RANDOM SAMPLE ANTENNA SELECTION WITH ANTENNA SWAPPING

by

Edmund Chun Yue Tam

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Abstract

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Wireless communications employing multiple transmit and receive antennas can bring promising improvements to link quality as well as system capacity. The potential gain in performance for a multiple-input multiple-output (MIMO) system is mitigated by the increased cost of the number of expensive radio-frequency (RF) hardware components. To reduce cost of deploying MIMO technology, a complexity reduction technique known as antenna selection can be applied. In antenna selection, only a subset of the full array of transmit and receive antennas is chosen based on a selection criterion. The antennas are connected to a limited number of RF chains by a low-cost RF switch. The resulting system enjoys many benefits offered by the full complexity MIMO system but with fewer RF resources.

This thesis proposes a novel and efficient iterative antenna selection algorithm based on a minimum bit error rate (BER) selection criterion for a zero-forcing (ZF) MIMO receiver. The proposed algorithm finds an efficient joint transmit and receive antenna selection solution that is close to the globally optimal antenna configuration with reduced complexity. The complexity and performance of the algorithm can be traded off. The proposed algorithm can also be used for transmit or receive only antenna selection as special cases.

The proposed algorithm introduces the concepts of random antenna selection (RAS)

and antenna swapping (AS). The startup processing involves the training and estimation of the MIMO channel for the subset of antennas connected to the available RF chains. The thesis also develops a fast method for antenna swapping based on rank-2 matrix modification, and the computational complexity of the algorithm is analyzed. The behavior of the RAS-AS algorithm with a random swapping sequence is modelled as a finite-state Markov chain, and the expected number of iterations is computed analytically.

The BER performance of the algorithm is simulated, and results show promising BER performance gains after only small numbers of RAS-AS iterations. The algorithm is applicable to both spatially uncorrelated or correlated MIMO channels, and similar BER performance improvements are observed for the case where transmit antennas are correlated.

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Contents

List of Tables

List of Figures

Acronyms

ABER	Approximate bit error rate
AS	Antenna selection
A/D	Analog-to-digital
BER	Bit error rate
BPSK	Binary phase shift keying
BS	Base station
CDF	Cumulative distribution function
CSI	Channel state information
DOF	Degrees of freedom
ES	Exhaustive search
EGC	Equal gain combining
i.i.d.	Independent identically distributed
MGF	Moment generating function
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MRC	Maximal ratio combining
MS	Mobile station
PAS	Power azimuth spectrum
PDF	Probability density function

RF	Radio frequency
RAS	Random antenna selection
RAS-AS	Random antenna selection with antenna swapping
SC	Selection combining
SNR	Signal-to-noise ratio
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
ZF	Zero-forcing

List of Important Symbols

$(\cdot)^T$	Matrix or vector transpose
$(\cdot)^*$	Complex conjugate
$(\cdot)^H$	Matrix or vector conjugate transpose
$\otimes$	Kronecker product
$\det(\cdot)$	Determinant of a matrix
$\mathrm{E}[\cdot]$	Expectation of random variables
$\boldsymbol{I}_N$	$N \times N$ identity matrix
L_{tx}	Number of transmit RF chains
L_{rx}	Number of receive RF chains
N_{tx}	Number of transmit antennas
N_{rx}	Number of receive antennas

Chapter 1

Introduction

In order to realize the goals of next-generation wireless communication systems, employing multiple antennas on both sides of the communication link is seen as a promising solution. These multiple-input multiple-output (MIMO) wireless systems have the potential to increase the capacity of the system [1] or improve the quality of the communication link [2]. The tradeoff is an increase in hardware cost and signal processing complexity. For each antenna, there would have to be an associated radio-frequency (RF) chain of expensive hardware, and these include modulators, analog-to-digital (A/D) convertors, mixers, and amplifiers. The system hardware complexity increases quickly with every antenna and RF chain added. On the other hand, antennas alone are relatively inexpensive compared to the components in the RF chain. A MIMO system can have a large number of antennas, while only requiring a small amount of RF chain hardware. It is found that by carefully selecting a subset of antennas and connecting through a low cost RF switch, many benefits of the full complexity MIMO system can be retained [3] [4]. This leads to the study of antenna selection, which is a complexity reduction scheme that can reduce the hardware requirement of MIMO systems by choosing a subset of antennas based on some required performance criterion.

1.1 Motivation

In order to reduce the complexity and hardware requirements for deploying MIMO systems, antenna selection is proposed as a complexity reduction scheme. Numerous antenna selection algorithms are proposed and studied in the literature. These algorithms can be categorized into transmit-side antenna selection, receive-side antenna selection, and joint transmit and receive antenna selection. It is noted in [5] that the problem of jointly finding a subset of transmit and receive antennas efficiently is still an open problem. Many of the algorithms proposed in the literature focus on antenna selection on one side of the communication link, and the study of joint transmit and receive antenna selection has been limited. It is also noted that many existing antenna selection algorithms require the full complexity MIMO channel to be estimated, and it would be beneficial if this requirement can be reduced. Motivated by these factors, the thesis proposes an iterative algorithm for joint transmit and receive antenna selection that has low computational complexity.

1.2 Thesis Outline

The following is an outline and organization of the thesis. In Chapter 2, existing literature on antenna selection is presented and reviewed. The MIMO channel model and bit error rate (BER) expressions used in the rest of the thesis is also introduced.

Chapter 3 presents an antenna selection algorithm based on the concept of random antenna selection (RAS), together with an antenna selection criterion. The potential of random antenna subset selection is also justified through analyzing the approximate bit error rate (ABER) outage probability, as well as the expected number of iterations required to obtain an certain ABER threshold. The pseudocode of the RAS algorithm is presented at the end of the chapter.

In Chapter 4, the concept of antenna swapping (AS) is introduced. The relationship between antenna swapping and rank-2k matrix modification is established, where k represents the number of pairs of antennas to be swapped. Two antenna swapping sequences are then introduced: deterministic and random. At the end of the chapter, a realization of the RAS algorithm from Chapter 3 is proposed with the concept of antenna swapping, and the two are related through a matrix inversion update expression. The resulting algorithm is the RAS-AS algorithm, and pseudocode is presented at the end of the chapter.

In Chapter 5, a fast and complexity reduced RAS-AS algorithm is presented through a simplification made possible by performing rank-2 matrix modifications. The complexity of the reduced algorithm is analyzed in terms of the initialization overhead and the number of multiplications and additions in each iteration. The expected number of iterations and average BER performance of the RAS-AS algorithm under uncorrelated and correlated channel conditions using both deterministic or random swapping sequences is also analyzed and simulated.

Chapter 6 summarizes and concludes the work in the thesis, and provides suggestions for future research.

1.3 Thesis Contributions

In this thesis, a novel joint transmit and receive antenna selection algorithm is proposed that uses the idea of random antenna selection and antenna swapping. The following summarizes the contributions of this thesis:

- The proposed random antenna selection with antenna swapping (RAS-AS) algorithm is a novel, efficient, joint transmit and receive antenna subset selection algorithm that reduces the computation of exhaustive search based on a minimum bit error rate (BER) selection criterion for a zero-forcing (ZF) multiple-input multiple-output (MIMO) receiver.

- The novel concept of random antenna swapping is introduced. The thesis establishes the relationship between antenna swapping with rank-$2k$ matrix modification for k pairs of antennas to be swapped.

- At the startup of the algorithm, instead of requiring the full complexity MIMO channel to be estimated, which involves all the antennas on both sides of the link, the proposed algorithm requires an amount of channel estimation and initial training corresponding to that of the number of available radio-frequency (RF) chains on both sides of the link. Additional channel estimation is spread over time and is performed only as the algorithm swaps in new antennas.

- The thesis models the behavior of the RAS-AS algorithm with a Markov chain model, and the expected number of iterations as well as the variance are analyzed. The computational requirements of the RAS-AS algorithm with a rank-2 simplification are determined, and the BER performance of the RAS-AS algorithm is also simulated.

- The proposed Fast RAS-AS algorithm significantly lowers complexity from exhaustive search while finding near optimal antenna configurations. Simulation results show that after the expected number of iterations for finding a near optimal set of antennas, close to optimal BER performance can be achieved most of the time.

- The proposed RAS-AS algorithm is suitable for systems with large numbers of antennas, and the algorithm is applicable to both spatially uncorrelated and correlated MIMO channels. The RAS-AS algorithm can also be used for transmit antenna selection only or receive antenna selection only as special cases.

Chapter 2

Background

This chapter first presents the background on MIMO systems and establishes the role of antenna selection in MIMO wireless communication. Following this, an overview of the existing antenna selection algorithms in the literature is presented. The MIMO channel model used in this thesis is presented in the last part of the chapter.

2.1 Multiple-Input Multiple-Output System

From the early work of Telatar [1] and Foschini [6], it is shown that employing multiple transmit and receive antennas has the potential to greatly increase the capacity in wireless communication systems. By exploiting the spatial dimension, capacity increases linearly with the minimum number of antennas on both sides of the link. This enables a system to achieve high spectral efficiency, and provide high data rate services that are envisioned in future generations of wireless communication systems.

The potential benefits of using MIMO systems is offset by the increase in hardware requirements and signal processing complexity. Each antenna is associated with a chain of expensive RF resources, and this includes modulators, mixers, analog-to-digital convertors, and power amplifiers, which dominate the cost of the system. With multiple antennas on both sides of the communication link, the amount of channel training and estimation increases significantly relative to a single-input single-output (SISO) system. This in turn increases the dimensionality of the signal processing problem, and increases the complexity of the algorithms required to capture the benefits of MIMO systems.

It is therefore desirable to reduce the amount of expensive RF chain hardware, while harvesting the many advantages of MIMO systems. Therefore, antenna selection is proposed as a complexity reduction technique to enable practical deployment of MIMO systems.

2.2 Antenna Selection

The idea of antenna selection stems from the fact that antennas are relatively inexpensive when compared with the rest of the RF chain hardware. Therefore, a system can deploy a large number of antennas while having only a small number of RF chains, and the two can be connected through a low-cost RF switch. This results in the formulation of the antenna selection problem, which tries to find the best subset of antennas to connect to the limited RF resources, based on some selection criterion. It is found that with the proper subset of antennas selected, many benefits of the full complexity MIMO system can be retained [3], such as the diversity order of the system. A system diagram of an antenna selection system is shown in Figure 2.1.

The goal is to find and connect the best L_{tx} transmit RF chains to the N_{tx} transmit antennas, and the best L_{rx} receive RF chains to the N_{rx} receive antennas. The best antennas will also vary with time and the selection process needs to be repeated periodically. The antenna selection, channel estimation, and MIMO signal detection are performed in the signal processing unit on the receiver side.

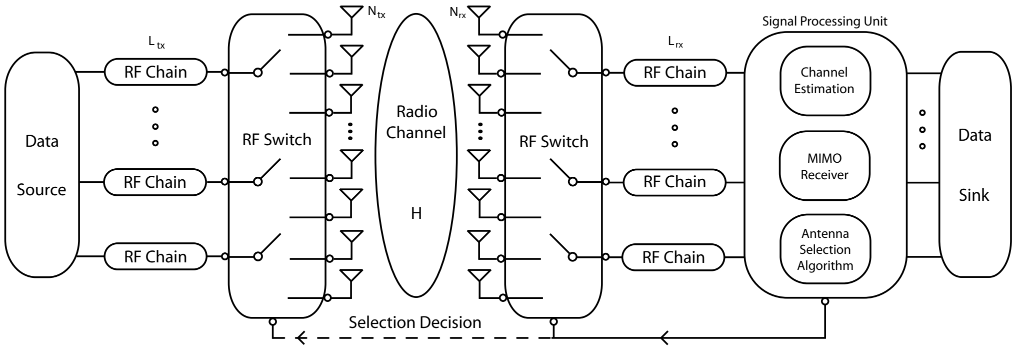

Figure 2.1: System with antenna selection.

Numerous antenna selection algorithms are proposed in the literature, varying in complexity, selection criteria, and optimality criteria. Antenna selection can also be broadly classified into transmit antenna selection, receive antenna selection, and joint transmit and receive antenna selection. MIMO systems can improve the link quality of the system through diversity methods, and/or improve data rate through spatial multiplexing. Therefore, the two antenna selection criteria typically considered in the literature are based on maximizing either diversity or system capacity [5].

The following sections first present antenna selection algorithms from the capacity point of view. Then, antenna selection based on a diversity point of view will be presented. Antenna section algorithms with a focus on capacity are suitable for spatial multiplexing systems that require high data rates. Antenna selection algorithms with a focus on diversity are suitable for systems that require robust link quality, which is also related to achieving high received signal-to-noise ratio (SNR), and low bit error rate (BER).

2.3 Antenna Selection based on System Capacity

For antenna selection algorithms that focus on capacity, the goal is to select a subset of antennas that maximize the following MIMO capacity expression [1]:

$$C(H) = log_2 \left[det \left(I_{N_{tx}} + \frac{E_s}{N_o} H^H H \right) \right] = log_2 \left[det \left(I_{N_{rx}} + \frac{E_s}{N_o} H H^H \right) \right] \qquad (2.1)$$

where $I_{N_{tx}}$ is the $N_{tx} \times N_{tx}$ identity matrix, $I_{N_{rx}}$ is the $N_{rx} \times N_{rx}$ identity matrix, H is the $N_{rx} \times N_{tx}$ MIMO channel matrix, E_s is the average symbol energy, and N_o is the noise energy. The following subsections review the algorithms proposed in the literature for receive antenna selection, transmit antenna selection, and joint transmit and receive antenna selection that maximize the system capacity.

2.3.1 Receive Antenna Selection

For receive antenna selection with a capacity maximization criterion, the objective of the algorithm is to select a subset of receive antennas so that the capacity expression is maximized. It is noted from [5] that there is no exact solution for finding the optimal receive antenna set without exhaustively searching through all the possible configurations. Suboptimal or complexity reduced algorithms have been proposed in [7] [8] [9] [10] [11].

In [7], an initial antenna configuration with all the receive antennas are used. The receive antenna that has the least impact on the capacity, or the antenna that results in minimum capacity loss is removed from the antenna set iteratively, until the desired number of receive antennas remains. In [8], an initial empty set of antennas is used, and the receive antennas that result in the largest capacity gain are added iteratively to the antenna set, until the desired number of receive antennas are chosen.

Two other iterative receive antenna selection algorithms are proposed in [9] and [10].

These algorithms maximize channel capacity by selecting antennas with minimal correlation.

Low computational complexity algorithms in [11] are norm-based, i.e., the antenna selection is based on maximizing the Forbenius norm or column norm of the channel matrix.

2.3.2 Transmit Antenna Selection

For transmit antenna selection with a focus on capacity maximization, the objective is the same as that of the receive antenna selection algorithms in the previous section, and both norm-based or iterative type selection algorithms can be applied [5] [12]. Algorithms using properties of determinants for positive definite Hermitian matrices are proposed in [13]. Transmit antenna selection also requires a feedback link. With full channel state information (CSI), the transmitter can achieve the maximum capacity of the channel via the water-filling strategy [5] [14]. Transmit antenna selection for low-rank channels has also been studied in [15].

2.3.3 Joint Transmit and Receive Antenna Selection

The authors in [16] propose a suboptimal algorithm for joint transmit and receive antenna selection based on a capacity maximization criterion, by performing separate transmit and receive antenna selections. The algorithm first performs antenna selection on one side of the link, while keeping the antennas at the other end of the link fixed. After the antennas for one side of the communication channel are selected, antenna selection is performed for the other side, while keeping the set of selected antennas fixed. Similar algorithms are proposed in [17] and [18]. However, optimal joint transmit and receive antenna selection is still an open problem [5], and can only be optimized using exhaustive search (ES).

2.4 Antenna Selection based on Diversity Selection

Antenna selection with a diversity maximization criterion focuses on improving communication link quality. Diversity combining can be achieved via three classical ways: selection combining (SC), maximal ratio combining (MRC), and equal gain combining (EGC) [19].

2.4.1 Receive Antenna Selection

For Single-Input Multiple-Output (SIMO) systems with $N_{tx} = 1$ transmit antenna, $L_{tx} = 1$ transmit RF chain, $N_{rx} > 1$ receive antennas, and $N_{tx} > L_{rx} \geq 1$ receive RF chains, a subset of these receive antennas can be selected, and their signals combined. This method is called generalized selection diversity [5] [20]. When MRC is used, this method is also known as hybrid selection/maximal ratio combining [4]. The combining process can also employ EGC. The optimal antenna subset for generalized selection diversity is one that contains the L_{rx} branches with the largest SNR, for both MRC or EGC [5]. For MIMO systems with $N_{tx} = L_{tx} > 1$, space-time block codes with receive antenna selection is studied in [21] [22] [23].

2.4.2 Transmit Antenna Selection

On the transmitter side, for Multiple-Input Single-Output (MISO) systems with N_{tx} transmit antennas and $N_{tx} > L_{tx} \geq 1$ transmit RF chains, and $N_{rx} = L_{rx} = 1$ receive antenna and receive RF chain, respectively, the equivalent antenna selection scheme to hybrid selection/maximal ratio combining on the receiver side, is known as hybrid maximal ratio transmission [5]. This scheme selects transmit antennas such that the superposition of the received signal gives maximum SNR, and it is found that the optimal set of transmit antennas are those with the largest channel gain [5]. Hybrid maximal ratio transmission for

$N_{rx} = L_{rx} > 1$ with receiver-side diversity combining is also studied in [24]. It is noted that maximal ratio transmission requires the feedback of estimated channel gains from the set of transmit antennas to the set of receive antennas. Another transmit antenna selection algorithm for MISO systems using space-time code is proposed in [23]. In [25], an optimal transmit antenna selection algorithm is proposed which minimizes the error rate by exhaustively searching through all antenna configurations.

2.4.3 Joint Transmit and Receive Antenna Selection

In this case the system has N_{tx} transmit antennas, L_{tx} transmit RF chains, N_{rx} receive antennas, and L_{rx} receive RF chains, with $N_{tx} > L_{tx} > 1$ and $N_{rx} > L_{rx} > 1$. In order to maximize diversity, space-time coding is used in [26], and the optimal antenna subset that minimizes the probability of error can be found by jointly selecting transmit and receive antennas with channel gains such that the Frobenius norm of the selected MIMO channel matrix is maximized through exhaustive search.

Another joint transmit and receive antenna selection algorithm based on the second order statistics of the channel is proposed in [27]. It is found that the optimal joint selection of the transmit and receive antennas can be decoupled and selected independently of each other [27]. For linear receivers, the selection criterion involves maximizing the singular values of the transmit covariance matrix and receive covariance matrix, by searching through all the transmit antenna configurations and receive antenna configurations independently [27].

It is noted in [5] that other than through exhaustive search, there are no existing fast, efficient, or systematic methods for finding the optimal joint transmit and receive antenna set that are not based on channel statistics.

The joint selection algorithm proposed in this thesis is based on random antenna selection with antenna swapping (RAS-AS). The RAS-AS algorithm is an iterative joint transmit and receive antenna subset selection algorithm. The RAS-AS algorithm provides an efficient way that can find a near optimal subset of transmit and receive antennas. The RAS-AS algorithm can also be used for transmit antenna selection or receive antenna selection as special cases.

2.5 MIMO Signal Model

For a communication system with multiple antennas at both ends, let N_{tx}, L_{tx}, N_{rx}, and L_{rx} represent the number of transmit antennas, available transmit RF chains, receive antennas, and available receive RF chains, respectively. The received signal vector $\underline{r}$ can be represented as

$$\underline{r} = H\underline{s} + \underline{n} \tag{2.2}$$

where $\underline{s}$ is the temporally and spatially white input signal vector of dimension $L_{tx} \times 1$ with $E[\underline{ss}^H] = E_s I_{L_{tx}}$ and E_s is the average symbol energy ; H is an $L_{rx} \times L_{tx}$ antenna selected MIMO channel with independent identically distributed (i.i.d.) complex Gaussian channel gains and flat Rayleigh quasi-static fading, where the channel is constant over a time frame and the channel realizations over different time frames are uncorrelated; and $\underline{n}$ is the temporally and spatially white additive Gaussian noise vector of dimension $L_{rx} \times 1$ with $E[\underline{nn}^H] = N_o I_{L_{rx}}$ and N_o is the noise energy. The input signal-to-noise ratio (SNR) is defined as $\gamma_o = E_s/N_o$.

