k$ can be written as $\mathbf{f}_k = \bar{\mathbf{f}}_k + \tilde{\mathbf{f}}_k,\ 1 \leq k \leq M$, where $\bar{\mathbf{f}}_k$ is an unbiased estimate of $\mathbf{f}_k$ and $\tilde{\mathbf{f}}_k$ is a zero mean random variable that represents the channel estimation error. We remark that the case of imperfect CSI with known channel error covariance will be presented in a forthcoming publication. For consistency, we use the same simulation model as [7]: for all simulations, we assume the transmission power from the M sources to be identical and 10dB above the noise level. The relative channel estimation error is set to a level of -20dB.

We investigate different scenarios with different numbers of sources, relays and receive antennas and same SINR threshold for all sources $\gamma = \gamma_1 = \ldots = \gamma_M$. Fig. 2 compares the minimum sum transmit power at the relays versus the SINR threshold γ with $M = 2$ sources, $N = 4$ receive antennas and $R = 4, 6, 8, 10$ relays respectively. It can be seen in Fig. 2 that as the number of relays increases, the required minimum sum transmit power at the relays decreases. It is noted that when the number of relays increases from 4 to 6, the reduction in sum power is largest, as the number of relays increases further until 12, the performance gain in the sum power decreases with an increase in the number of relays.

In Fig. 3, the cases of 3 sources, with the numbers of relays and receive antennas each fixed at $R = N = 4, 6, 8, 10, 12$ are compared. It is obvious from the plots that the minimum sum transmit power at the relays decreases as more relays and receiver antennas are added.

Fig. 4 compares algorithm performance as a function of the number of iterations for the case of $M = 3, R = N = 4$. It is show that after about 5 iterations, a fixed point is being approached. Also it is noted that the first two iterations result in the largest gain, and that performance gain diminishes as the number of iterations increases. Comparing Figs. 2 and 3 we observe that as the numbers of relays and receive antennas increase, the required minimum sum transmit power at the relays decreases, which is achieved by the increased diversity at relays and receive antenna arrays.

We note that existing distributed beamforming systems [7] [8] do not exploit a multi-antenna destination and thus generally require a larger number of relays (for example 20 relays reported in [7] and [8]) to support a smaller number of sources. As the computational complexity of semidefinite programming is O($(M + R^2)^{3.5}$) in the worst case where R

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is the number of relays, the larger number of relays results in a high cost to compute $\mathbf{W}$. In the system proposed in this paper, with 6-10 relays, the system can support up to 8 sources, reducing computational cost and increasing the number of supported sources.

VI. Conclusion

In this paper we study the scenario of a multi-access system through a network of relays. For fixed relay weights, the optimum linear decoder is derived. For a fixed linear decoder at the destination, the relay weights are optimized using semidefinite programming relaxation. We also propose an iterative algorithm to optimize the decoder at the destination and the relay weights. Compared with existing systems in the literature, the proposed system can support more users with fewer relays and hence reduces computational complexity.

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior. information theory," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, 2004.

[2] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2425, 2003.

[3] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE Journal on Selected Areas on Communications*, vol. 22, pp. 1099–1109, Aug. 2004.

[4] M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: space-time transmission and iterative decoding," *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 362–371, Feb. 2004.

[5] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, 2006.

[6] T. Kiran and B. S. Rajan, "Partial-coherent distributed space-time codes with differential encoder and decoder," *IEEE Journal on Selected Areas on Communications*, vol. 25, pp. 426–433, 2007.

[7] V. H. Nassab, S. Shahbazpanahi, A. Grami, and Z. Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. Signal Processing*, vol. 56, pp. 4306–4316, Sept. 2008.

[8] S. Fazeli-Dehkordy, S. Shahbazpanahi, and S. Gazor, "Multiple peer-to-peer communications using a network of relays," *IEEE Trans. Signal Processing*, vol. 57, no. 4, pp. 3053 – 3062, 2009.

[9] Z. Ding, W. H. Chin, and K. K. Leung, "Distributed beamforming and power allocation for cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1817–1822, May 2008.

[10] Y. Zheng and S. D. Blostein, "Downlink distributed beamforming through relay network," *Proc. IEEE Globecom, Honolulu, Hawii,*, 2009.

[11] S. Serbetli and A. Yener, "Transceiver optimization for multiuser MIMO system," *IEEE Trans. Signal Processing*, vol. 52, pp. 214–226, Jan. 2004.

[12] Y. Jing and B. Hassibi, "Network beamforming using relays with perfect channel information," *Int. Conf. Acoustics, Speech,Signal Processing (ICASSP)*, pp. 473–476, Apr. 2007.

[13] K. S. Gomadam and S. A. Jafar, "Optimal distributed beamforming in relay networks with common interference," *Proc. Global Telecommunication Conference*, pp. 3868–3872, Apr. 2007.

[14] J. F. Sturm, "a matlab toolbox for optimization over symmetric cones. optimization methods and software," pp. 625–653, 1999.

[15] Y. Huang and S. Zhang, "Complex matrix decomposition and quadratic programming," *Mathematics of Operations Research*, vol. 32, pp. 758–768, 2007.

[16] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting. signal processing," *IEEE Trans. Signal Processing*, vol. 54, pp. 2239–2251, June 2006.

[17] P. Tseng, "Further results on approximating nonconvex quadratic optimization by semidefinite programming relaxation," *SIAM Journal on Optimization*, vol. 14, pp. 268–283, July 2003.

[18] S. Zhang, "Quadratic maximization and semidefinite relaxation," *Math. Program.*, pp. 453–465, May 2000.