2.6 MIMO Channel Model

The proposed algorithm is also applicable to antenna correlated MIMO channels, and the channel matrix can be modeled as follows [28] [29]

$$H = R_r^{\frac{1}{2}} H_w R_t^{\frac{1}{2}} \tag{2.3}$$

where H_w is the $L_{rx} \times L_{tx}$ MIMO channel matrix with i.i.d. complex Gaussian channel gains, R_t is the $L_{tx} \times L_{tx}$ covariance matrix of the rows of H, and R_r is the $L_{rx} \times L_{rx}$ covariance matrix of the columns of H. The $(.)^{\frac{1}{2}}$ represents the square root of a matrix. The following assumes a MIMO channel with only transmit antenna correlation or only receive antenna correlation, respectively:

$$H = H_w R_t^{\frac{1}{2}} \ , \tag{2.4}$$

or

$$H = R_r^{\frac{1}{2}} H_w \ . \tag{2.5}$$

This thesis will focus on MIMO channels with correlation at only one side of the link. This models the scenario where the base station (BS) is positioned on top of a tall building with few surrounding scatterers, and the mobile station (MS) is located in an environment with many surrounding scatterers. Therefore, the signal received at the BS antenna array would experience some degree of correlation. The signal arriving at the MS would be uncorrelated due to the rich scattering environment.

Assuming the BS to be the transmitter and the MS to be the receiver, the channel model in (2.4) can be used. The uplink situation where the BS is the receiver and the MS is the transmitter can be handled similarly.

In [30], it is found that a Power Azimuth Spectrum (PAS) with a truncated Laplacian distribution best fits measurement results in urban and rural environments. Therefore, the truncated Laplacian PAS model in [30] is used to generate the coefficients in the correlation matrix in (2.4).

The following defines the parameters for the model in [30]. Let N_c represent the number of scattering clusters, $\phi_{0,k}$ represent the angle of incident from cluster k, σ_k represent the angle spread of the signal from cluster k, and $\Delta\phi_k$ represent the truncation range in the PAS, for $k = 1, \ldots, N_c$. Let the normalized distance be defined as $\frac{d}{\lambda}$ and $D = 2\pi\frac{d}{\lambda}$, where d is the physical spacing between antenna elements, and define λ to be the wavelength of the signal.

Let x be the real part of the complex baseband signal, and y be the imaginary part of the complex baseband signal. The cross correlation between the real parts and the cross correlation between the real and imaginary parts of the complex baseband signal at two antenna elements that are a distance d apart is given in [30], and are given as follows:

$$\begin{aligned} R_{xx}(D) &= J_0(D) + 4\sum_{k=1}^{N_c} \frac{Q_k}{\sigma_k\sqrt{2}} \sum_{m=1}^{\infty} \frac{J_{2m}(D)}{(\sqrt{2}/\sigma_k)^2 + (2m)^2} \cos(2m\phi_{0,k}) \\ & \left\{ \frac{\sqrt{2}}{\sigma_k} + \exp\left(-\frac{\Delta\phi_k\sqrt{2}}{\sigma_k} \left[2m\sin(2m\Delta\phi_k) - \frac{\sqrt{2}}{\sigma_k}\cos(2m\Delta\phi_k) \right] \right) \right\} \end{aligned} \tag{2.6}$$

$$\begin{aligned} R_{xy}(D) &= 4\sum_{k=1}^{N_c} \frac{Q_k}{\sigma_k\sqrt{2}} \sum_{m=0}^{\infty} \frac{J_{2m+1}(D)}{(\sqrt{2}/\sigma_k)^2 + (2m+1)^2} \sin((2m+1)\phi_{0,k}) \\ & \left\{ \frac{\sqrt{2}}{\sigma_k} - \exp\left(-\frac{\Delta\phi_k\sqrt{2}}{\sigma_k} [(2m+1)\sin((2m+1)\Delta\phi_k) \right.\right. \\ & \left.\left. + \frac{\sqrt{2}}{\sigma_k}\cos((2m+1)\Delta\phi_k)] \right) \right\} , \end{aligned} \tag{2.7}$$

where $J_m(.)$ is the m^{th} order Bessel function of the first kind. From [30], the complex correlation coefficient is

$$R(D) = R_{xx}(D) + jR_{xy}(D) \ . \tag{2.8}$$

Let a MIMO antenna selection system be denoted with the notation (N_{tx}:N_{rx},L_{tx}:L_{rx}) = (4:8,2:4), representing 4 transmit and 8 receive antennas, and 2 transmit and 4 receive RF chains, respectively. For antenna elements that are positioned a distance $\frac{\lambda}{2}$ apart from one another, and geometry where $N_c = 2$, $\phi_{0,1} = -\frac{\pi}{2}$, $\phi_{0,2} = \frac{\pi}{2}$, $\sigma_1 = \sigma_2 = \frac{\pi}{6}$, $\Delta\phi_1 = \Delta\phi_2 = \frac{\pi}{3}$, and with the signal coming from the second cluster having half the power as the signal from the first cluster, the correlation matrix for a (4:8,2:4) system that has 4 transmit antennas is found to be

$$R_t = \begin{pmatrix} R(D)|_{d=0} & R(D)|_{d=\frac{\lambda}{2}} & R(D)|_{d=\lambda} & R(D)|_{d=\frac{3\lambda}{2}} \\ R(D)^H|_{d=\frac{\lambda}{2}} & R(D)|_{d=0} & R(D)|_{d=\frac{\lambda}{2}} & R(D)|_{d=\lambda} \\ R(D)^H|_{d=\lambda} & R(D)^H|_{d=\frac{\lambda}{2}} & R(D)|_{d=0} & R(D)|_{d=\frac{\lambda}{2}} \\ R(D)^H|_{d=\frac{3\lambda}{2}} & R(D)^H|_{d=\lambda} & R(D)^H|_{d=\frac{\lambda}{2}} & R(D)|_{d=0} \end{pmatrix} \tag{2.9}$$

$$R_t = \begin{pmatrix} 1.000 & -0.924-0.065i & 0.767+0.083i & -0.647-0.097i \\ -0.924+0.065i & 1.000 & -0.924-0.065i & 0.767+0.083i \\ 0.767-0.083i & -0.924+0.065i & 1.000 & -0.924-0.065i \\ -0.647+0.097i & 0.767-0.083i & -0.924+0.065i & 1.000 \end{pmatrix} \tag{2.10}$$

Figure 2.2 plots the magnitude of the correlation coefficients as a function of normalized distance $\frac{d}{\lambda}$, with $N_c = 2$, $\phi_{0,1} = -\frac{\pi}{2}$, $\phi_{0,2} = \frac{\pi}{2}$, $\sigma_1 = \sigma_2 = \frac{\pi}{6}$, $\Delta\phi_1 = \Delta\phi_2 = \frac{\pi}{3}$. The plot in Figure 2.2 matches that obtained in [30].

2.7 Zero Forcing Receiver

Due to its relative simplicity, a zero forcing (ZF) MIMO receiver will be used in the development of the antenna selection algorithm in this thesis. The sufficient conditions for the existence of a ZF solution is when the number of antennas and RF chains on the transmit side is less than or equal to the number of antennas and RF chains on the receive side (i.e. $N_{tx} \leq N_{rx}$, $L_{tx} \leq L_{rx}$, $L_{tx} \leq N_{tx}$, and $L_{rx} \leq N_{rx}$). A practical scenario under these conditions

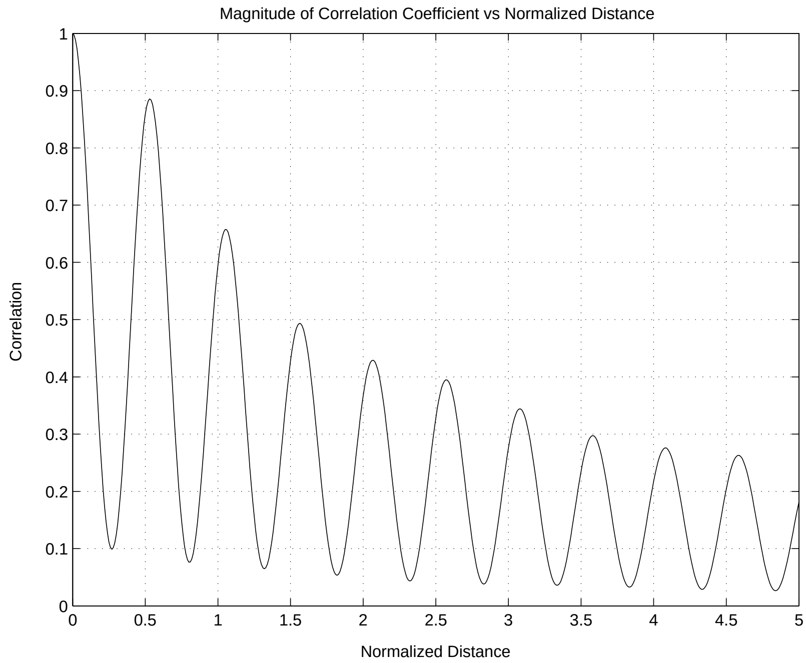

Figure 2.2: Magnitude of correlation vs normalized distance.

includes fixed wireless applications where the mobile stations can have the same number or more antennas than the base station. The estimate of the transmitted signal at the output of the ZF receiver is

$$\underline{\widetilde{s}} = H^{\dagger}\underline{r} = \underline{s} + H^{\dagger}\underline{n} \tag{2.11}$$

where $(.)^{\dagger}$ denotes the pseudoinverse of a matrix. The post-processing SNR γ_k for the k^{th} data stream is given by [31]

$$\gamma_k = \frac{\gamma_o}{\left[\left(H^H H\right)^{-1}\right]_{k,k}} = \gamma_o g_k^2 \; ; \; g_k^2 = \left[\left(H^H H\right)^{-1}\right]_{k,k}^{-1} \tag{2.12}$$

where g_k^2 can be defined as the power gain.

2.8 BER Expressions

The following presents the closed form BER expression for an antenna selected MIMO system with a zero-forcing receiver and, without loss of generality, binary phase shift keying (BPSK) modulation. The instantaneous average BER across the data streams for a certain antenna configuration conditioned on the channel realization is given by [31]

$$BER_{avg} = \frac{1}{L_{tx}} \sum_{k=1}^{L_{tx}} Q(\sqrt{2\gamma_o g_k^2}) \tag{2.13}$$

where $\gamma_o = E_s/N_o$, with E_s equal to the average symbol energy, and N_o equal to the noise energy, $g_k^2 = [(H_{L_{rx}\times L_{tx}}^H H_{L_{rx}\times L_{tx}})^{-1}]_{k,k}^{-1}$, and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} dy$.

It is assumed that the receiver estimates the channel, while the transmitter has no channel knowledge. Therefore, the antenna selection algorithm will be implemented at the receiver side. During antenna selection, the indices of the selected transmit antennas will be fed back to the transmitter.

From (2.13), it is noted that calculating g_k^2 involves matrix inversion, which is one of the most expensive operations when evaluating the BER for different antenna selected MIMO channels. Therefore, the subsequent chapters in the thesis will present methods that can speed up this operation and facilitate the swapping of antennas in the selection algorithm.

Chapter 3

Random Antenna Selection

This chapter presents an algorithm based on the concept of random antenna selection (RAS) and its selection criterion. The outage probability for RAS is also analyzed to justify the potential of using RAS as a method for antenna selection.

3.1 Concept

This section describes the concept of random antenna selection, and how it can be used as a means for antenna selection. With random selection, a subset of transmit and receive antennas are selected randomly and connected to the available RF resources. The antenna selection criterion based on the performance of the system is evaluated, and the process of randomly choosing an independent subset of antennas can be repeated until the globally optimal or a good enough antenna configuration is found.

The introduction of randomness into an antenna selection algorithm is novel and the randomness can prevent the algorithm from finding a local rather than global minimum cost solution. On the other hand, if the algorithm was deterministic and greedy, locally optimum solutions would result, though likely at a lower computational complexity.

3.2 Selection Criterion

Antenna selection algorithms that use a capacity maximization criterion can find a subset of antennas that has the potential to support the highest data rate possible in a communication link. However, system capacity is an information-theoretic measure and this data rate may not be realizable with limited resources and finite processing delay. Therefore, a capacity measure may not reflect actual system performance. A more realistic measure of the system performance is based on average BER of the system. Antenna selection algorithms that use the average BER minimization criterion can find a subset of antennas that give low error rate and good link quality. Therefore, in this thesis, minimizing the average BER expression of the system in (2.13) is chosen as the antenna selection criterion.

For different types of modulations, similar average BER expressions can be used as the selection criterion. The selection criterion can also be adapted to other types of receivers with different definitions of the power gain, g_k^2.

Uniform transmit power allocation is assumed in this work. However, transmit power allocation can also be jointly optimized with antenna selection by using an approximate average BER expression [31] as the selection criterion. The approximate minimum bit error rate (AMBER) power allocation scheme [31] can be applied to the selected antenna subset in each iteration of the algorithm to jointly optimize the transmit power with the selected antennas.

3.3 Random Antenna Selection Algorithm

At the startup of the algorithm, a subset of L_{tx} transmit and L_{rx} receive antennas are selected at random and connected to the available RF chains, and channel estimation is performed for the MIMO system using these antennas. The average BER performance of the antenna

Table 3.1: RAS algorithm pseudocode.

	Initialization:
1	Randomly select L_{tx} transmit, L_{rx} receive antennas to form H_0
2	Calculate $\left(H_0^H H_0\right)^{-1}$ and g_k^2 for $k = 1, \ldots, L_{tx}$ using (2.12)
3	Initialize $\left(H^H H\right)_{best}^{-1} = \left(H_0^H H_0\right)^{-1}$ and $\text{AvgBER}_{best} = \text{AvgBER}_0$ using (2.13)
	Main Loop (n^{th} Step) {
4	Randomly select an independent subset of transmit and receive antennas to form H_n
5	Calculate the inverse $\left(H_n^H H_n\right)^{-1}$
6	Calculate g_k^2 for $k = 1, \ldots, L_{tx}$ using (2.12)
7	Calculate AvgBER_n using (2.13)
8	if ($\text{AvgBER}_n < \text{AvgBER}_{best}$) then
9	Current antenna configuration is the best : Set $\left(H^H H\right)_{best}^{-1} = \left(H_n^H H_n\right)^{-1}$
10	$\text{AvgBER}_{best} = \text{AvgBER}_n$
	end if
	}

selected system is then evaluated. In each subsequent iteration of the algorithm, an independent subset of transmit and receive antennas is chosen, additional channel estimation is performed for the new antennas, and the average BER performance of the system using these independent subsets of antennas is evaluated. The algorithm keeps track of the antenna configuration that results in the best performance, and terminates when either all antenna configurations are tested or when a desired average BER performance is obtained. As the algorithm cycles through the possible antenna configurations, the algorithm converges to the globally optimal antenna configuration that provides the best average BER performance. The pseudocode of the RAS algorithm is presented in detail in Table 3.1.

From Table 3.1, it is noted that each iteration requires computationally expensive matrix inversion in step 5, when a different random subset of antennas is chosen. The following chapters address this issue and present an iterative method to reduce computational complexity. The rest of this chapter investigates the merits of performing RAS by examining the outage probability of the system and the expected number of iterations. The statistics of the received SNR is first presented in the following section, and from this distribution, the statistics of the error rate for the different data streams can be determined.

3.4 Statistics of the Received SNR

This section presents the statistics of the received SNR, $\gamma_k = \gamma_o g_k^2$ in the average BER expression in (2.13) . Using the statistics of the received SNR, the outage probability of the system is then determined.

From [28], the SNR on the k^{th} data stream, γ_k, for a $L_{rx} \geq L_{tx}$ MIMO channel with a zero forcing receiver is weighted Chi-square distributed, with $2(L_{rx} - L_{tx} + 1)$ degrees of freedom (DOF) and has a weight of $\frac{\gamma_o}{2\sigma_k^2}$. The probability density function (PDF) of the k^{th} stream for $\gamma_k \geq 0$ is [28]

$$f_{\Gamma_k}(\gamma_k) = \frac{\sigma_k^2 e^{-\gamma_k \sigma_k^2/\gamma_o}}{\gamma_o (L_{rx} - L_{tx})!} \left(\frac{\gamma_k \sigma_k^2}{\gamma_o} \right)^{L_{rx} - L_{tx}} \tag{3.1}$$

where σ_k^2 represents the k^{th} diagonal entry in the inverse channel correlation matrix, Σ^{-1}. For uncorrelated channels, $\Sigma = I_{L_{tx} \times L_{tx}}$, and for transmit antenna correlated channel, $\Sigma = R_t$.

The notation $\gamma_k \sim \chi^2(n, w)$ is used to indicate that γ_k is weighted Chi-square distributed, with weight w, and n DOF. The PDF, cumulative distribution function (CDF) and the moment generating function (MGF) of the Chi-square and w-weighted Chi-square random variable with n DOF are provided in Appendices A and B, respectively.

3.4.1 Approximate BER Expression

In order to find a closed form expression for the outage probability of the antenna selected system, an approximation to the average BER of the system is used. The BER expression in (2.13) can be approximated as $p(\gamma) \approx \frac{1}{5}exp\{-c\gamma\}$ [31], where c is a constant depending on the signal constellation. The average approximate BER (ABER) of the system is therefore

$$ABER_{avg} = \frac{1}{5L_{tx}} \sum_{k=1}^{L_{tx}} e^{-c\gamma_o g_k^2} . \qquad (3.2)$$

For BPSK, the constant $c = 1$ [31]. Higher order modulations can be analyzed using different values of c. From (3.2) , the ABER for the k^{th} data stream can be defined to be

$$ABER_{k-stream} = \frac{1}{5} e^{-\gamma_o g_k^2} . \qquad (3.3)$$

The average ABER expression can also be used as a selection criterion, and it provides an uniform framework for different types of modulations by using different constants.

3.4.2 Distribution of the ABER of the k^{th} data stream

The following derives the PDF of the ABER of the k^{th} data stream using the PDF of the k^{th} received SNR from the previous section. This can provide an approximation to the distribution of the average BER of the k^{th} data stream at high SNR. The ABER of the k^{th} data stream is given by (3.3), where

$$ABER_{k-stream} = b_k = \frac{1}{5} e^{-\gamma_k} = g(\gamma_k) . \qquad (3.4)$$

Solving for γ_k results in

$$\gamma_{k,solution} = ln\left(\frac{1}{5b_k}\right) . \qquad (3.5)$$

Taking the derivative of (3.4) results in

$$g'(\gamma_k) = \frac{-1}{5} e^{-\gamma_k} . \qquad (3.6)$$

Therefore, together with (3.1), (3.5), and (3.6), the PDF of the ABER of the k^{th} stream can be found to be

$$\begin{aligned}
f_{ABER_{k-stream}}(b_k) &= \frac{f_{\Gamma_k}(\gamma_{k,solution})}{|g'(\gamma_{k,solution})|} \\
&= \left.\frac{e^{-\frac{\gamma_{k,solution}}{2w}}\gamma_{k,solution}^{\frac{n}{2}-1}}{(2w)^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)}\frac{1}{|\frac{-1}{5}e^{-\gamma_{k,solution}}|}\right|_{n=2(L_{rx}-L_{tx}+1),w=\frac{\gamma_o}{2\sigma_k^2}} \\
&= \left.\frac{e^{-\frac{-ln(5b_k)}{2w}}[-ln(5b_k)]^{\frac{n}{2}-1}}{(2w)^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)}\frac{1}{|\frac{-1}{5}e^{-(-ln(5b_k))}|}\right|_{n=2(L_{rx}-L_{tx}+1),w=\frac{\gamma_o}{2\sigma_k^2}} \\
&= \left.\frac{(5b_k)^{\frac{1}{2w}}(-1)^{\frac{n}{2}-1}[ln(5b_k)]^{\frac{n}{2}-1}}{(2w)^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)}\frac{1}{b_k}\right|_{n=2(L_{rx}-L_{tx}+1),w=\frac{\gamma_o}{2\sigma_k^2}} \\
&= \left.\frac{(5)^{\frac{1}{2w}}(b_k)^{\frac{1}{2w}-1}(-1)^{\frac{n}{2}-1}[ln(5b_k)]^{\frac{n}{2}-1}}{(2w)^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)}\right|_{n=2(L_{rx}-L_{tx}+1),w=\frac{\gamma_o}{2\sigma_k^2}} \\
f_{ABER_{k-stream}}(b_k) &= \frac{(-1)^{L_{rx}-L_{tx}}(5)^{\frac{\sigma_k^2}{\gamma_o}}}{(L_{rx}-L_{tx})!}\left(\frac{\sigma_k^2}{\gamma_o}\right)^{L_{rx}-L_{tx}+1}(b_k)^{\frac{\sigma_k^2}{\gamma_o}-1}[ln(5b_k)]^{L_{rx}-L_{tx}}. \quad (3.7)
\end{aligned}$$

For $0 < \gamma_k < \infty$, the PDF of the ABER of the k^{th} stream is valid for $\frac{1}{5} > b_k > 0$.

3.5 ABER Outage Probability

The merits of random selection are shown for the simple case in which all antennas are randomly selected independently in each iteration. The probability of an outage is defined as the probability when all received $ABER_k$, $k = 1,...,L_{tx}$, in the different data streams go above a certain threshold. The set of $ABER_k$ are functions of the power gain g_k^2 for $k = 1,...,L_{tx}$, which are correlated random variables due to the inversion operation in (2.12). To enable traceable mathematical analysis, independent coding and decoding across the data streams is assumed. Therefore, the set of $ABER_k$ for $k = 1,...,L_{tx}$ would be modeled as independent random variables, and their joint PDF is the product of their individual marginal PDFs. The outage probability for a given antenna configuration is therefore

$$\begin{aligned} Pr_{outage} &= Pr(ABER_1 > T_1, \ldots, ABER_{L_{tx}} > T_{L_{tx}}) \\ &= \prod_{k=1}^{L_{tx}} Pr(ABER_k > T_k) \; . \end{aligned}$$

Through integration of (3.7), $Pr(ABER_k > T_k)$ can be found as

$$\begin{aligned} Pr(ABER_k > T_k) &= \int_{T_k}^{\frac{1}{5}} f_{ABER_{k-stream}}(b_k) db_k \\ &= \int_{T_k}^{\frac{1}{5}} \frac{(-1)^{L_{rx}-L_{tx}} (5)^{\frac{\sigma_k^2}{\gamma_o}}}{(L_{rx}-L_{tx})!} \left(\frac{\sigma_k^2}{\gamma_o}\right)^{L_{rx}-L_{tx}+1} (b_k)^{\frac{\sigma_k^2}{\gamma_o}-1} [ln(5b_k)]^{L_{rx}-L_{tx}} db_k \; . \end{aligned}$$

Apply the following change of variables

$$y_k = ln(5b_k) \quad ; \quad b_k = \tfrac{1}{5}e^{y_k} \quad ; \quad \tfrac{1}{5}e^{y_k}dy_k = db_k \quad , \tag{3.8}$$

together with the new integration limits

$$\begin{array}{rcl} b_k = T_k & \Rightarrow & y_k = ln(5T_k) \\ b_k = \frac{1}{5} & \Rightarrow & y_k = ln(5(\frac{1}{5})) = ln(1) = 0 \end{array} \quad , \tag{3.9}$$

the integration becomes

$$\begin{aligned} Pr(ABER_k > T_k) &= \int_{ln(5T_k)}^{0} \frac{(-1)^{L_{rx}-L_{tx}} (5)^{\frac{\sigma_k^2}{\gamma_o}}}{(L_{rx}-L_{tx})!} \left(\frac{\sigma_k^2}{\gamma_o}\right)^{L_{rx}-L_{tx}+1} \left(\frac{1}{5}e^{y_k}\right)^{\frac{\sigma_k^2}{\gamma_o}-1} (y_k)^{L_{rx}-L_{tx}} \frac{1}{5}e^{y_k} dy_k \\ &= \int_{ln(5T_k)}^{0} \frac{(-1)^{L_{rx}-L_{tx}}}{(L_{rx}-L_{tx})!} \left(\frac{\sigma_k^2}{\gamma_o}\right)^{L_{rx}-L_{tx}+1} \left(e^{y_k \frac{\sigma_k^2}{\gamma_o}}\right) (y_k)^{L_{rx}-L_{tx}} dy_k \\ &= C_k \int_{ln(5T_k)}^{0} (e^{a_k y_k}) \, (y_k)^m dy_k \; , \end{aligned}$$

where

$$m = L_{rx} - L_{tx} \quad a_k = \frac{\sigma_k^2}{\gamma_o} \quad C_k = \frac{(-1)^m}{m!} (a_k)^{m+1} \quad . \tag{3.10}$$

The above integral can be evaluated as

$$\int_{ln(5T_k)}^{0} (e^{a_k y_k}) \, (y_k)^m dy_k = e^{a_k y_k} \sum_{r=0}^{m} \frac{(-1)^r y_k^{m-r} m!}{a_k^{r+1}(m-r)!} \Bigg|_{ln(5T_k)}^{0} \quad . \tag{3.11}$$

Therefore,

$$Pr(ABER_k > T_k) = C_k \left(e^{a_k y_k} \sum_{r=0}^{m} \frac{(-1)^r y_k^{m-r} m!}{a_k^{r+1}(m-r)!} \Bigg|_{ln(5T_k)}^{0} \right) . \tag{3.12}$$

The outage probability can be found to be

$$Pr_{outage} = \prod_{k=1}^{L_{tx}} C_k \left(\sum_{r=0}^{m} \frac{e^{a_k y_k}(-1)^r y_k^{m-r} m!}{(m-r)! a_k^{r+1}} \Bigg|_{y_k=ln(5T_k)}^{y_k=0} \right) . \tag{3.13}$$

The probability of still being in outage after K iterations is specified by $Pr_{failure}$ and is calculated as $(Pr_{outage})^K$, and

$$K = \frac{log(Pr_{failure})}{log(Pr_{outage})} \tag{3.14}$$

is the expected number of RAS iterations for finding a non-outage set of antennas given a maximum $Pr_{failure}$.

3.5.1 Numerical Results

This section presents the ABER outage probability for a MIMO system with (N_{tx}:N_{rx},L_{tx}:L_{rx}) = (4:8,2:4). The expression in (3.13) is evaluated to obtain the ABER outage probability for an uncorrelated MIMO channel, with $\sigma_k^2 = 1$ for $k = 1, ..., L_{tx}$. Figure 3.1 shows the ABER outage probability of the (4:8,2:4) uncorrelated antenna selected system for different required ABER thresholds. Together with the ABER outage probability, the expected number of RAS iterations calculated using (3.14) are tabulated in Table 3.2 for a given $Pr_{failure}$ of 0.01 and SNR from 0dB to 10dB.

From Figure 3.1 it can be seen that the outage probability for an ABER threshold of 10^{-3} is smaller than the outage probability for lower ABER thresholds of 10^{-4}, 10^{-5}, or 10^{-6} over all SNRs. This makes sense intuitively, as there is a smaller chance of finding a set of antennas that all fail to meet a high ABER threshold than when a low ABER threshold is to be met.

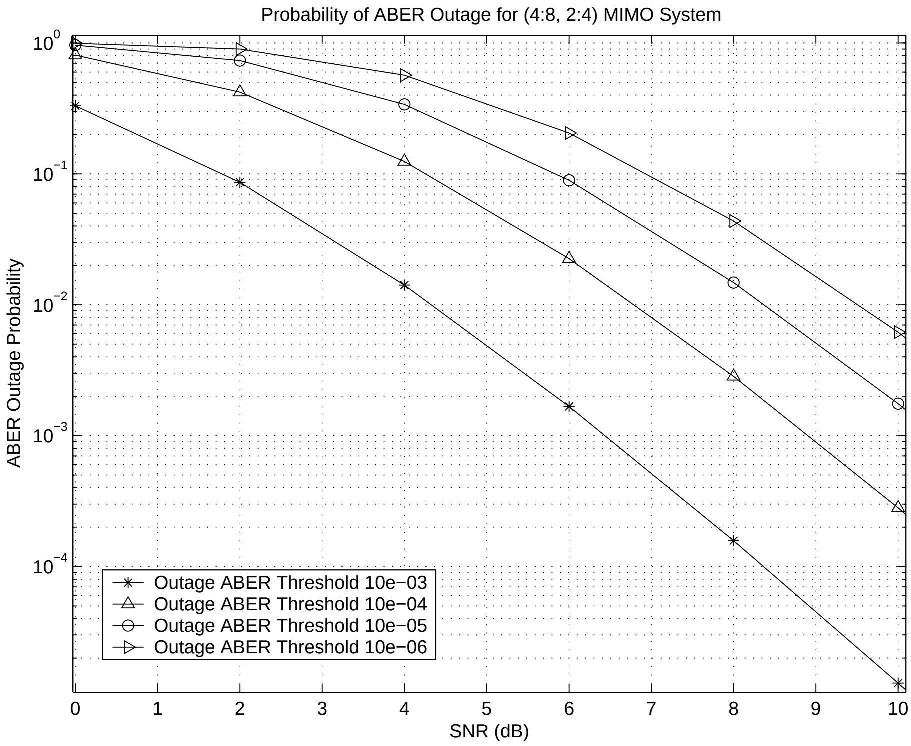

Figure 3.1: ABER outage probability of a (4:8,2:4) system.

Table 3.2: Expected number of RAS iterations versus SNR.

$ABER_{Threshold}$	0dB	2dB	4dB	6dB	8dB	10dB
10^{-3}	4.17	1.88	1.08	0.72	0.53	0.41
10^{-4}	21.48	5.33	2.21	1.21	0.79	0.56
10^{-5}	121.69	14.93	4.26	1.91	1.09	0.73

From Table 3.2, the expected number of iterations in order to find a non-outage set of antennas decreases rapidly for high SNR. At a low SNR of 4dB, the RAS algorithm requires only 1.08, 2.21, and 4.26 iterations to find a non-outage set of antennas for ABER thresholds of 10^{-3}, 10^{-4}, and 10^{-5} respectively. Therefore, after a small number of expected RAS iterations, the system would not be in outage. These numerical results illustrate the merits of performing random antenna subset selection.

3.6 Summary

This chapter presented the concept of RAS based on a minimum average BER selection criterion. From the statistics of the received SNR, the ABER outage probability using the RAS is determined. Numerical results suggest that after a small number of RAS iterations the system would not be in outage, and this shows the potential merit of RAS.

Chapter 4

Random Antenna Selection with Antenna Swapping

In the previous chapter, the use of RAS for antenna selection is presented. It is noted that in each RAS iteration, the evaluation of the computationally expensive matrix inversion operation in the selection criterion is required when a different random subset of antennas is chosen. This motivates the search for a method to reduce the computational complexity of the RAS algorithm.

This chapter presents the concept of antenna swapping (AS) to address this problem, and shows that antenna swapping is equivalent to performing matrix modification. Receive antenna swapping is first presented, followed by transmit antenna swapping. A deterministic swapping sequence and random swapping sequence is then introduced. Near the end of the chapter, a method to update the inverse of a modified matrix is presented, and this enables the combination of RAS with AS to form an efficient joint transmit and receive antenna selection algorithm.

4.1 Antenna Swapping

The antenna swapping technique can be applied to facilitate an efficient implementation of the RAS algorithm. In each iteration, the algorithm can swap a pair of transmit or receive antennas. An antenna from the selected set of antennas is swapped out, and is replaced by an antenna that does not belong to the set of selected antennas during that iteration. The combination of the RAS algorithm with antenna swapping is referred to as the RAS-AS algorithm.

The computational requirements of the RAS-AS algorithm are dominated by the computational complexity of the transmit and receive antenna swapping operations. These operations will be further investigated in the later sections of the chapter.

4.1.1 Matrix Modification for Receive Antenna Swapping

This section considers the case of swapping a pair of receive antennas. Each receive antenna is represented by a row of channel gains in the MIMO channel matrix. Let H_n be the MIMO channel matrix of dimension $L_{rx} \times L_{tx}$, and it is represented below

$$H_n = \begin{bmatrix} \underline{h}_1^H \\ \underline{h}_2^H \\ \ldots \\ \underline{h}_{L_{rx}}^H \end{bmatrix} . \tag{4.1}$$

Therefore,

$$H_n^H H_n = \begin{bmatrix} \underline{h}_1 & \underline{h}_2 & \ldots & \underline{h}_{L_{rx}} \end{bmatrix} \begin{bmatrix} \underline{h}_1^H \\ \underline{h}_2^H \\ \ldots \\ \underline{h}_{L_{rx}}^H \end{bmatrix} = \underline{h}_1 \underline{h}_1^H + \underline{h}_2 \underline{h}_2^H + \ldots + \underline{h}_{L_{rx}} \underline{h}_{L_{rx}}^H . \tag{4.2}$$

From the above, it can be seen that each row's contribution to $H_n^H H_n$ is of the form $\underline{h}_j \underline{h}_j^H$. Therefore, if the j^{th} row is swapped out and the i^{th} row is swapped in, then the contribution from the j^{th} row can be subtracted and the contribution from the i^{th} row can be added to $H_n^H H_n$ to obtain $H_{n+1}^H H_{n+1}$ as follows

$$\begin{aligned} H_{n+1}^H H_{n+1} &= H_n^H H_n + \underline{h}_i \underline{h}_i^H - \underline{h}_j \underline{h}_j^H \\ &= H_n^H H_n + UV^H \end{aligned}$$

where

$$U = \begin{bmatrix} \underline{h}_i & \underline{h}_j \end{bmatrix} \tag{4.3}$$

$$V = \begin{bmatrix} \underline{h}_i & -\underline{h}_j \end{bmatrix} \tag{4.4}$$

so that

$$UV^H = \begin{bmatrix} \underline{h}_i & \underline{h}_j \end{bmatrix} \begin{bmatrix} \underline{h}_i & -\underline{h}_j \end{bmatrix}^H = \begin{bmatrix} \underline{h}_i & \underline{h}_j \end{bmatrix} \begin{bmatrix} \underline{h}_i^H \\ -\underline{h}_j^H \end{bmatrix} = \underline{h}_i \underline{h}_i^H - \underline{h}_j \underline{h}_j^H .$$

Therefore, swapping a pair of receive antennas has the effect of introducing a rank-2 modification to $H_n^H H_n$ in the n^{th} step of the algorithm. The modification matrix is formed by multiplying two $L_{tx} \times 2$ rectangular matrices given in (4.3) and (4.4) above. The modification matrix has at most rank 2, and this occurs when the rows and columns of the UV^H matrix are linearly independent. Rank 1 modification can occur when the channel gains to be swapped out and the channel gains to be swapped in are highly correlated. In general, by modeling the channel gains as independent random variables, the modification matrix is of rank 2.

The above result can be extended to swapping more than one pair of receive antennas at a time. The general form of U and V would be as follows

$$U = \begin{bmatrix} (\text{set of antennas to swap in } \underline{h}_i\text{'s}) & (\text{set of antennas to swap out } \underline{h}_j\text{'s}) \end{bmatrix} \tag{4.5}$$

$$V = \left[\begin{array}{cc} (\text{set of antennas to swap in } \underline{h}_i\text{'s}) & -(\text{set of antennas to swap out } \underline{h}_j\text{'s}) \end{array} \right] \quad (4.6)$$

This would result in a rank-2k modification to the $H_n^H H_n$ matrix, where k is the number of receive antenna pairs to be swapped.

4.1.2 Matrix Modification for Transmit Antenna Swapping

This section considers the case of swapping a pair of transmit antennas. Each transmit antenna is represented by a column of channel gains in the MIMO channel matrix. The effect of swapping a pair of transmit antennas is equivalent to modifying a column in the H_n matrix. Let H_n represent the MIMO channel matrix of dimension $L_{rx} \times L_{tx}$, and

$$H_n = \left[\begin{array}{cccc} \underline{h}_1 & \underline{h}_2 & \dots & \underline{h}_{L_{tx}} \end{array} \right] . \quad (4.7)$$

Therefore,

$$\begin{aligned} H_n^H H_n &= \left[\begin{array}{cccc} \underline{h}_1 & \underline{h}_2 & \dots & \underline{h}_{L_{tx}} \end{array} \right]^H \left[\begin{array}{cccc} \underline{h}_1 & \underline{h}_2 & \dots & \underline{h}_{L_{tx}} \end{array} \right] \\ &= \left[\begin{array}{c} \underline{h}_1^H \\ \underline{h}_2^H \\ \dots \\ \underline{h}_{L_{tx}}^H \end{array} \right] \left[\begin{array}{cccc} \underline{h}_1 & \underline{h}_2 & \dots & \underline{h}_{L_{tx}} \end{array} \right] \\ &= \left[\begin{array}{cccc} \underline{h}_1^H \underline{h}_1 & \underline{h}_1^H \underline{h}_2 & \dots & \underline{h}_1^H \underline{h}_{L_{tx}} \\ \underline{h}_2^H \underline{h}_1 & \underline{h}_2^H \underline{h}_2 & \dots & \underline{h}_2^H \underline{h}_{L_{tx}} \\ \dots & \dots & \dots & \dots \\ \underline{h}_{L_{tx}}^H \underline{h}_1 & \underline{h}_{L_{tx}}^H \underline{h}_2 & \dots & \underline{h}_{L_{tx}}^H \underline{h}_{L_{tx}} \end{array} \right] . \end{aligned} \quad (4.8)$$

When the i^{th} column in H_n is changed, it will affect the i^{th} row and i^{th} column in $H_n^H H_n$. Therefore, changing the i^{th} column in H_n, requires simultaneous changes to the i^{th} row and i^{th} column of $H_n^H H_n$. From above, it is noted that the i^{th} row and i^{th} column in $H_n^H H_n$ are symmetric with respect to each other. Therefore, a result on the simultaneous change of a symmetric column and row in [32] can be applied, and is presented below.