Appendix I: Randomization method details:

$$\begin{aligned}\mathbf{w}_l^H\mathbf{D}\mathbf{w} &= \mathbf{e}_l^H\mathbf{\Sigma}^{1/2}\mathbf{U}^H\mathbf{D}^{-1/2}\mathbf{D}\mathbf{D}^{-1/2}\mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{e}_l \\ &= \mathbf{e}_l^H\mathbf{\Sigma}\mathbf{e}_l \\ &= e^{-j\theta_{l,1}}\mathbf{\Sigma}_{1,1}e^{j\theta_{l,1}} + \ldots + e^{-j\theta_{l,R}}\mathbf{\Sigma}_{R,R}e^{j\theta_{l,R}} \\ &= \mathbf{\Sigma}_{1,1} + \ldots + \mathbf{\Sigma}_{R,R} \\ &= trace\left(\mathbf{\Sigma}\right) \\ &= trace\left(\mathbf{U}\mathbf{\Sigma}\mathbf{U}^H\right) \quad . \end{aligned} \tag{24}$$

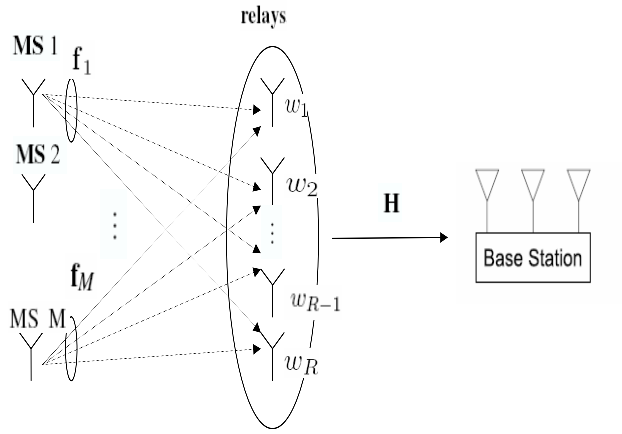

Fig. 1. Uplink distributed beamforming system.

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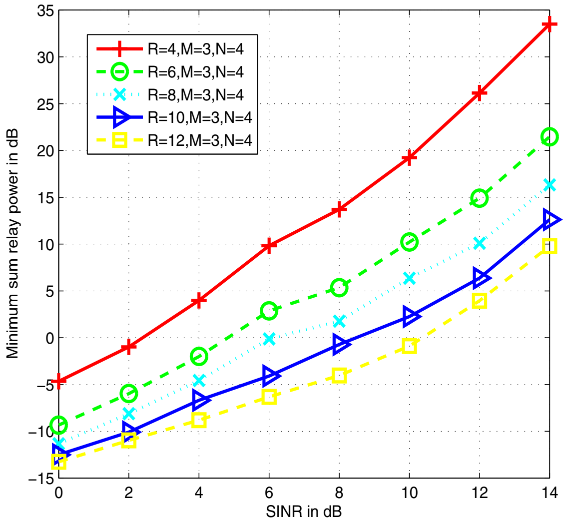

Fig. 2. Minimum sum relay transmit power versus SINR threshold γ for 2 sources and 4 receive antennas for different number of relays.

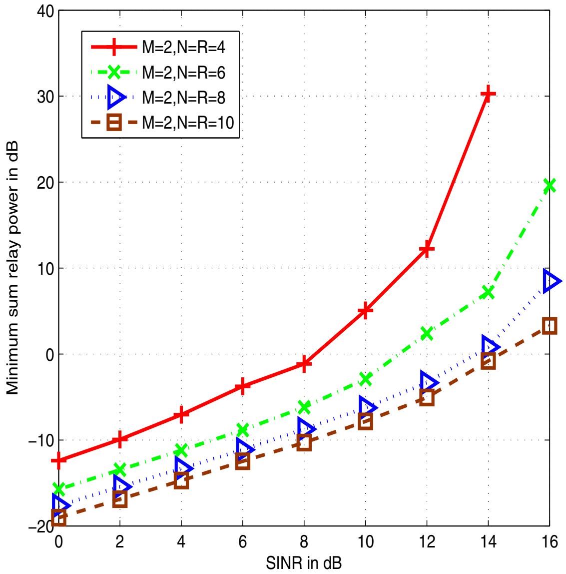

Fig. 3. Minimum sum relay transmit power versus SINR threshold γ for 3 sources, R relays and N receive antennas for R=N=4,6,8,10,12.

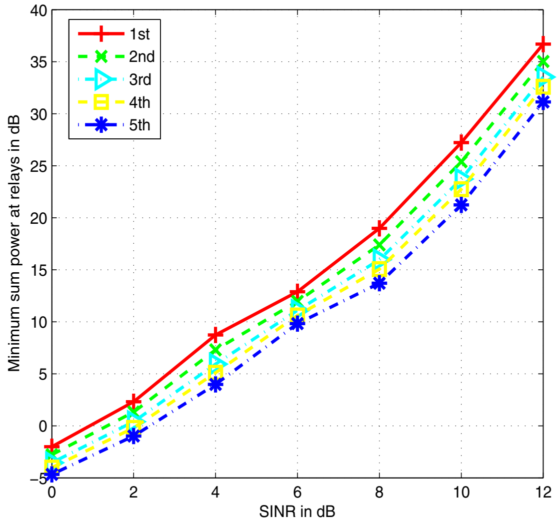

Fig. 4. Minimum sum relay transmit power versus SINR threshold γ for 3 sources, 4 relays and 4 receive antennas for different number of iterations.

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