In order to simultaneously change the symmetric i^{th} row and i^{th} column in $H_n^H H_n$, a modification matrix C would be of the form

$$C = \begin{bmatrix} 0 & \dots & 0 & c_{1,i} & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ c_{i,1} & \dots & \dots & c_{i,i} & \dots & \dots & \dots & c_{i,L_{tx}} \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & c_{L_{tx},i} & 0 & \dots & \dots & 0 \end{bmatrix}$$

where only the i^{th} row and i^{th} column have non-zero values. Comparing to (4.8) above, in order to remove the i^{th} transmit antenna and insert the j^{th} transmit antenna, the modification matrix would have to be

$$C = \begin{bmatrix} 0 & \dots & 0 & \underline{h}_1^H \underline{h}_j - \underline{h}_1^H \underline{h}_i & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ \underline{h}_j^H \underline{h}_1 - \underline{h}_i^H \underline{h}_1 & \dots & \dots & \underline{h}_j^H \underline{h}_j - \underline{h}_i^H \underline{h}_i & \dots & \dots & \dots & \underline{h}_j^H \underline{h}_{L_{tx}} - \underline{h}_i^H \underline{h}_{L_{tx}} \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & \underline{h}_{L_{tx}}^H \underline{h}_j - \underline{h}_{L_{tx}}^H \underline{h}_i & 0 & \dots & \dots & 0 \end{bmatrix}. \quad (4.9)$$

The modification matrix C has at most rank 2 and can be represented by the product of two rectangular matrices of dimension $L_{tx} \times 2$ and $2 \times L_{tx}$ respectively [32]. Let these two rectangular matrices be U and V, and they are defined as follows

$$U = \begin{bmatrix} \underline{u} & \underline{e}_i \end{bmatrix} \tag{4.10}$$

$$V = \begin{bmatrix} \underline{e}_i & \underline{u} \end{bmatrix} \tag{4.11}$$

where $\underline{u}$ contains the adjustment values to $H_n^H H_n$, and $\underline{e}_i$ is a vector with a 1 in the i^{th} position and zeros everywhere else. Then

$$C = UV^H = \begin{bmatrix} \underline{u} & \underline{e}_i \end{bmatrix} \begin{bmatrix} \underline{e}_i & \underline{u} \end{bmatrix}^H = \begin{bmatrix} \underline{u} & \underline{e}_i \end{bmatrix} \begin{bmatrix} \underline{e}_i^H \\ \underline{u}^H \end{bmatrix} = \underline{u}\underline{e}_i^H + \underline{e}_i\underline{u}^H \ . \tag{4.12}$$

Expanding the above results in

$$\begin{aligned} C &= \begin{bmatrix} u_{1,1} \\ \dots \\ u_{i,1} \\ \dots \\ u_{L_{tx},1} \end{bmatrix} \begin{bmatrix} 0_{1,1} \\ \dots \\ 1_{i,1} \\ \dots \\ 0_{L_{tx},1} \end{bmatrix}^H + \begin{bmatrix} 0_{1,1} \\ \dots \\ 1_{i,1} \\ \dots \\ 0_{L_{tx},1} \end{bmatrix} \begin{bmatrix} u_{1,1} \\ \dots \\ u_{i,1} \\ \dots \\ u_{L_{tx},1} \end{bmatrix}^H \\ &= \begin{bmatrix} u_{1,1} \\ \dots \\ u_{i,1} \\ \dots \\ u_{L_{tx},1} \end{bmatrix} \begin{bmatrix} 0_{1,1} & \dots & 1_{i,1} & \dots & 0_{L_{tx},1} \end{bmatrix} + \begin{bmatrix} 0_{1,1} \\ \dots \\ 1_{i,1} \\ \dots \\ 0_{L_{tx},1} \end{bmatrix} \begin{bmatrix} u_{1,1}^H & \dots & u_{i,1}^H & \dots & u_{L_{tx},1}^H \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 0 & \dots & 0 & u_{1,1} & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & u_{i,1} & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & u_{L_{tx},1} & 0 & \dots & \dots & 0 \end{bmatrix} + \begin{bmatrix} 0 & \dots & 0 & 0 & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ u_{1,1}^H & \dots & \dots & u_{i,1}^H & \dots & \dots & \dots & u_{L_{tx},1}^H \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & 0 & 0 & \dots & \dots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \dots & 0 & u_{1,1} & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ u_{1,1}^H & \dots & \dots & u_{i,1} + u_{i,1}^H & \dots & \dots & \dots & u_{L_{tx},1}^H \\ 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ \dots & 0 & \dots & \dots & \dots & 0 & 0 & \dots \\ 0 & \dots & 0 & u_{L_{tx},1} & 0 & \dots & \dots & 0 \end{bmatrix} .$$

Comparing to (4.9) above, the values for $\underline{u}$ can be determined as follows

$$u_{k,1} = \underline{h}_k^H \underline{h}_j - \underline{h}_k^H \underline{h}_i \tag{4.13}$$

for $k = 1, ..., (i-1), (i+1), ..., L_{tx}$. From

$$u_{i,1} + u_{i,1}^H = \underline{h}_j^H \underline{h}_j - \underline{h}_i^H \underline{h}_i \tag{4.14}$$

the i^{th} element in $\underline{u}$ can be determined as

$$u_{i,1} = \frac{1}{2} \left(\underline{h}_j^H \underline{h}_j - \underline{h}_i^H \underline{h}_i \right) . \tag{4.15}$$

Therefore, using (4.13) and (4.15), the matrices U and V from (4.10) and (4.11) above can be determined. With the modification matrix C defined, the updated $H_{n+1}^H H_{n+1}$ can be computed as

$$H_{n+1}^H H_{n+1} = H_n^H H_n + C = H_n^H H_n + UV^H \ . \tag{4.16}$$

From the above, it can be seen that swapping a pair of transmit antennas has the effect of performing a rank-2 modification on the $H_{n+1}^H H_{n+1}$ matrix. Similar to the case of receive antenna swapping, the modification matrix has at most rank 2. In general, by modeling the channel gains as independent random variables, the modification matrix is of rank 2. To swap k pairs of transmit antennas, a modification matrix of rank-2k and appropriate adjustments to the values in $\underline{u}$ would be required.

4.2 Antenna Swapping Sequence

The antenna swapping sequence is defined as the set of antenna configurations that the algorithm iterates through to calculate the average BER performance of the system. The following presents two types of swapping sequences. The first type is a deterministic swapping sequence, and the second type is a random swapping sequence.

4.2.1 Deterministic Swapping Sequence

A deterministic swapping sequence of antenna configurations is a sequence where each neighboring configuration differs by one transmit or receive antenna. The swapping sequence can be decoupled into a transmit antenna swapping sequence and a receive antenna swapping sequence. The two sequences can then be combined to form a single transmit and receive antenna swapping sequence with all the combinations. In order to keep the number of swapping operations to a minimum, the antenna sequence with antenna configurations

Table 4.1: Receive antenna swapping sequence.

1 2 3 4	1 4 7 8	1 5 6 7	2 3 4 6	1 4 5 8	3 4 5 8	3 5 6 8
1 2 3 5	1 5 7 8	1 4 6 7	2 3 4 7	1 4 5 7	3 4 5 7	2 5 6 8
1 2 3 6	1 6 7 8	1 3 6 7	2 3 4 8	1 3 5 7	3 4 5 6	2 4 6 8
1 2 3 7	2 6 7 8	1 2 6 7	2 3 5 8	1 3 5 8	2 3 5 6	2 4 5 8
1 2 3 8	3 6 7 8	1 2 5 7	2 3 6 8	1 5 6 8	2 3 5 7	2 4 5 7
1 2 4 8	4 6 7 8	1 2 4 7	2 3 7 8	1 4 6 8	2 3 6 7	2 4 5 6
1 2 5 8	5 6 7 8	1 2 4 6	2 4 7 8	1 3 6 8	2 4 6 7	1 2 5 6
1 2 6 8	4 5 6 7	1 2 4 5	2 5 7 8	1 3 4 8	3 4 6 7	1 4 5 6
1 2 7 8	3 5 6 7	1 3 4 5	3 5 7 8	1 3 4 7	3 4 6 8	1 3 5 6
1 3 7 8	2 5 6 7	2 3 4 5	4 5 7 8	3 4 7 8	4 5 6 8	1 3 4 6

that differ from the neighboring configurations by one transmit or one receive antenna will be used.

The swapping sequence is cyclic. A possible transmit and receive antenna sequence is presented below for a (4:8,2:4) system. Table 4.1 presents a receiver antenna swapping sequence that cycles through all the subset antenna configurations of size 4 in a receive antenna array with 8 antennas. The sequence goes from top to bottom, left to right. Each element in the sequence differs from its neighbor by one antenna element. The last antenna configuration, [1 3 4 6], wraps around to the first antenna configuration, [1 2 3 4], in the table, and the two differ by one antenna element. Table 4.2 presents a transmit antenna swapping sequence that cycles through all the size 2 subset antenna configurations for a transmit antenna array with 4 antennas.

The above sequences are not unique and there exist other cyclic swapping sequences. The combination of the transmit and receive antenna swapping sequences represents all the

Table 4.2: Transmit antenna swapping sequence.

1 2	1 3	1 4	2 4	3 4	2 3

possible antenna configurations of the system.

4.2.2 Random Swapping Sequence

A random swapping sequence is one that randomly selects a antenna, and exchanges it with another antenna that is not selected in that iteration. In each iteration, a random pair of transmit or receive antenna is swapped, and each side is equally likely to have its antenna swapped. This is discussed further in Section 4.4. In a random swapping sequence, it is possible for the same antenna configurations to be tested more than once.

4.3 Inversion Update for Modified Matrix

From previous sections, it is shown that antenna swapping is equivalent to performing matrix modification. Therefore, this section will present the Sherman-Morrison formula and Woodbury formula that can be used to update the inverse of a modified matrix.

4.3.1 Sherman-Morrison Formula

The Sherman-Morrison formula allows the inverse of a modified matrix to be computed from the inverse of the unmodified matrix. The following presents the details of this matrix inversion update.

Let A be a $N \times N$ invertible square matrix, with $\underline{u}$ and $\underline{v}$ representing two $N \times 1$ dimension vectors, and β is any arbitrary scalar. Then, let $\widetilde{A} = A + \beta \underline{uv}^H$ be the rank-1 modified version of the original matrix A, where the quantity $\beta \underline{uv}^H$ represents a rank-1 modification.

Then the inverse of the modified matrix can be related to the inverse of the original matrix as follows

$$(\widetilde{A})^{-1} = (A + \beta \underline{u}\underline{v})^{-1} = A^{-1} - \frac{\beta}{\lambda} A^{-1} \underline{u}\underline{v}^H A^{-1} \tag{4.17}$$

where $\lambda = 1 + \beta \underline{v}^H A^{-1} \underline{u}$ [32]. When $\beta = 1$, (4.17) becomes the Sherman-Morrison formula [32].

4.3.2 The Woodbury Formula

A generalization of the Sherman-Morrison formula is the Woodbury formula and it is presented below. Let D be a $N \times N$ invertible square matrix, with P and Q representing two $N \times k$ dimensional matrices $(k < N)$, and ξ is any arbitrary scalar. Then, let $\widetilde{D} = D + \xi PQ^H$ be the rank-k modified version of the original matrix D, where the quantity ξPQ^H represents a rank-k modification. Then the inverse of the modified matrix can be related to the inverse of the original matrix as follows

$$(\widetilde{D})^{-1} = (D + \xi PQ^H)^{-1} = D^{-1} - \xi D^{-1} P \Sigma^{-1} Q^H D^{-1} \tag{4.18}$$

where $\Sigma = I_k + \xi Q^H D^{-1} P$, and I_k is the $k \times k$ identity matrix [32]. When $k = 1$, the expression in (4.18) simplifies to (4.17), with $\Sigma = \lambda$.

4.4 Random Antenna Selection with Antenna Swapping Algorithm

With the matrix inversion update expressions from the previous section, the inverse of the modified matrix after swapping a pair of antennas can be computed. This can facilitate an efficient method to compute the power gain required in the evaluation of the selection criterion. The realization of the RAS algorithm with AS is referred as the RAS-AS algorithm.

The RAS-AS algorithm starts off by randomly selecting L_{tx} transmit antennas and L_{rx} receive antennas. The algorithm swaps a pair of transmit or receive antennas in each iteration, and keeps track of the antenna configuration that provides the best average BER performance. In each iteration, there is a probability of p_{swap} that a pair of transmit antennas is swapped, or a pair of receive antennas is swapped with probability $(1 - p_{swap})$. The p_{swap} parameter will be further discussed in subsequent chapters. The algorithm terminates when either all antenna configurations are tested or when a desired average BER performance is achieved. While most existing algorithms require the full complexity MIMO channel to be estimated, the RAS-AS algorithm only requires $L_{rx} \times L_{tx}$ complex channel gains to be estimated at startup. Channel estimation is performed when new antennas are swapped. This reduces the amount of initial training and spreads the overall channel training and estimation over time, making the RAS-AS algorithm practical for systems with large numbers of antennas. This thesis considers temporally uncorrelated channels, and for channels with temporal correlation, the RAS-AS algorithm can provide the flexibility to facilitate small updates to maintain or improve performance over time, starting with the best antenna configuration found from the previous time slot. The pseudocode of the RAS-AS algorithm is presented in Table 4.3.

In the next chapter, further simplifications will be introduced to the transmit and receive antenna swapping operations, and these computational complexity reductions can be used to perform the matrix inversion update in step 6 of the pseudocode in Table 4.3.

4.5 Summary

This chapter presents the concept of antenna swapping, and relates it to performing a rank-$2k$ matrix modification when k pairs of antennas are swapped. From this relationship,

Table 4.3: RAS-AS algorithm pseudocode.

	Initialization:
1	Randomly select L_{tx} transmit, L_{rx} receive antennas to form H_0
2	Calculate $\left(H_0^H H_0\right)^{-1}$ and g_k^2 for $k = 1, \ldots, L_{tx}$ using (2.12)
3	Initialize $\left(H^H H\right)_{best}^{-1} = \left(H_0^H H_0\right)^{-1}$ and $\text{AvgBER}_{best} = \text{AvgBER}_0$ using (2.13)
	Main Loop:
4	Iterate through the antenna configurations (n^{th} Step) $\{$
5	Swap a pair of transmit antennas with probability p_{swap} or
	swap a pair of receive antennas with probability $(1 - p_{swap})$ to form H_n
6	Update $\left(H_n^H H_n\right)^{-1}$ with (4.18)
7	Calculate g_k^2 for $k = 1, \ldots, L_{tx}$ using (2.12)
8	Calculate AvgBER_n using (2.13)
9	if ($\text{AvgBER}_n < \text{AvgBER}_{best}$) then
10	Current antenna configuration is the best : Set $\left(H^H H\right)_{best}^{-1} = \left(H_n^H H_n\right)^{-1}$
11	$\text{AvgBER}_{best} = \text{AvgBER}_n$
	end if $\}$

the Woodbury formula for matrix inversion update is applied to the modified matrix to compute the power gains required for the evaluation of the selection criterion. This chapter also presents a deterministic and random swapping sequence and the pseudocode for the RAS-AS algorithm is presented at the end of the chapter.

Chapter 5

Fast Random Antenna Selection with Antenna Swapping

This chapter presents further complexity simplification of the RAS-AS algorithm based on rank-2 matrix modification from the swapping of a pair of transmit or receive antennas. The simplified algorithm is analyzed in terms of the required number of multiplications and additions. The order of computational complexity is also derived. The behavior of the RAS-AS algorithm is also analyzed in terms of the expected number of iterations using either a deterministic or random swapping sequence. A greedy version of the algorithm is also introduced for comparison purposes. Simulations of the average BER performance for the RAS-AS algorithm conclude this chapter.

5.1 Rank-2 Complexity Reduction

In Sections 4.1.1 and 4.1.2, it is noted that the swapping of a pair of receive antennas or transmit antennas is equivalent to introducing a rank-2 modification to the $H^H H$ matrix. The inverse of the perturbed $H^H H$ matrix can be updated using the Woodbury formula in (4.18) for calculating g_k^2. As a result of the rank-2 modification matrix, the expressions for the swapping of a pair of transmit antennas and the swapping of a pair of receive antennas can be simplified. The following applies a complexity reduction scheme based on [32] for evaluating the matrix inverse after the transmit antenna swapping and receive antenna swapping operations.

5.1.1 Reduced Complexity Transmit Antenna Swapping

When a pair of transmit antennas is swapped, a row and column in the $H^H H$ matrix has to be modified. The rank-2 nature of the modification matrix and the structure of the modification matrix allows for a more efficient computation of the matrix inversion update equation. From the previous chapter, the update equation is

$$H_{n+1}^H H_{n+1} = H_n^H H_n + C = H_n^H H_n + UV^H \tag{5.1}$$

where U and V^H are two rectangular matrices of dimension $L_{tx} \times 2$ and $2 \times L_{tx}$, respectively, and they are defined in (4.10) and (4.11) as

$$U = \begin{bmatrix} \underline{u} & \underline{e}_i \end{bmatrix} \tag{5.2}$$

$$V = \begin{bmatrix} \underline{e}_i & \underline{u} \end{bmatrix} \tag{5.3}$$

where $\underline{u}$ contains the adjustment values to $H_n^H H_n$, and $\underline{e}_i$ is a vector with a 1 in the i^{th} position and zeros everywhere else. Applying the Woodbury formula (4.18) to (5.1) above results in

$$(H_{n+1}^H H_{n+1})^{-1} = (H_n^H H_n)^{-1} - (H_n^H H_n)^{-1} U \left(I_2 + V^H (H_n^H H_n)^{-1} U \right)^{-1} V^H (H_n^H H_n)^{-1}$$

where I_2 is the 2×2 identity matrix. Substituting U and V from (4.10) and (4.11) into the above expression, and letting $B_n = H_n^H H_n$, $B_{n+1} = H_{n+1}^H H_{n+1}$, results in

$$B_{n+1}^{-1} \quad = \quad B_n^{-1} - B_n^{-1} \begin{bmatrix} \underline{u} & \underline{e}_i \end{bmatrix} \left(I_2 + \begin{bmatrix} \underline{e}_i & \underline{u} \end{bmatrix}^H B_n^{-1} \begin{bmatrix} \underline{u} & \underline{e}_i \end{bmatrix} \right)^{-1} \begin{bmatrix} \underline{e}_i & \underline{u} \end{bmatrix}^H B_n^{-1}$$

$$
\begin{aligned}
&= B_n^{-1} - \begin{bmatrix} B_n^{-1}\underline{u} & B_n^{-1}\underline{e}_i \end{bmatrix} \left(I_2 + \begin{bmatrix} \underline{e}_i^H B_n^{-1} \\ \underline{u}^H B_n^{-1} \end{bmatrix} \begin{bmatrix} \underline{u} & \underline{e}_i \end{bmatrix} \right)^{-1} \begin{bmatrix} \underline{e}_i^H B_n^{-1} \\ \underline{u}^H B_n^{-1} \end{bmatrix} \\
&= B_n^{-1} - \begin{bmatrix} B_n^{-1}\underline{u} & B_n^{-1}\underline{e}_i \end{bmatrix} \begin{bmatrix} 1+\underline{e}_i^H B_n^{-1}\underline{u} & \underline{e}_i^H B_n^{-1}\underline{e}_i \\ \underline{u}^H B_n^{-1}\underline{u} & 1+\underline{u}^H B_n^{-1}\underline{e}_i \end{bmatrix}^{-1} \begin{bmatrix} \underline{e}_i^H B_n^{-1} \\ \underline{u}^H B_n^{-1} \end{bmatrix} \qquad (5.4)
\end{aligned}
$$

Because of the $\underline{e}_i$ vector, the elements in the above expression can be simplified. It can be seen that $B_n^{-1}\underline{e}_i$ is the i^{th} column of B_n^{-1}, $\underline{e}_i^H B_n^{-1}$ is the i^{th} row of B_n^{-1}, and $\underline{e}_i^H B_n^{-1}\underline{e}_i$ is the (i,i) element in B_n^{-1}. Let these be denoted $\underline{B}_{n,i_{col}}^{-1}$, $\underline{B}_{n,i_{row}}^{-1}$, and $\beta_{i,i}$ respectively. Let Y be the middle matrix in the second term of (5.4), therefore

$$
Y = \begin{bmatrix} 1+\underline{e}_i^H B_n^{-1}\underline{u} & \underline{e}_i^H B_n^{-1}\underline{e}_i \\ \underline{u}^H B_n^{-1}\underline{u} & 1+\underline{u}^H B_n^{-1}\underline{e}_i \end{bmatrix} = \begin{bmatrix} 1+\underline{B}_{n,i_{row}}^{-1}\underline{u} & \beta_{i,i} \\ \underline{u}^H B_n^{-1}\underline{u} & 1+\underline{u}^H \underline{B}_{n,i_{col}}^{-1} \end{bmatrix} = \begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix}
$$

where

$$
\begin{aligned}
y_1 &= 1+\underline{B}_{n,i_{row}}^{-1}\underline{u} \\
y_2 &= \beta_{i,i} \\
y_3 &= \underline{u}^H B_n^{-1}\underline{u} \\
y_4 &= 1+\underline{u}^H \underline{B}_{n,i_{col}}^{-1}
\end{aligned}
$$

From above, Y is a 2×2 matrix, and its inverse can be found to be

$$
Y^{-1} = \frac{1}{y_1 y_4 - y_2 y_3} \begin{bmatrix} y_4 & -y_2 \\ -y_3 & y_1 \end{bmatrix} = \frac{1}{d} \begin{bmatrix} y_4 & -y_2 \\ -y_3 & y_1 \end{bmatrix} .
$$

Therefore, (5.4) becomes

$$
\begin{aligned}
B_{n+1}^{-1} &= B_n^{-1} - \begin{bmatrix} B_n^{-1}\underline{u} & \underline{B}_{n,i_{col}}^{-1} \end{bmatrix} Y^{-1} \begin{bmatrix} \underline{B}_{n,i_{row}}^{-1} \\ \underline{u}^H B_n^{-1} \end{bmatrix} \\
&= B_n^{-1} - \begin{bmatrix} B_n^{-1}\underline{u} & \underline{B}_{n,i_{col}}^{-1} \end{bmatrix} \frac{1}{d} \begin{bmatrix} 1+\underline{u}^H \underline{B}_{n,i_{col}}^{-1} & -\beta_{i,i} \\ -\underline{u}^H B_n^{-1}\underline{u} & 1+\underline{B}_{n,i_{row}}^{-1}\underline{u} \end{bmatrix} \begin{bmatrix} \underline{B}_{n,i_{row}}^{-1} \\ \underline{u}^H B_n^{-1} \end{bmatrix} \qquad (5.5)
\end{aligned}
$$

with

$$d = y_1 y_4 - y_2 y_3 = \left(1 + \underline{B}_{n,i_{row}}^{-1} \underline{u}\right) \left(1 + \underline{u}^H \underline{B}_{n,i_{col}}^{-1}\right) - \left(\beta_{i,i}\right) \left(\underline{u}^H B_n^{-1} \underline{u}\right) . \tag{5.6}$$

To further simplify the above, the following relationship in matrix multiplication is used: let W and Z be $n \times 2$ matrices, and let T be a 2×2 matrix. Then

$$WTZ^H = \begin{bmatrix} \underline{w}_1 & \underline{w}_2 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} \underline{z}_1^H \\ \underline{z}_2^H \end{bmatrix} = \sum_{i=1}^{2} \sum_{j=1}^{2} \underline{w}_i T_{ij} \underline{z}_j^H . \tag{5.7}$$

Therefore, letting

$$\begin{aligned} W &= \begin{bmatrix} B_n^{-1}\underline{u} & \underline{B}_{n,i_{col}}^{-1} \end{bmatrix} \\ T &= \begin{bmatrix} 1 + \underline{u}^H \underline{B}_{n,i_{col}}^{-1} & -\beta_{i,i} \\ -\underline{u}^H B_n^{-1} \underline{u} & 1 + \underline{B}_{n,i_{row}}^{-1} \underline{u} \end{bmatrix} \\ Z^H &= \begin{bmatrix} \underline{B}_{n,i_{row}}^{-1} \\ \underline{u}^H B_n^{-1} \end{bmatrix} , \end{aligned}$$

expanding the second term in (5.5) using (5.7) results in

$$\begin{aligned} B_{n+1}^{-1} &= B_n^{-1} - \frac{1}{d} WTZ^H \\ &= B_n^{-1} - \frac{1}{d} \left(\sum_{i=1}^{2} \sum_{j=1}^{2} \underline{w}_i T_{ij} \underline{z}_j^H \right) \\ &= B_n^{-1} - \frac{1}{d} \left(\underline{w}_1 T_{11} \underline{z}_1^H + \underline{w}_1 T_{12} \underline{z}_2^H + \underline{w}_2 T_{21} \underline{z}_1^H + \underline{w}_2 T_{22} \underline{z}_2^H \right) \\ &= B_n^{-1} - \frac{1}{d} \begin{pmatrix} \left(B_n^{-1}\underline{u}\right) \left(1 + \underline{u}^H \underline{B}_{n,i_{col}}^{-1}\right) \left(\underline{B}_{n,i_{row}}^{-1}\right) & + \\ \left(B_n^{-1}\underline{u}\right) \left(-\beta_{i,i}\right) \left(\underline{u}^H B_n^{-1}\right) & + \\ \left(\underline{B}_{n,i_{col}}^{-1}\right) \left(-\underline{u}^H B_n^{-1} \underline{u}\right) \left(\underline{B}_{n,i_{row}}^{-1}\right) & + \\ \left(\underline{B}_{n,i_{col}}^{-1}\right) \left(1 + \underline{B}_{n,i_{row}}^{-1} \underline{u}\right) \left(\underline{u}^H B_n^{-1}\right) & \end{pmatrix} . \end{aligned}$$

Collecting similar terms results in

$$B_{n+1}^{-1} = B_n^{-1} - \frac{1}{d} \left(\begin{array}{cc} \left(B_n^{-1}\underline{u}\right) \left[\begin{array}{c} \left(1+\underline{u}^H \underline{B}_{n,i_{col}}^{-1}\right)\left(\underline{B}_{n,i_{row}}^{-1}\right) + \\ \left(-\beta_{i,i}\right)\left(\underline{u}^H B_n^{-1}\right) \end{array} \right] & + \\ \left(\underline{B}_{n,i_{col}}^{-1}\right) \left[\begin{array}{c} \left(-\underline{u}^H B_n^{-1}\underline{u}\right)\left(\underline{B}_{n,i_{row}}^{-1}\right) + \\ \left(1+\underline{B}_{n,i_{row}}^{-1}\underline{u}\right)\left(\underline{u}^H B_n^{-1}\right) \end{array} \right] & \end{array} \right) . \quad (5.8)$$

In (5.8), variables similar to those in [32] are defined to reduce the amount of redundant calculations, with

$$\underline{n} = B_n^{-1}\underline{u} \quad (5.9)$$

$$m = \underline{u}^H \underline{B}_{n,i_{col}}^{-1} = \underline{B}_{n,i_{row}}^{-1}\underline{u} \quad (5.10)$$

$$k_1 = 1 + m \quad (5.11)$$

$$k_2 = -\underline{u}^H \underline{n} \quad (5.12)$$

$$k_3 = -\beta_{i,i} \quad (5.13)$$

$$d = |k_1|^2 - k_2 k_3 . \quad (5.14)$$

Therefore, (5.8) can be simplified to

$$B_{n+1}^{-1} = B_n^{-1} - \frac{1}{d}\left[\underline{B}_{n,i_{col}}^{-1}\left(k_1\underline{n}^H + k_2\underline{B}_{n,i_{row}}^{-1}\right) + \underline{n}\left(k_3\underline{n}^H + k_1\underline{B}_{n,i_{row}}^{-1}\right)\right] . \quad (5.15)$$

The expression in (5.15) is the resulting simplified rank-2 matrix inversion update formula that can be used when a pair of transmit antennas are swapped.

Another rank-2 simplification is given in [32] based on a different definition of the U matrix. Instead of dividing by 2 to correct the $c_{i,i} = 2u_{i,1}$ term in the modification matrix C in (5.1), another method is to pre-subtract the extra contribution of $u_{i,1}$ with the following modification to the U matrix [32]

$$U = \left[\begin{array}{cc} \underline{u} - u_{i,1}\underline{e}_i & \underline{e}_i \end{array} \right] . \quad (5.16)$$

The definition for V is unchanged. In this case, the modification matrix C becomes

$$C = UV^H = \underline{u}\underline{e}_i^H + \underline{e}_i\underline{u}^H - u_{i,1}\underline{e}_i\underline{e}_i^H \ .$$

With the new definition for U, the simplified rank-2 matrix inversion update expression has the same form as (5.15) with k_2 defined differently as

$$k_2 = -\left(\underline{u}^H\underline{n} + u_{i,1}\right) \tag{5.17}$$

and the other variables from (5.9) to (5.11), (5.13), and (5.14) are the same. The computational difference is an extra addition in the k_2 term, but a multiplication by $\frac{1}{2}$ is avoided when modifying the $c_{i,i}$ term as noted in [32].

5.1.2 Reduced Complexity Receive Antenna Swapping

When a pair of receive antennas are swapped, the $H^H H$ matrix is corrected as follows

$$H_{n+1}^H H_{n+1} = H_n^H H_n + \underline{h}_{in}\underline{h}_{in}^H - \underline{h}_{out}\underline{h}_{out}^H = H_n^H H_n + S \tag{5.18}$$

where $\underline{h}_{in}$ are the channel gains associated with the receive antenna that is to be swapped in, and $\underline{h}_{out}$ are the channel gains associated with the receive antenna to be swapped out. This can be accomplished with the following rank-2 modification matrix $S = QE^H$, where Q and E are $L_{tx} \times 2$ matrices, and are defined as

$$Q = \begin{bmatrix} \underline{h}_{in} & \underline{h}_{out} \end{bmatrix} \tag{5.19}$$

$$E = \begin{bmatrix} \underline{h}_{in} & -\underline{h}_{out} \end{bmatrix} \ . \tag{5.20}$$

Due to the rank-2 nature of the modification matrix, an efficient implementation and simplification of the matrix inverse update equation can be obtained. Starting with the update equation from (5.18) and applying the Woodbury formula (4.18), results in

$$(H_{n+1}^H H_{n+1})^{-1} = (H_n^H H_n)^{-1} - (H_n^H H_n)^{-1} Q \left(I_2 + E^H (H_n^H H_n)^{-1} Q\right)^{-1} E^H (H_n^H H_n)^{-1}$$

where I_2 is the 2×2 identity matrix. Substituting Q and E from (5.19) and (5.20) into the above expression, and letting $F_n = H_n^H H_n$, $F_{n+1} = H_{n+1}^H H_{n+1}$, results in the following

$$F_{n+1}^{-1} = F_n^{-1} - F_n^{-1}\begin{bmatrix} \underline{h}_{in} & \underline{h}_{out} \end{bmatrix}\left(I_2 + \begin{bmatrix} \underline{h}_{in}^H \\ -\underline{h}_{out}^H \end{bmatrix} F_n^{-1}\begin{bmatrix} \underline{h}_{in} & \underline{h}_{out} \end{bmatrix}\right)^{-1}\begin{bmatrix} \underline{h}_{in}^H \\ -\underline{h}_{out}^H \end{bmatrix} F_n^{-1}$$

$$F_{n+1}^{-1} = F_n^{-1} - \begin{bmatrix} F_n^{-1}\underline{h}_{in} & F_n^{-1}\underline{h}_{out} \end{bmatrix}\left(I_2 + \begin{bmatrix} \underline{h}_{in}^H F_n^{-1} \\ -\underline{h}_{out}^H F_n^{-1} \end{bmatrix}\begin{bmatrix} \underline{h}_{in} & \underline{h}_{out} \end{bmatrix}\right)^{-1}\begin{bmatrix} \underline{h}_{in}^H F_n^{-1} \\ -\underline{h}_{out}^H F_n^{-1} \end{bmatrix}$$

$$F_{n+1}^{-1} = F_n^{-1} - \begin{bmatrix} F_n^{-1}\underline{h}_{in} & F_n^{-1}\underline{h}_{out} \end{bmatrix}\begin{bmatrix} 1+\underline{h}_{in}^H F_n^{-1}\underline{h}_{in} & \underline{h}_{in}^H F_n^{-1}\underline{h}_{out} \\ -\underline{h}_{out}^H F_n^{-1}\underline{h}_{in} & 1-\underline{h}_{out}^H F_n^{-1}\underline{h}_{out} \end{bmatrix}^{-1}\begin{bmatrix} \underline{h}_{in}^H F_n^{-1} \\ -\underline{h}_{out}^H F_n^{-1} \end{bmatrix}. \tag{5.21}$$

Let X be the middle matrix in the second term of (5.21), therefore

$$X = \begin{bmatrix} 1+\underline{h}_{in}^H F_n^{-1}\underline{h}_{in} & \underline{h}_{in}^H F_n^{-1}\underline{h}_{out} \\ -\underline{h}_{out}^H F_n^{-1}\underline{h}_{in} & 1-\underline{h}_{out}^H F_n^{-1}\underline{h}_{out} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

where

$$\begin{aligned} x_1 &= 1+\underline{h}_{in}^H F_n^{-1}\underline{h}_{in} \\ x_2 &= \underline{h}_{in}^H F_n^{-1}\underline{h}_{out} \\ x_3 &= -\underline{h}_{out}^H F_n^{-1}\underline{h}_{in} \\ x_4 &= 1-\underline{h}_{out}^H F_n^{-1}\underline{h}_{out}\ . \end{aligned}$$

From above, X is a 2×2 matrix, and its inverse can be found to be

$$X^{-1} = \frac{1}{x_1x_4 - x_2x_3}\begin{bmatrix} x_4 & -x_2 \\ -x_3 & x_1 \end{bmatrix} = \frac{1}{p}\begin{bmatrix} x_4 & -x_2 \\ -x_3 & x_1 \end{bmatrix}$$

Therefore, (5.21) becomes

$$F_{n+1}^{-1} = F_n^{-1} - \begin{bmatrix} F_n^{-1}\underline{h}_{in} & F_n^{-1}\underline{h}_{out} \end{bmatrix} X^{-1}\begin{bmatrix} \underline{h}_{in}^H F_n^{-1} \\ -\underline{h}_{out}^H F_n^{-1} \end{bmatrix}$$

$$F_{n+1}^{-1} = F_n^{-1} - \frac{1}{p}\left[\begin{array}{cc} F_n^{-1}\underline{h}_{in} & F_n^{-1}\underline{h}_{out} \end{array}\right]\left[\begin{array}{cc} 1-\underline{h}_{out}^H F_n^{-1}\underline{h}_{out} & -\underline{h}_{in}^H F_n^{-1}\underline{h}_{out} \\ \underline{h}_{out}^H F_n^{-1}\underline{h}_{in} & 1+\underline{h}_{in}^H F_n^{-1}\underline{h}_{in} \end{array}\right]\left[\begin{array}{c} \underline{h}_{in}^H F_n^{-1} \\ -\underline{h}_{out}^H F_n^{-1} \end{array}\right] \tag{5.22}$$

with

$$p = \left(1+\underline{h}_{in}^H F_n^{-1}\underline{h}_{in}\right)\left(1-\underline{h}_{out}^H F_n^{-1}\underline{h}_{out}\right) - \left(\underline{h}_{in}^H F_n^{-1}\underline{h}_{out}\right)\left(-\underline{h}_{out}^H F_n^{-1}\underline{h}_{in}\right)$$

To reduce the number of redundant calculations, the following vectors are defined

$$\underline{r}_1 = F_n^{-1}\underline{h}_{in} \tag{5.23}$$

$$\underline{r}_2 = F_n^{-1}\underline{h}_{out} \ . \tag{5.24}$$

Using (5.23) and (5.24), (5.22) becomes

$$F_{n+1}^{-1} = F_n^{-1} - \frac{1}{p}\left[\begin{array}{cc} \underline{r}_1 & \underline{r}_2 \end{array}\right]\left[\begin{array}{cc} 1-\underline{h}_{out}^H\underline{r}_2 & -\underline{h}_{in}^H\underline{r}_2 \\ \underline{h}_{out}^H\underline{r}_1 & 1+\underline{h}_{in}^H\underline{r}_1 \end{array}\right]\left[\begin{array}{c} \underline{r}_1^H \\ -\underline{r}_2^H \end{array}\right] \tag{5.25}$$

and p becomes

$$p = \left(1+\underline{h}_{in}^H\underline{r}_1\right)\left(1-\underline{h}_{out}^H\underline{r}_2\right) - \left(\underline{h}_{in}^H\underline{r}_2\right)\left(-\underline{h}_{out}^H\underline{r}_1\right) \ . \tag{5.26}$$

Further expanding (5.25) with the matrix relationship in (5.7) results in

$$\begin{aligned} F_{n+1}^{-1} &= F_n^{-1} - \frac{1}{p}\left[\begin{array}{c} (\underline{r}_1)(1-\underline{h}_{out}^H\underline{r}_2)(\underline{r}_1^H) \quad + \\ (\underline{r}_1)(\underline{h}_{in}^H\underline{r}_2)(\underline{r}_2^H) \quad + \\ (\underline{r}_2)(\underline{h}_{out}^H\underline{r}_1)(\underline{r}_1^H) \quad + \\ (\underline{r}_2)(-1)(1+\underline{h}_{in}^H\underline{r}_1)(\underline{r}_2^H) \end{array}\right] \\ &= F_n^{-1} - \frac{1}{p}\left[\underline{r}_1\left(j_1\underline{r}_1^H + j_2\underline{r}_2^H\right) + \underline{r}_2\left(j_3\underline{r}_1^H + j_4\underline{r}_2^H\right)\right] \end{aligned} \tag{5.27}$$

where j_1, j_2, j_3, and j_4 are constants defined as

$$j_1 = 1-\underline{h}_{out}^H\underline{r}_2 \tag{5.28}$$

$$j_2 = \underline{h}_{in}^H\underline{r}_2 \tag{5.29}$$

$$j_3 = \underline{h}_{out}^H \underline{r}_1 \tag{5.30}$$

$$j_4 = -(1 + \underline{h}_{in}^H \underline{r}_1) \; . \tag{5.31}$$

The determinant p can be expressed in terms of the variables as

$$p = j_2 j_3 - j_1 j_4 \; . \tag{5.32}$$

The expression (5.27) is the resulting simplified rank-2 matrix inversion update formula that can be used when a pair of receive antennas are swapped.

The amount of simplification is less than that for the case of swapping transmit antennas, because the structure of the modification matrix in the transmit antenna swapping contains the $\underline{e}_i$ vector, that allows the final expression (5.15) to be further simplified.

5.2 Fast Random Antenna Selection with Antenna Swapping Algorithm

In the previous sections, simplified expressions for the transmit and receiving operations are presented. These expressions can be used to update the matrix inverse in step 6 of the RAS-AS algorithm in the previous chapter, and this version is called the Fast RAS-AS algorithm. The pseudocode of the Fast RAS-AS algorithm is presented in Table 5.1. The Fast RAS-AS algorithm achieves the same average BER performance and expected number of iterations as the RAS-AS algorithm, and these will be examined in the later sections of the chapter.

Table 5.1: Fast RAS-AS algorithm pseudocode.

	Initialization:
1	Randomly select L_{tx} transmit, L_{rx} receive antennas to form H_0
2	Calculate $\left(H_0^H H_0\right)^{-1}$ and g_k^2 for $k = 1, \ldots, L_{tx}$ using (2.12)
3	Initialize $\left(H^H H\right)_{best}^{-1} = \left(H_0^H H_0\right)^{-1}$ and $\text{AvgBER}_{best} = \text{AvgBER}_0$ using (2.13)
	Main Loop:
4	Iterate through the antenna configurations (n^{th} Step) $\{$
5	Swap a pair of transmit antennas with probability p_{swap} or
	swap a pair of receive antennas with probability $(1 - p_{swap})$ to form H_n
6	Update $\left(H_n^H H_n\right)^{-1}$ with (5.15) or (5.27) for transmit antenna swapping
	or receive antenna swapping, respectively.
7	Calculate g_k^2 for $k = 1, \ldots, L_{tx}$ using (2.12)
8	Calculate AvgBER_n using (2.13)
9	if ($\text{AvgBER}_n < \text{AvgBER}_{best}$) then
10	Current antenna configuration is the best : Set $\left(H^H H\right)_{best}^{-1} = \left(H_n^H H_n\right)^{-1}$
11	$\text{AvgBER}_{best} = \text{AvgBER}_n$
	end if $\}$

5.3 Greedy Fast Random Antenna Selection with Antenna Swapping Algorithm

A greedy version of the Fast RAS-AS algorithm is presented in this section, and it is introduced as a generic greedy algorithm mainly for comparison purposes to the Fast RAS-AS algorithm. The greedy algorithm first selects a random subset of antennas, and alternates between swapping transmit and receive antennas. The algorithm swaps through all the different pairs of antennas on one side and the antenna selection criterion is evaluated. If the average BER performance of the system improves after swapping a pair of antennas, the algorithm keeps track of this configuration and sets it as the best antenna configuration. The new antenna is swapped back, resulting in the original antenna configuration, and another antenna is swapped in. After swapping through all the different pairs of antennas, the algorithm uses the best antenna configuration, and the same process is repeated for the antennas on the other side of the communication link. The greedy algorithm terminates when both sides cannot find an antenna that would improve the average BER performance of the system after swapping through all pairs of transmit and receive antennas. The pseudocode of the Greedy Fast RAS-AS algorithm is presented in Table 5.2.

Similar to the Fast RAS-AS algorithm, the computational complexity of the greedy algorithm is dominated by the number of matrix inversion update operations after fast antenna swapping. The computation required for the fast antenna swapping operations are presented in the following sections, and the expected number of iterations of the greedy algorithm is determined through simulations. The average BER performance of the greedy algorithm will be presented in the later sections of the chapter.

Table 5.2: Greedy Fast RAS-AS algorithm pseudocode.

	Initialization:
1	Randomly select L_{tx} transmit, L_{rx} receive antennas to form H_0
2	Calculate $\left(H_0^H H_0\right)^{-1}$ and g_k^2 for $k = 1, ..., L_{tx}$ using (2.12)
3	Initialize $\left(H^H H\right)_{best}^{-1} = \left(H_0^H H_0\right)^{-1}$ and $\text{AvgBER}_{best} = \text{AvgBER}_0$ using (2.13)
	Main Loop: (Alternate between transmit and receive antennas) {
4	$Transmit_{foundBetter}$ = 0, $Receive_{foundBetter}$ = 0.
5	Loop: Swap through all pairs of antennas for each side (n^{th} Step) {
6	Swap a pair of antennas.
7	Update $\left(H_n^H H_n\right)^{-1}$ with (5.15) or (5.27) for transmit antenna swapping
8	Calculate g_k^2 for $k = 1, ..., L_{tx}$ using (2.12)
9	Calculate AvgBER_n using (2.13)
10	if ($\text{AvgBER}_n < \text{AvgBER}_{best}$) then
11	Current antenna configuration is the best : Set $\left(H^H H\right)_{best}^{-1} = \left(H_n^H H_n\right)^{-1}$
12	$\text{AvgBER}_{best} = \text{AvgBER}_n$
13	$Transmit_{foundBetter}$ = 1 for transmit antenna swapping, or
	$Receive_{foundBetter}$ = 1 for receive antenna swapping.
	end if
14	Swap back to original configuration in the n^{th} step of the algorithm.
	}
15	if ($Transmit_{foundBetter}$ == 0 and $Receive_{foundBetter}$ == 0) then
16	Cannot find better antenna to swap in, terminate algorithm.
	end if
	}

5.4 Computational Complexity

This section analyzes the computational complexity of the Fast RAS-AS algorithm in terms of the number of additions and multiplications. Let a complex(real) multiplication and complex(real) addition be denoted $\mathscr{C}_m(\mathscr{R}_m)$ and $\mathscr{C}_a(\mathscr{R}_a)$, respectively. The analysis will first identify the initial overhead required at the start up of the algorithm, and then the required computation of performing the transmit or receive antenna swapping operation will be analyzed.

5.4.1 Initialization Overhead

At the startup of the Fast RAS-AS algorithm, a random set of L_{tx} transmit antennas, and L_{rx} receive antennas are chosen to form the $L_{rx} \times L_{tx}$ MIMO channel matrix, H. The dominant initialization overhead is the computation of the inverse of the $H^H H$ matrix. Once the initial inverse is computed, the simplified update equations in (5.15) or (5.27) can be used to update the matrix inverse as antennas are swapped.

5.4.2 Transmit Antenna Swapping Computation

This section will examine the required number of computation for the transmit antenna swapping with the rank-2 simplification. Starting with the expression in (5.15), the update equation for the inverse after swapping a pair of transmit antennas is

$$B_{n+1}^{-1} = B_n^{-1} - \frac{1}{d}\left[\underline{B}_{n,i_{col}}^{-1}\left(k_1 \underline{n}^H + k_2 \underline{B}_{n,i_{row}}^{-1}\right) + \underline{n}\left(k_3 \underline{n}^H + k_1 \underline{B}_{n,i_{row}}^{-1}\right)\right]$$

with variables $\underline{n}$, m, k_1, k_2, k_3, and d defined in (5.9) to (5.14). The amount of computation for the variables are tabulated in Table 5.3. The variables are computed in the order given.

The number of multiplications and additions for m is zero because its components can

Table 5.3: Transmit antenna swap - variable computation.

Variable	Dimension	Multiplications	Additions
$\underline{u}$ from (4.13) , (4.15)	$(L_{tx} \times 1)$ $\underline{u}$	$\mathscr{R}_m$	$(L_{tx}-1)\mathscr{C}_a+\mathscr{R}_a$
$\underline{n} = B_n^{-1}\underline{u}$	$(L_{tx} \times L_{tx})$ B_n^{-1} $(L_{tx} \times 1)$ $\underline{u}$	$(L_{tx}^2-2L_{tx}+1)\mathscr{C}_m$ $+(4L_{tx}-3)\mathscr{R}_m$	$(L_{tx}^2-L_{tx}-1)\mathscr{C}_a$ $+\mathscr{R}_a$
$m = \underline{u}^H \underline{B}_{n,i_{col}}^{-1}$ $= \underline{B}_{n,i_{row}}^{-1}\underline{u}$	$(1 \times L_{tx})$ $\underline{u}^H$ $(L_{tx} \times 1)$ $\underline{B}_{n,i_{col}}^{-1}$	0	0
$k_1 = 1+m$	(1×1) m	0	$\mathscr{R}_a$
$k_2 = -\underline{u}^H\underline{n}$	$(1 \times L_{tx})$ $\underline{u}^H$ $(L_{tx} \times 1)$ $\underline{n}$	$(2L_{tx}-1)\mathscr{R}_m$	$2(L_{tx}-1)\mathscr{R}_a$
$k_3 = -\beta_{i,i}$	(1×1) $\beta_{i,i}$	0	0
$d = \lvert k_1\rvert^2 - k_2k_3$	(1×1) k_1, k_2, k_3	$3\mathscr{R}_m$	$2\mathscr{R}_a$
Total		$(L_{tx}^2-2L_{tx}+1)\mathscr{C}_m$ $+6L_{tx}\mathscr{R}_m$	$(L_{tx}^2-2)\mathscr{C}_a$ $+(2L_{tx}+3)\mathscr{R}_a$

be obtained after $\underline{n}$ has been computed. The last row of Table 5.3 shows the total computation required for the variables. Using the above computational requirements of the variables, Table 5.4 shows the total amount of computation required for updating the inverse expression for transmit antenna swapping. The $\frac{1}{d}$ constant in expression (5.15) is expanded into the brackets as follows in order to reduce the number of multiplications

$$B_{n+1}^{-1} = B_n^{-1} - \left[\underline{B}_{n,i_{col}}^{-1} \left(\frac{k_1}{d} \underline{n}^H + \frac{k_2}{d} \underline{B}_{n,i_{row}}^{-1} \right) + \underline{n} \left(\frac{k_3}{d} \underline{n}^H + \frac{k_1}{d} \underline{B}_{n,i_{row}}^{-1} \right) \right] .$$

The computational requirement for the different components in (5.15) are broken up and computed in the order in Table 5.4.

From previous section, $B_n = H_n^H H_n$ and is a Hermitian matrix. Therefore, its inverse, B_n^{-1} is also Hermitian. Taking this into account, only the diagonal and upper triangular elements have to be computed for variables j, k, l, and B_{n+1}^{-1}. The computational complexity analysis also takes into account the real elements when multiplying and adding vectors and matrices.

The total number of multiplications and additions required to update the matrix inverse is given in the last row of Table 5.4, which are $(L_{tx}^2 + L_{tx} - 1)\mathscr{C}_m + (8L_{tx} + 5)\mathscr{R}_m$ and $(L_{tx}^2 + L_{tx} - 2)\mathscr{C}_a + (4L_{tx} + 2)\mathscr{R}_a$, respectively.

The Q-function in the average BER expression can be implemented efficiently in the form of a look-up table. An additional $L_{tx} - 1$ additions and 1 more multiplication are required to calculate the average BER from the L_{tx} data streams using expression (2.13).

Table 5.5 summarizes the computational complexity for the cases of calculating the variables and updating the matrix inverse. In each case, the computational complexity is of order $O(L_{tx}^2)$.

Table 5.4: Transmit antenna swap - inverse update computation.

Variable	Dimension	Multiplications	Additions
$a = \frac{k_1}{d}$	$(1 \times 1)\ d, k_1$	$2\mathscr{R}_m$	0
$b = \frac{k_2}{d}$	$(1 \times 1)\ d, k_2$	$\mathscr{R}_m$	0
$c = \frac{k_3}{d}$	$(1 \times 1)\ d, k_3$	$\mathscr{R}_m$	0
$\underline{d} = a\underline{n}^H$	$(1 \times 1)\ a,\ (1 \times L_{tx})\ \underline{n}^H$	$L_{tx}\mathscr{C}_m$	0
$\underline{e} = b\underline{B}_{n,i_{row}}^{-1}$	$(1 \times 1)\ b,\ (1 \times L_{tx})\ \underline{B}_{n,i_{row}}^{-1}$	$(2L_{tx} - 1)\mathscr{R}_m$	0
$\underline{f} = c\underline{n}^H$	$(1 \times 1)\ c,\ (1 \times L_{tx})\ \underline{n}^H$	$2L_{tx}\mathscr{R}_m$	0
$\underline{g} = a\underline{B}_{n,i_{row}}^{-1}$	$(1 \times 1)\ a,\ (1 \times L_{tx})\ \underline{B}_{n,i_{row}}^{-1}$	$(L_{tx} - 1)\mathscr{C}_m + 2\mathscr{R}_m$	0
$\underline{h} = \underline{d} + \underline{e}$	$(1 \times L_{tx})\ \underline{d}, \underline{e}$	0	$(L_{tx} - 1)\mathscr{C}_a + \mathscr{R}_a$
$\underline{i} = \underline{f} + \underline{g}$	$(1 \times L_{tx})\ \underline{f}, \underline{g}$	0	$(L_{tx} - 1)\mathscr{C}_a + \mathscr{R}_a$
$j = \underline{B}_{n,i_{col}}^{-1}\underline{h}$	$(L_{tx} \times 1)\ \underline{B}_{n,i_{col}}^{-1},\ (1 \times L_{tx})\ \underline{h}$	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_m + 2L_{tx}\mathscr{R}_m$	$L_{tx}\mathscr{R}_a$
$k = \underline{n}\underline{i}$	$(L_{tx} \times 1)\ \underline{n},\ (1 \times L_{tx})\ \underline{i}$	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_m + 2L_{tx}\mathscr{R}_m$	$L_{tx}\mathscr{R}_a$
$l = j + k$	$(L_{tx} \times L_{tx})\ j, k$	0	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_a + L_{tx}\mathscr{R}_a$
$B_{n+1}^{-1} = B_n^{-1} - l$	$(L_{tx} \times L_{tx})\ B_n^{-1}, l$	0	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_a + L_{tx}\mathscr{R}_a$
Total		$(L_{tx}^2 + L_{tx} - 1)\mathscr{C}_m$ $+(8L_{tx} + 5)\mathscr{R}_m$	$(L_{tx}^2 + L_{tx} - 2)\mathscr{C}_a$ $+(4L_{tx} + 2)\mathscr{R}_a$

Table 5.5: Transmit antenna swap - computation summary.

Calculation	Multiplications	Additions
Variables (5.9) to (5.14)	$(L_{tx}^2 - 2L_{tx} + 1)\mathscr{C}_m + 6L_{tx}\mathscr{R}_m$	$(L_{tx}^2 - 2)\mathscr{C}_a + (2L_{tx} + 3)\mathscr{R}_a$
Matrix Inverse (5.15)	$(L_{tx}^2 + L_{tx} - 1)\mathscr{C}_m + (8L_{tx} + 5)\mathscr{R}_m$	$(L_{tx}^2 + L_{tx} - 2)\mathscr{C}_a + (4L_{tx} + 2)\mathscr{R}_a$
Total	$(2L_{tx}^2 - L_{tx})\mathscr{C}_m + (14L_{tx} + 5)\mathscr{R}_m$	$(2L_{tx}^2 + L_{tx} - 4)\mathscr{C}_a + (6L_{tx} + 5)\mathscr{R}_a$

Table 5.6: Receive antenna swap - variable computation.

Variable	Dimension	Multiplications	Additions
$\underline{r}_1 = F_n^{-1}\underline{h}_{in}$	$(L_{tx} \times L_{tx})\ F_n^{-1}$ $(L_{tx} \times 1)\ \underline{h}_{in}$	$L_{tx}(L_{tx}-1)\mathscr{C}_m + 2L_{tx}\mathscr{R}_m$	$L_{tx}(L_{tx}-1)\mathscr{C}_a$
$\underline{r}_2 = F_n^{-1}\underline{h}_{out}$	$(L_{tx} \times L_{tx})\ F_n^{-1}$ $(L_{tx} \times 1)\ \underline{h}_{out}$	$L_{tx}(L_{tx}-1)\mathscr{C}_m + 2L_{tx}\mathscr{R}_m$	$L_{tx}(L_{tx}-1)\mathscr{C}_a$
$j_1 = 1 - \underline{h}_{out}^H\underline{r}_2$	$(1 \times L_{tx})\ \underline{h}_{out}^H$, $(L_{tx} \times 1)\ \underline{r}_2$	$2L_{tx}\mathscr{R}_m$	$2L_{tx}\mathscr{R}_a$
$j_2 = \underline{h}_{in}^H\underline{r}_2$	$(1 \times L_{tx})\ \underline{h}_{in}^H$, $(L_{tx} \times 1)\ \underline{r}_2$	$L_{tx}\mathscr{C}_m$	$(L_{tx}-1)\mathscr{C}_a$
$j_3 = \underline{h}_{out}^H\underline{r}_1$	$(1 \times L_{tx})\ \underline{h}_{out}^H$, $(L_{tx} \times 1)\ \underline{r}_1$	0	0
$j_4 = \underline{h}_{in}^H\underline{r}_1 - 1$	$(1 \times L_{tx})\ \underline{h}_{in}^H$, $(L_{tx} \times 1)\ \underline{r}_1$	$2L_{tx}\mathscr{R}_m$	$2L_{tx}\mathscr{R}_a$
$p = j_2 j_3 - j_1 j_4$	$(1 \times 1)\ j_1,\ j_2,\ j_3,\ j_4$	$3\mathscr{R}_m$	$2\mathscr{R}_a$
Total		$(2L_{tx}^2 - L_{tx})\mathscr{C}_m$ $+(8L_{tx}+3)\mathscr{R}_m$	$(2L_{tx}^2 - L_{tx} - 1)\mathscr{C}_a$ $+(4L_{tx}+2)\mathscr{R}_a$

5.4.3 Receive Antenna Swapping Computation

This section examines the required amount of computation for receive antenna swapping with rank-2 simplification. Starting with the expression in (5.27), the update equation for the inverse after swapping a pair of transmit antennas is

$$F_{n+1}^{-1} = F_n^{-1} - \frac{1}{p}\left[\underline{r}_1\left(j_1\underline{r}_1^H + j_2\underline{r}_2^H\right) + \underline{r}_2\left(j_3\underline{r}_1^H + j_4\underline{r}_2^H\right)\right]$$

with variables $\underline{r}_1$, $\underline{r}_2$, j_1, j_2, j_3, j_4, and p defined in (5.23), (5.24), and (5.28) to (5.32). The amount of computation for the variables are tabulated in the Table 5.6. The variables are computed in the order given.

The number of multiplications and additions for j_3 is zero because j_3 is the conjugate of j_2. The last row of Table 5.6 shows the total computation required for the variables. Using the above computational requirements, Table 5.7 shows the total amount of computation required for updating the inverse expression for receive antenna swapping. The $\frac{1}{p}$ constant in expression (5.27) is expanded into the brackets as follows in order to reduce the number of multiplications

$$F_{n+1}^{-1} = F_n^{-1} - \left[\underline{r}_1\left(\frac{j_1}{p}\underline{r}_1^H + \frac{j_2}{p}\underline{r}_2^H\right) + \underline{r}_2\left(\frac{j_3}{p}\underline{r}_1^H + \frac{j_4}{p}\underline{r}_2^H\right)\right] .$$

The computational requirement for the different components in (5.27) are broken up and computed in the order in Table 5.7.

From previous section, $F_n = H_n^H H_n$ and is a Hermitian matrix. Therefore, its inverse, F_n^{-1} is also Hermitian. Taking this into account, only the diagonal and upper triangular elements have to be computed for the variables k, l, m, and F_{n+1}^{-1}. The computational complexity analysis also takes into account the real elements when multiplying and adding vectors and matrices.

The total number of multiplications and additions required to update the matrix inverse is given in the last row of Table 5.7, which are $(L_{tx}^2 + L_{tx})\mathscr{C}_m + (8L_{tx} + 4)\mathscr{R}_m$ and $(L_{tx}^2 + L_{tx})\mathscr{C}_a + 4L_{tx}\mathscr{R}_a$, respectively. After updating the matrix inverse, the set of g_k^2 for $k = 1, \ldots, L_{tx}$ can be obtained by inverting the (k,k) elements in the updated inverse.

The Q-function in the average BER expression can be implemented efficiently in the form of a look-up table. An additional $L_{tx} - 1$ additions and 1 more multiplication are required to calculate the average BER from the L_{tx} data streams using expression (2.13).

Table 5.8 summarizes the computational complexity for the cases of calculating the variables and updating the matrix inverse. In each case, the computational complexity is of order $O(L_{tx}^2)$.

Table 5.7: Receive antenna swap - inverse update computation.

Variable	Dimension	Multiplications	Additions
$a = \frac{j_1}{p}$	$(1\times1)\ p,\ j_1$	$\mathscr{R}_m$	0
$b = \frac{j_2}{p}$	$(1\times1)\ p,\ j_2$	$2\mathscr{R}_m$	0
$c = \frac{j_3}{p}$	$(1\times1)\ p,\ j_3$	0	0
$d = \frac{j_4}{p}$	$(1\times1)\ p,\ j_4$	$\mathscr{R}_m$	0
$\underline{e} = a\underline{r}_1^H$	$(1\times1)\ a,\ (1\times L_{tx})\ \underline{r}_1^H$	$2L_{tx}\mathscr{R}_m$	0
$\underline{f} = b\underline{r}_2^H$	$(1\times1)\ b,\ (1\times L_{tx})\ \underline{r}_2^H$	$L_{tx}\mathscr{C}_m$	0
$\underline{g} = c\underline{r}_1^H$	$(1\times1)\ c,\ (1\times L_{tx})\ \underline{r}_1^H$	$L_{tx}\mathscr{C}_m$	0
$\underline{h} = d\underline{r}_2^H$	$(1\times1)\ d,\ (1\times L_{tx})\ \underline{r}_2^H$	$2L_{tx}\mathscr{R}_m$	0
$\underline{i} = \underline{e} + \underline{f}$	$(1\times L_{tx})\ \underline{e},\ \underline{f}$	0	$L_{tx}\mathscr{C}_a$
$\underline{j} = \underline{g} + \underline{h}$	$(1\times L_{tx})\ \underline{g},\ \underline{h}$	0	$L_{tx}\mathscr{C}_a$
$k = \underline{r}_1\underline{i}$	$(L_{tx}\times1)\ \underline{r}_1,\ (1\times L_{tx})\ \underline{i}$	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_m + 2L_{tx}\mathscr{R}_m$	$L_{tx}\mathscr{R}_a$
$l = \underline{r}_2\underline{j}$	$(L_{tx}\times1)\ \underline{r}_2,\ (1\times L_{tx})\ \underline{j}$	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_m + 2L_{tx}\mathscr{R}_m$	$L_{tx}\mathscr{R}_a$
$m = k + l$	$(L_{tx}\times L_{tx})\ k,\ l$	0	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_a + L_{tx}\mathscr{R}_a$
$F_{n+1}^{-1} = F_n^{-1} - m$	$(L_{tx}\times L_{tx})\ F_n^{-1},\ m$	0	$\frac{L_{tx}(L_{tx}-1)}{2}\mathscr{C}_a + L_{tx}\mathscr{R}_a$
Total		$(L_{tx}^2 + L_{tx})\mathscr{C}_m$ $+(8L_{tx}+4)\mathscr{R}_m$	$(L_{tx}^2 + L_{tx})\mathscr{C}_a$ $+4L_{tx}\mathscr{R}_a$

Table 5.8: Receive antenna swap - computation summary.

Calculation	Multiplications	Additions
Variables (5.23) to (5.24), (5.28) to (5.32)	$(2L_{tx}^2 - L_{tx})\mathscr{C}_m + (8L_{tx}+3)\mathscr{R}_m$	$(2L_{tx}^2 - L_{tx} - 1)\mathscr{C}_a + (4L_{tx}+2)\mathscr{R}_a$
Matrix Inverse (5.27)	$(L_{tx}^2 + L_{tx})\mathscr{C}_m + (8L_{tx}+4)\mathscr{R}_m$	$(L_{tx}^2 + L_{tx})\mathscr{C}_a + 4L_{tx}\mathscr{R}_a$
Total	$(3L_{tx}^2)\mathscr{C}_m + (16L_{tx}+7)\mathscr{R}_m$	$(3L_{tx}^2 - 1)\mathscr{C}_a + (8L_{tx}+2)\mathscr{R}_a$

Table 5.9: Gauss-Jordan elimination computational complexity.

Multiplications	Additions
$(L_{tx}^3)\mathscr{C}_m$	$(L_{tx}^3 - 2L_{tx}^2 + L_{tx})\mathscr{C}_a$

5.5 Matrix Inversion by Gauss-Jordan Elimination

In the case that the inverse of the matrix required to find the power gains g_k^2 in (2.12) is not updated via antenna swapping, the matrix inversion can be found using Gauss-Jordan elimination. Table 5.9 presents the computational complexity of the Gauss-Jordan elimination method required for calculating the inverse of a $L_{tx} \times L_{tx}$ matrix [33]. It can be seen from Table 5.9 that finding the inverse of a matrix using the Gauss-Jordan elimination method is of $O(L_{tx}^3)$ complexity [33].

In order to compare the computational complexity of the Fast RAS-AS algorithm to the computation required in performing a full complexity ES using the Gauss-Jordan elimination method for finding matrix inverses, the complex multiplication and addition expressions in Table 5.5, 5.8, and 5.9 are expressed in terms of real multiplications and additions. Each complex multiplication involves $4\mathscr{R}_m$ and $2\mathscr{R}_a$, and each complex addition involves $2\mathscr{R}_a$. Table 5.10 presents the computational complexity of the transmit antenna swapping operation, receive antenna swapping operation, and the matrix inversion using Gauss-Jordan elimination in terms of real multiplications and additions.

After expressing the computational complexity in terms of real multiplications and additions, Table 5.11, 5.12, and 5.13 present the computation required to perform transmit antenna swapping, receive antenna swapping, and Gauss-Jordan elimination in one Fast RAS-AS or full complexity ES iteration for different values of L_{tx}.

In subsequent sections, the expected number of iterations of the RAS-AS algorithm will be analyzed. Using the expected number of iterations and the computational complexity in

Table 5.10: Computation summary of transmit antenna swapping, receive antenna swapping, and Gauss-Jordan elimination.

	Multiplications	Additions
Transmit Antenna Swapping	$(8L_{tx}^2 + 10L_{tx} + 5)\mathscr{R}_m$	$(8L_{tx}^2 + 6L_{tx} - 3)\mathscr{R}_a$
Receive Antenna Swapping	$(12L_{tx}^2 + 16L_{tx} + 7)\mathscr{R}_m$	$(12L_{tx}^2 + 8L_{tx})\mathscr{R}_a$
Gauss-Jordan Elimination	$(4L_{tx}^3)\mathscr{R}_m$	$(5L_{tx}^3 - 4L_{tx}^2 + 2L_{tx})\mathscr{R}_a$

Table 5.11: Transmit antenna swapping computation for different L_{tx}.

L_{tx}	Transmit Antenna Swap Multiplications	Transmit Antenna Swap Additions
2	57	41
3	107	87
4	173	149
5	255	227
6	353	321

Table 5.12: Receive antenna swapping computation for different L_{tx}.

L_{tx}	Receive Antenna Swap Multiplications	Receive Antenna Swap Additions
2	87	64
3	163	132
4	263	224
5	387	340
6	535	480

Table 5.13: Gauss-Jordan elimination computation for different L_{tx}.

L_{tx}	Gauss-Jordan Multiplications	Gauss-Jordan Additions
2	32	28
3	108	105
4	256	264
5	500	535
6	864	948

Table 5.11, 5.12, and 5.13, the computational savings of the Fast RAS-AS algorithm over a full complexity ES will be examined.

5.6 Performance of a Deterministic Swapping Sequence

In a deterministic swapping sequence, a pair of transmit or receive antennas are swapped according to a predefined swapping pattern in each iteration. Let a MIMO system with N_{tx} number of transmit antennas, N_{rx} number of receive antennas, L_{tx} number of transmit RF chains, and L_{rx} number of receive RF chains be denoted (N_{tx}:N_{rx},L_{tx}:L_{rx}). A possible transmit and receive antenna swapping sequence for a (4:8,2:4) system was presented in Tables 4.2 and 4.1, respectively. In the following, the expected number of iterations for the RAS-AS algorithm using a deterministic swapping sequence to find the optimal antenna configuration is first analyzed, and the average BER performance is simulated for the cases of a MIMO channel with uncorrelated antennas and a MIMO channel with correlation between the transmit antennas.

5.6.1 Expected Number of Iterations

For a given channel realization, one of the antenna configurations in the deterministic swapping sequence has the lowest average BER, which we term the optimal configuration. For RAS-AS, a random configuration is selected initially. The expected distance between the starting and optimal configurations refers to the expected number of iterations.

Let $N = \binom{N_{tx}}{L_{tx}}\binom{N_{rx}}{L_{rx}}$ represent the total number of possible antenna configurations. Assume each antenna configuration is equally likely to be selected as the starting configuration, and therefore, each configuration has a $1/N$ probability of being selected. Assuming the configurations are labeled by indices, the probability of selecting the antenna configuration n is $P_{config}(n) = 1/N$, where $n = 1,...,N$. The possible distances from the optimal configuration to any starting antenna configuration would be in the range of 0 to $N-1$. Therefore, on average, the expected distance to the optimal configuration from any antenna configuration, or the expected number of iterations is

$$E[iteration] = \sum_{k=0}^{N-1} kP_{config}(k) = \frac{1}{N}\sum_{k=0}^{N-1} k = \frac{1}{N}\frac{1}{2}(N-1)((N-1)+1) = \frac{N-1}{2}\ . \quad (5.33)$$

Therefore, the expected number of iterations for the RAS-AS algorithm with a deterministic swapping sequence is $(N-1)/2$, where $N = \binom{N_{tx}}{L_{tx}}\binom{N_{rx}}{L_{rx}}$ is the total number of antenna configurations. This result is expected and it makes sense intuitively.

5.6.2 Simulation Results

For a (4:8,2:4) MIMO system, the average BER performance of the RAS-AS algorithm is simulated using 1000 channel realizations. Figure 5.1 presents the average BER performance of the RAS-AS algorithm when a deterministic swapping sequence is used. It is observed in Figure 5.1 that after performing 50% of the exhaustive search iterations, which is the expected number of iterations for a deterministic swapping sequence, the RAS-AS

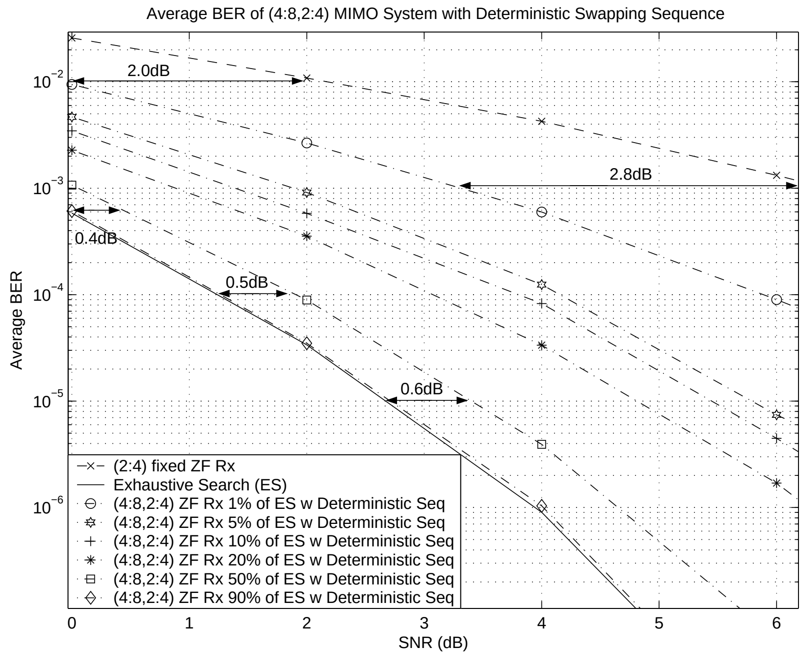

Figure 5.1: Average BER of (4:8,2:4) MIMO system with deterministic swapping sequences.

algorithm is able to find an antenna configuration that can achieve an average BER performance that is about 0.4dB, 0.5dB, and 0.6dB away from the optimal performance achieved with exhaustive search for average BERs of 10^{-3}, 10^{-4}, and 10^{-5}, respectively. Therefore, after performing the expected number of iterations found from the analysis, it is possible for the RAS-AS algorithm to find a set of antennas that can achieve close to optimal performance. It is also observed that a 2.0dB to 2.8dB gain in average BER performance is achieved over using a fixed subset of antennas, by performing only a small percentage of possible RAS-AS iterations (1%), at average BERs of 10^{-2} and 10^{-3}, respectively.

The correlation matrix in (2.10) is used in the simulation of the performance of the RAS-AS algorithm under correlated transmit antenna channel conditions. Figure 5.2 presents the RAS-AS average BER performance for a (4:8,2:4) system using a deterministic swapping sequence with different percentages of exhaustive search iterations.

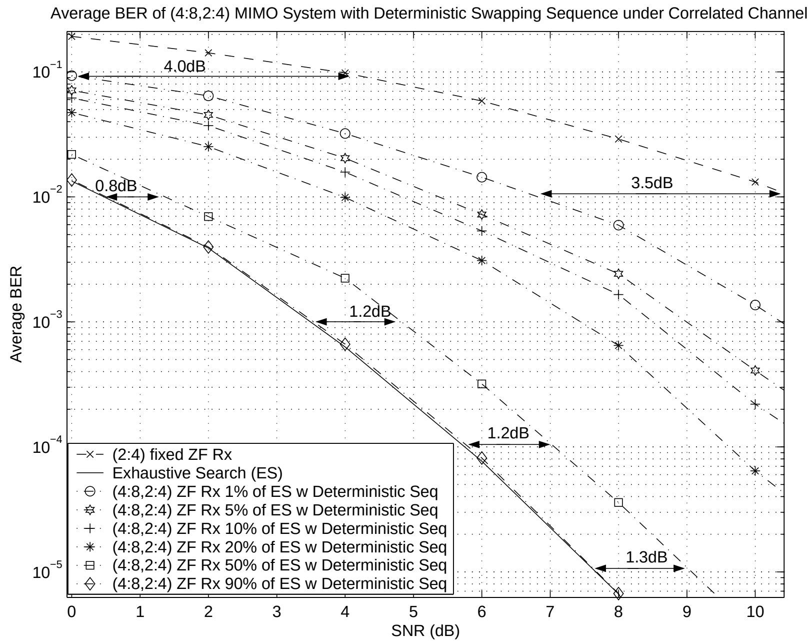

Figure 5.2: Average BER of (4:8,2:4) MIMO system with deterministic swapping sequence under spatially correlated channels.

From Figure 5.2, it can be seen that under a correlated transmit antenna condition, the system exhibits a higher average BER across all SNRs when compared with the performance under an uncorrelated channel in Figure 5.1. The slopes of the average BER curves are less steep, as the diversity order is reduced due to correlation between antennas. It is observed that the 50% average BER curve for a deterministic swapping sequence is about 0.8dB, 1.2dB, 1.2dB, and 1.3dB away from the exhaustive search performance under transmit antenna correlation for average BERs of 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} respectively. This suggests that the RAS-AS algorithm behaves similarly in terms of computation versus performance under uncorrelated and correlated channel conditions. Similar to the uncorrelated case, a large average BER performance gain is observed from performing only 1% of the ES iterations when compared with the average BER achieved using a fixed subset of antennas. The observed gain for the correlated case is 4.0dB at an average BER of 10^{-1} and

3.5dB at an average BER of 10^{-2}.

5.7 Performance of a Random Swapping Sequence

For a random swapping sequence, a single pair of transmit or receive antennas are randomly swapped in each iteration. The behavior of the RAS-AS algorithm using random swapping can be modelled as a random walk on a homogeneous stationary Markov chain. The following sections will develop this model, and the expected number of iterations of the RAS-AS algorithm using a random swapping sequence to the different states in the Markov chain model is found. The average BER performance of the RAS-AS algorithm using a random swapping sequence is also simulated at the end of the chapter for both uncorrelated and correlated transmit antenna MIMO channels.

5.7.1 Markov Chain Model for Analysis

The combinatorial problem of random antenna selection at either the transmit or receive side can be classically described as one of selecting balls from an urn, where an initial set of L_{tx} or L_{rx} balls are randomly selected, respectively. For random antenna swapping, each iteration would randomly replace one of the selected balls with another randomly selected ball from the urn. Assume the optimal set of balls is colored red and all the other balls are colored white. Therefore, there would be two urns, one for the transmit side and one for the receive side, each having L_{tx} red balls and $N_{tx} - L_{tx}$ white balls, and L_{rx} red balls and $N_{rx} - L_{rx}$ white balls, respectively.

The following discussion will develop the model for the urn on the transmit side. A similar development can be used for the receive side. For an urn with N_{tx} balls, with L_{tx} of them being red, there can be $L_{tx}+1$ different states, each having different numbers of red

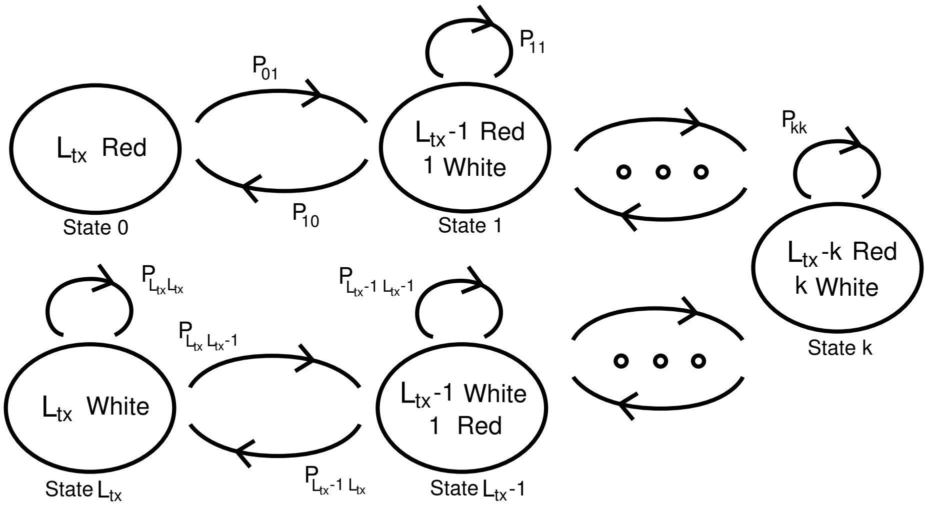

Figure 5.3: Transmit side RAS-AS Markov chain model.

and white balls. The states are connected by edges with different transition probabilities, p_{ij}, and is illustrated in Figure 5.3.

With random selection, the algorithm can start in any of the states with different starting probabilities. Each edge indicates an allowed state transition. Each state has a probability to remain in the same state after each swapping iteration. The state with L_{tx} red balls and 0 white ball is the state with the optimal configuration (State 0). State k is the state with k white balls and $L_{tx} - k$ red balls. The system in Figure 5.3 is a discrete-time finite state Markov chain with transition probabilities that depend only on the current time instant, regardless of the behavior of the algorithm in previous time instances. The Markov chain is also homogeneous in time with stationary transition probabilities, and is also irreducible as any state is reachable from any other state with different numbers of state transitions. The transition matrix of the above Markov chain has the following form.

$$P_{tx} = \begin{pmatrix} 0 & p_{01} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ p_{10} & p_{11} & p_{12} & 0 & \cdots & \dots & \dots & \dots & 0 \\ \vdots & & & & & & & & \vdots \\ 0 & \cdots & 0 & p_{kk-1} & p_{kk} & p_{kk+1} & 0 & \dots & 0 \\ \vdots & & & & & & & & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & p_{L_{tx}-1L_{tx}-2} & p_{L_{tx}-1L_{tx}-1} & p_{L_{tx}-1L_{tx}} \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & p_{L_{tx}L_{tx}-1} & p_{L_{tx}L_{tx}} \end{pmatrix}. \tag{5.34}$$

Each state can move to its neighboring states with some nonzero transition probability. Other than the optimal state (State 0), all the other states have a probability of remaining in the same state after a swapping operation. The elements of the transition matrix are given as follows

$$p_{i,i} = \left(\frac{L_{tx}-i}{L_{tx}}\right)\left(\frac{i}{N_{tx}-L_{tx}}\right) + \left(\frac{N_{tx}-L_{tx}-i}{N_{tx}-L_{tx}}\right)\left(\frac{i}{L_{tx}}\right) \tag{5.35}$$

$$p_{i,i+1} = \left(\frac{L_{tx}-i}{L_{tx}}\right)\left(\frac{N_{tx}-L_{tx}-i}{N_{tx}-L_{tx}}\right) \tag{5.36}$$

$$p_{i,i-1} = \left(\frac{i}{L_{tx}}\right)\left(\frac{i}{N_{tx}-L_{tx}}\right) \tag{5.37}$$

with i representing the number of white balls, and $i = 0, \dots, L_{tx}$. For any row of the transition matrix, the sum of the transition probabilities is 1. The above expressions are derived in the following paragraphs.

The transition probability $p_{i,i}$ represents the probability of remaining in the same state with i white balls after a swapping operation. This event can happen when a white ball is swapped out and another white ball is swapped in, or when a red ball is swapped out and another red ball is swapped in. The first term in (5.35) is the probability of selecting a red ball to swap out and selecting another red ball to swap in, when there are i white balls in the urn. The probability of selecting a red ball to swap out is $\left(\frac{L_{tx}-i}{L_{tx}}\right)$ and the probability of selecting another red ball to swap in is $\left(\frac{i}{N_{tx}-L_{tx}}\right)$. The second term in (5.35) is the probability of selecting a while ball to swap out and selecting another while ball to swap in, when there are i white balls in the urn. The probability of selecting a white ball to swap out is $\left(\frac{i}{L_{tx}}\right)$ and the probability of selecting another white ball to swap in is $\left(\frac{N_{tx}-L_{tx}-i}{N_{tx}-L_{tx}}\right)$. The two events are mutually exclusive, therefore, the transition probability $p_{i,i}$ is the summation of the probabilities of the two events.

The transition probability $p_{i,i+1}$ represents the probability of gaining a white ball after a swapping operation. This event can happen when a red ball is swapped out and a white ball is swapped in. The term in (5.36) is the probability of selecting a red ball to swap out and selecting a white ball to swap in, when there are i white balls in the urn. The probability of selecting a red ball to swap out is $\left(\frac{L_{tx}-i}{L_{tx}}\right)$ and the probability of selecting a white ball to swap in is $\left(\frac{N_{tx}-L_{tx}-i}{N_{tx}-L_{tx}}\right)$.

The transition probability $p_{i,i-1}$ represents the probability of losing a white ball after a swapping operation. This event can happen when a white ball is swapped out and a red ball is swapped in. The term in (5.37) is the probability of selecting a white ball to swap out and selecting a red ball to swap in, when there are i white balls in the urn. The probability of selecting a white ball to swap out is $\left(\frac{i}{L_{tx}}\right)$ and the probability of selecting a red ball to swap in is $\left(\frac{i}{N_{tx}-L_{tx}}\right)$.

The probability of randomly choosing a configuration that belongs to one of the states, or the initial state distribution of the Markov chain can be found as follows: let i represent the number of white balls. The probability of starting in a state with i white balls is

$$\lambda_i = \frac{\binom{L_{tx}}{L_{tx}-i}\binom{N_{tx}-L_{tx}}{i}}{\binom{N_{tx}}{L_{tx}}} \tag{5.38}$$

for $i = 0, \ldots, L_{tx}$. The term $\binom{N_{tx}}{L_{tx}}$ represents all the possible combinations of choosing L_{tx} balls from a urn with N_{tx} balls. The term $\binom{L_{tx}}{L_{tx}-i}$ represents all the possible combinations of choosing $L_{tx} - i$ red balls from a total of L_{tx} red balls in the urn. The term $\binom{N_{tx}-L_{tx}}{i}$ represents all the possible combinations of choosing i white balls from a total of $N_{tx} - L_{tx}$ white balls in the urn. Therefore, the total number of combinations of choosing L_{tx} balls of which i of them are white and $L_{tx} - i$ of them are red is the product of $\binom{L_{tx}}{L_{tx}-i}$ and $\binom{N_{tx}-L_{tx}}{i}$. The probability of this event is equal to the ratio of all its combinations over all the possible combinations from choosing L_{tx} balls from N_{tx} balls.

Collecting all the λ_i into a column vector Λ forms the initial state distribution for the system. The steady state distribution vector of the system, Π, can be found when $\Pi = \Pi P$, and the summation of the elements of Π is 1.

For random swapping of both transmit and receive antennas, the algorithm can be analyzed with a Markov chain with $(L_{tx}+1)(L_{rx}+1)$ states. The state diagram is presented in Figure 5.4. As seen from Figure 5.4, each neighboring state differs by one red or white ball in the transmit side or receive side. A horizontal transition represents transmit antenna swapping, and a vertical transition represents receive antenna swapping. The probability of swapping an antenna on the transmit side or receive side is determined by the parameter p_{swap} in step 5 of the pseudocode in Table 4.3 and Table 5.1. For each iteration, there is a probability of p_{swap} to swap a pair of transmit antennas, or a probability of $(1 - p_{swap})$ to swap a pair of receive antennas. Without further information, p_{swap} is set to $\frac{1}{2}$ in this thesis, so that there is an equal probability of swapping an antenna on the transmit side or receive side in each iteration. This represents the case of equally likely random swapping of transmit and receive antennas.

The transition probability matrix for transmit and receive antenna swapping is a combination of the individual transmit swapping and receive swapping transition matrices. Let P_{tx} represent the transmit swapping transition matrix in (5.34), and similarly, let P_{rx} represent receive transition matrix. The transition matrix for combined transmit and receive

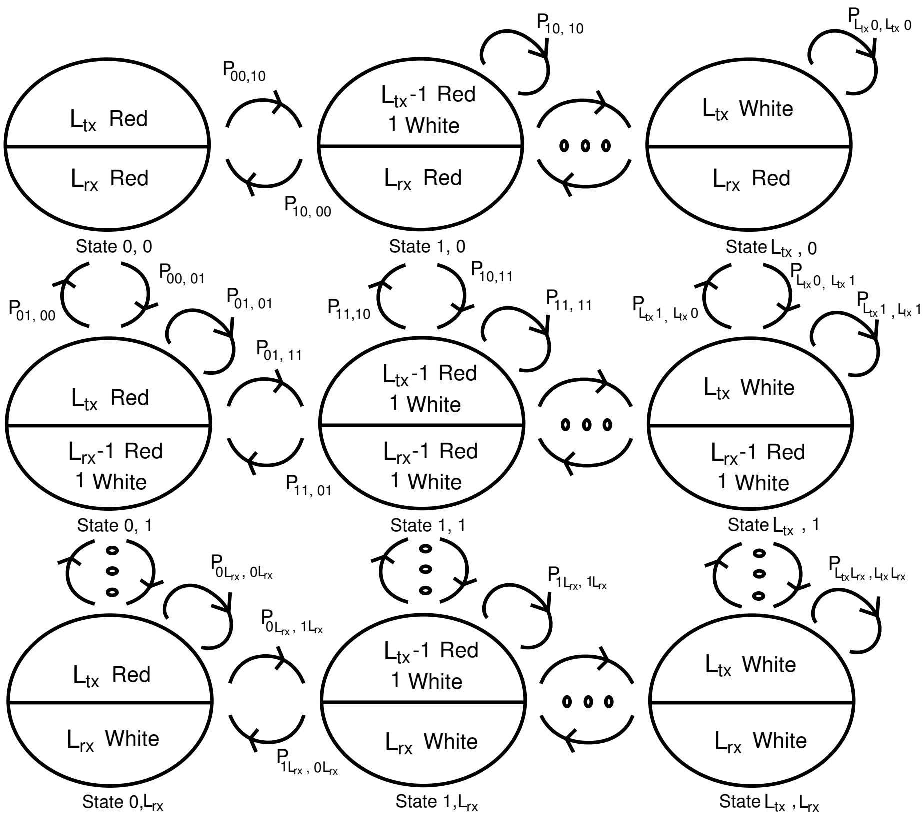

Figure 5.4: Transmit and receive side RAS-AS Markov chain model.

antenna swapping is found to be

$$P_{tx,rx} = I_{L_{rx}+1} \otimes p_{swap} P_{tx} + (1 - p_{swap}) P_{rx} \otimes I_{L_{tx}+1} \tag{5.39}$$

where $\otimes$ represents the Kronecker product. The above transition matrix has a structure, where the P_{tx} matrix is copied on the diagonals of the identity matrix $I_{L_{rx}+1}$. This represents the swapping of transmit antennas, while keeping the receive antennas fixed. The latter Kronecker product provides transition probabilities into the different P_{tx} on the diagonal when a receive antenna is swapped while keeping the transmit antennas fixed. Because the swapping of transmit or receive antennas are equally likely, transitions in P_{tx} and P_{rx} occur with probability p_{swap} and $(1 - p_{swap})$, respectively. For any row of the transition matrix, the sum of the transition probabilities is 1.

The probability of randomly choosing a configuration that belongs to one of the states or the initial distribution of the states can be found as follows: let i and k represent the numbers of white balls in the transmit urn and receive urn, respectively. The probability of starting in a state with i transmit white balls and k receive white balls is

$$\lambda_{i,k} = \frac{\binom{L_{tx}}{L_{tx}-i}\binom{N_{tx}-L_{tx}}{i}\binom{L_{rx}}{L_{rx}-k}\binom{N_{rx}-L_{rx}}{k}}{\binom{N_{tx}}{L_{tx}}\binom{N_{rx}}{L_{rx}}} \tag{5.40}$$

for $i = 0, \ldots, L_{tx}$, $k = 0, \ldots, L_{rx}$. The expression in (5.40) is a straightforward extension of expression (5.38). The initial state distribution of the system can be formed by collecting all the $\lambda_{i,k}$ into a column vector, $\widetilde{\Lambda}$. Similarly, the steady state distribution vector of the system, $\widetilde{\Pi}$, can be found when $\widetilde{\Pi} = \widetilde{\Pi}P$, and the summation of the elements of $\widetilde{\Pi}$ is 1.

5.7.2 First Passage Probability

The first passage probability from one state to another state in n steps is presented in this section, and is used to find the expected number of iterations required to reach the different states in the Markov chain model. Starting from state i, the probability of reaching state j in n steps for the first time can be defined as [34]

$$f_{ij}^{(n)} = Pr[s_n = j, s_m \neq j, \text{ for } 0 < m < n | s_0 = i] \tag{5.41}$$

where s_n represents the state of the system in step n. A relationship between the first passage probability, $f_{ij}^{(n)}$, and the n-step transition probability, $p_{ij}^{(n)}$, is given by [34]

$$p_{ij}^{(n)} = \sum_{g=1}^{n} f_{ij}^{(g)} p_{jj}^{(n-g)} \qquad n \geq 1 \tag{5.42}$$

where the n-step transition probability, $p_{ij}^{(n)}$, is the i^{th} and j^{th} element in the n^{th} power of the transition matrix P, i.e.,

$$p_{ij}^{(n)} = [P^n]_{i,j} \ . \tag{5.43}$$

For $i \neq j$, the following holds:

$$f_{ij}^{(0)} = 0 \tag{5.44}$$

$$p_{jj}^{(0)} = 1 \tag{5.45}$$

$$p_{ij}^{(0)} = 0 \tag{5.46}$$

$$f_{ij}^{(1)} = p_{ij}^{(1)} \ . \tag{5.47}$$

For $i = j$, the following holds:

$$\begin{aligned} f_{ii}^{(n)} &= 1 \quad \text{for } n = 0 \\ f_{ii}^{(n)} &= 0 \quad \text{for } n \neq 0 \ . \end{aligned} \tag{5.48}$$

The moment generating function (MGF) of the sequences $\{p_{ij}^{(n)}\}$ and $\{f_{ij}^{(n)}\}$ can be defined as [34]:

$$P_{ij}(z) = \sum_{n=0}^{\infty} p_{ij}^{(n)} z^n \tag{5.49}$$

$$F_{ij}(z) = \sum_{n=0}^{\infty} f_{ij}^{(n)} z^n \ . \tag{5.50}$$

Substituting the MGFs in (5.49) and (5.50) to the transition probabilities in (5.42), the relationship between the first passage probability and the n-step transition probability can be expressed as [34]

$$
\begin{aligned}
P_{ij}(z) &= \sum_{n=0}^{\infty} p_{ij}^{(n)} z^n \\
&= p_{ij}^{(0)} + \sum_{n=1}^{\infty} \sum_{g=1}^{n} f_{ij}^{(g)} p_{jj}^{(n-g)} z^n \\
&= p_{ij}^{(0)} + \sum_{g=1}^{\infty} f_{ij}^{(g)} z^g \sum_{r=0}^{\infty} p_{jj}^{(r)} z^r \\
&= p_{ij}^{(0)} + \sum_{g=0}^{\infty} f_{ij}^{(g)} z^g \sum_{r=0}^{\infty} p_{jj}^{(r)} z^r \\
P_{ij}(z) &= p_{ij}^{(0)} + F_{ij}(z) P_{jj}(z) \; . \qquad (5.51)
\end{aligned}
$$

To find the sequence of first passage probabilities, the expression in (5.51) can be re-arranged as

$$
\begin{aligned}
F_{ij}(z) &= \frac{P_{ij}(z) - p_{ij}^{(0)}}{P_{jj}(z)} \\
&= \frac{\sum_{n=0}^{\infty} p_{ij}^{(n)} z^n - p_{ij}^{(0)}}{\sum_{n=0}^{\infty} p_{jj}^{(n)} z^n} \\
F_{ij}(z) &= \frac{\sum_{n=1}^{\infty} p_{ij}^{(n)} z^n}{\sum_{n=0}^{\infty} p_{jj}^{(n)} z^n} \; . \qquad (5.52)
\end{aligned}
$$

The coefficients of $F_{ij}(z)$ are the first passage probabilities of reaching state j for the first time in $n = 0, 1, ..., \infty$ numbers of steps, when starting in state i. The coefficients of the polynomials $P_{ij}(z)$ and $P_{jj}(z)$ can be found from the n^{th} power of the transition matrix in (5.43) numerically. The coefficients of $F_{ij}(z)$ can then be calculated numerically by long division of the polynomials $P_{ij}(z)$ and $P_{jj}(z)$. The length of the polynomials $P_{ij}(z)$ and $P_{jj}(z)$ are chosen such that all the significant first passage probabilities for $n \geq 0$ are captured.

5.7.3 Expected Number of Iterations

This section will present the expected number of iterations for the algorithm with a random swapping sequence to reach the different states in the Markov chain model using the first passage probabilities from the previous section. As noted in [34], the first passage probability is a proper probability distribution, where

$$f_{ij} = \sum_{n=0}^{\infty} f_{ij}^{(n)} = 1 \; . \tag{5.53}$$

For transmit-antenna-only or receive-antenna-only swapping, let n_i be a discrete non-negative integer-valued random variable representing the number of steps in order to reach state p for the first time starting from state i. The associated PDF for the random variable n_i is the first passage probabilities $f_{ip}^{(n)}$ for $n = 0, \ldots, \infty$. Therefore, the expected number of iterations to reach state p for the first time starting from state i is

$$E\left[n_i\right] = \sum_{n=0}^{\infty} f_{ip}^{(n)} n \; . \tag{5.54}$$

where $E\left[.\right]$ is the expectation operator.

Similarly, for transmit and receive antenna swapping, let n_{ik} be a discreet non-negative integer-valued random variable representing the number of steps in order to reach state p,q for the first time starting from state i,k. The associated PDF for the random variable n_{ik} is the first passage probabilities $f_{ik,pq}^{(n)}$ for $n = 0, \ldots, \infty$. Therefore, the expected number of iterations to reach state p,q for the first time starting from state i,k is

$$E\left[n_{ik}\right] = \sum_{n=0}^{\infty} f_{ik,pq}^{(n)} n \; . \tag{5.55}$$

The algorithm can start in each of the states with initial state distribution probabilities defined in (5.38) or (5.40). Therefore, using the initial state distribution the average number of iterations required to reach state p or state p,q starting from any state can be found. Let

N represent this quantity and it is defined as

$$N = \sum_{i=0}^{S} \lambda_i n_i \tag{5.56}$$

where S is the last state which is equal to L_{tx} or L_{rx}, for transmit-antenna-only or receive-antenna-only swapping, respectively. From (5.56), it can be seen that N is a sum of the set of random variables n_i weighted by the initial state distribution probabilities. For transmit and receive antenna swapping, N is defined as

$$N = \sum_{k=0}^{L_{rx}} \sum_{i=0}^{L_{tx}} \lambda_{i,k} n_{i,k} \tag{5.57}$$

where $\lambda_{i,k}$ is the initial state distribution in (5.40).

The following presents the expected number of iterations required for the algorithm to reach state p or state p,q on average starting from any state, as well as the variance of the average number of iterations. For transmit-antenna-only or receive-antenna-only swapping, the expected value of N in (5.56) is

$$N_{mean} = \sum_{i=0}^{S} \lambda_i E[n_i] \tag{5.58}$$

where S is equal to L_{tx} or L_{rx} for transmit-antenna-only or receive-antenna-only swapping, respectively. Similarly, for transmit and receive antenna swapping, the expected value of N in (5.57) is

$$N_{mean} = \sum_{k=0}^{L_{rx}} \sum_{i=0}^{L_{tx}} \lambda_{i,k} E[n_{i,k}] \,. \tag{5.59}$$

The expressions in (5.58) and (5.59) are the expected number of iterations required for the RAS-AS algorithm, using a random swapping sequence, to find an antenna configuration belonging to state p or state p,q on average.

To find the variance in the number of iterations, the variance of the set of random variables n_i are averaged and weighted by the initial state distribution. Therefore, for transmit-antenna-only or receive-antenna-only swapping, the variance on average is

$$N_{var} = \sum_{i=0}^{S} \lambda_i Var[n_i] \tag{5.60}$$

where $Var[.]$ is the variance operator, and S is equal to L_{tx} or L_{rx} for transmit-antenna-only or receive-antenna-only swapping, respectively. Similarly, the variance on average for transmit and receive antenna swapping is

$$N_{var} = \sum_{k=0}^{L_{rx}} \sum_{i=0}^{L_{tx}} \lambda_{i,k} Var[n_{i,k}] \; . \tag{5.61}$$

The following section will apply the above analysis to a MIMO system to predict its expected number of iterations.

5.7.4 Analysis Example

As an example, the following applies the analysis from the previous sections to a (4:8,2:4) system. Consider the case of performing receive-antenna-only swapping. The case of performing transmit-antenna-only swapping can be analyzed similarly. The system can be modeled as in Figure 5.3 and the transition matrix is as follows.

$$P_{rx} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0.0625 & 0.3750 & 0.5625 & 0 & 0 \\ 0 & 0.2500 & 0.5000 & 0.2500 & 0 \\ 0 & 0 & 0.5625 & 0.3750 & 0.0625 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} . \tag{5.62}$$

The initial state distribution vector of the system can be calculated according to (5.38), and is presented in Table 5.14. The system reaches steady state with a state distribution vector

State	0	1	2	3	4
Probability	0.0143	0.2286	0.5143	0.2286	0.0143

Table 5.14: Receive antenna swapping initial distribution.

State	0	1	2	3	4
Probability	0.1137	0.3033	0.3412	0.2275	0.0142

Table 5.15: Receive antenna swapping steady state distribution.

presented in Table 5.15. It can be seen that during steady state, there is a 0.11 probability that the system will be in the optimal state (State 0).

For both transmit and receive antenna swapping, the system can be modelled as in Figure 5.4. The receive transition matrix P_{rx} for a (4:8,2:4) system is presented in (5.62) and the (4:8,2:4) P_{tx} transmit transition matrix is presented below.

$$P_{tx} = \begin{pmatrix} 0 & 1 & 0 \\ 0.25 & 0.50 & 0.25 \\ 0 & 1 & 0 \end{pmatrix}. \tag{5.63}$$

The combined state transition probability matrix for both transmit and receive antenna swapping can be found according to equation (5.39) and is presented in Appendix C.

The initial state distribution vector of the system can be calculated according to (5.40), and is presented in Table 5.16 for the (4:8,2:4) system. The system reaches steady state with a state distribution vector presented in Table 5.17. It can be seen that during steady state, there is a 0.0325 probability that the system will be in the optimal state (State 0,0).

Using the expression in (5.59), the expected number of iterations to reach the different states for the first time for a (4:8,2:4) system using the RAS-AS algorithm with a random swapping sequence is presented in Table 5.18. The first passage probability is used to find the expected number of iterations in order to reach the different states in the system, and

State	0	1	2
0	0.0024	0.0095	0.0024
1	0.0381	0.1524	0.0381
2	0.0857	0.3429	0.0857
3	0.0381	0.1524	0.0381
4	0.0024	0.0095	0.0024

Table 5.16: Transmit and receive antenna swapping initial distribution.

State	0	1	2
0	0.0325	0.0650	0.0162
1	0.0867	0.1733	0.0433
2	0.0975	0.1950	0.0487
3	0.0650	0.1300	0.0325
4	0.0041	0.0081	0.0020

Table 5.17: Transmit and receive antenna swapping steady state distribution.

State	2 $TxAnt_{opt}$	1 $TxAnt_{opt}$	0 $TxAnt_{opt}$
4 $RxAnt_{opt}$	473.53 (445.99)	174.23 (174.94)	473.53 (475.20)
3 $RxAnt_{opt}$	36.09 (36.96)	12.05 (13.13)	36.09 (37.29)
2 $RxAnt_{opt}$	14.87 (15.40)	2.93 (4.20)	14.87 (15.87)
1 $RxAnt_{opt}$	36.09 (36.75)	12.05 (12.25)	36.09 (36.79)
0 $RxAnt_{opt}$	473.53 (491.50)	174.23 (178.03)	473.53 (474.58)

Table 5.18: Expected number of iterations for each state of the (4:8,2:4) system. The total number of iterations for exhaustive search is 420. The simulated number of iterations are presented in brackets.

the full range of the first passage probability function that has significant probabilities is used.

The set of optimal transmit antennas and optimal receive antennas is denoted by $TxAnt_{opt}$ and $RxAnt_{opt}$, respectively. Each entry in Table 5.18 shows the expected number of iterations to reach a particular state, which is indicated by the number of optimal antennas in the top row and leftmost column of the table. The expected number of iterations obtained from the analysis is presented above the simulated number of iterations, which is presented in brackets for each entry in Table 5.18. The optimal state is located at the upper left hand corner, and the three surrounding states are the boundary states. For the (4:8,2:4) system, the total number of iterations by exhaustive search (ES) is $\binom{N_{tx}}{L_{tx}}\binom{N_{rx}}{L_{rx}} = \binom{4}{2}\binom{8}{4} = 420$. From

Table 5.18, it can be seen that a random swapping sequence would take 473.53 iterations, more than ES number of iterations, to reach the optimal state on average. This is expected as the random swapping sequence may revisit the same antenna configurations in the course of finding the optimal state. As a result, the calculated entries in the table represent an average upper bound on the computation of the swapping operations, as no computation is required for antenna configurations that are revisited.

It is observed that the expected number of iterations required to reach the boundary states with near optimal transmit and receive antennas are significantly lower than performing ES. For example, for the (4:8,2:4) system, only 36.09 iterations or 8.59% of the ES iterations are required to find a configuration that has all the optimal transmit antennas and three of the four optimal receive antennas. This is equivalent to a computational saving of 91.41% when compared to the full complexity ES.

The expected number of iterations for the (4:8,2:4) system are further verified through Monte Carlo simulations, and the results are presented in brackets under the expected number of iterations for each state in Table 5.18. The number of iterations to find the first antenna configuration that belongs to the different states are recorded for each channel realization. This process is repeated for 1000 channel realizations, and the number of iterations to reach each state are averaged. The results are presented for a SNR of 4dB, and similar simulated number of iterations are observed for other SNRs. From Table 5.18, it can be seen that the simulation results match closely with the results from the analysis.

Other system configurations are also considered in this thesis, and these include the (6:6,3:3), (5:7,2:3), (5:9,2:4), (8:8,4:4), and (9:9,4:4) systems. These systems vary in the number of antennas and RF chains on both the transmit and receive side. This leads to different numbers of exhaustive search iterations and RAS-AS computational complexity. Tables 5.19 to 5.23 presents the expected number of iterations to reach the different states

State	3 $TxAnt_{opt}$	2 $TxAnt_{opt}$	1 $TxAnt_{opt}$	0 $TxAnt_{opt}$
3 $RxAnt_{opt}$	431.39 (435.69)	64.90 (65.10)	64.90 (67.40)	431.39 (434.19)
2 $RxAnt_{opt}$	64.90 (66.59)	6.96 (7.16)	6.96 (6.82)	64.90 (64.75)
1 $RxAnt_{opt}$	64.90 (62.86)	6.96 (6.74)	6.96 (6.82)	64.90 (64.42)
0 $RxAnt_{opt}$	431.39 (420.70)	64.90 (63.41)	64.90 (64.51)	431.39 (444.22)

Table 5.19: Expected number of iterations for each state of the (6:6,3:3) system. The total number of iterations for exhaustive search is 400. The simulated number of iterations are presented in brackets.

for the first time for the different systems.

Similarly, the expected number of iterations for the different systems are further verified through Monte Carlo simulations, and the results are presented in brackets under the expected number of iterations for each state in Tables 5.19 to 5.23. From the tables, it can be seen that the simulated number of iterations match closely with the number of iterations found using the analysis.

From Tables 5.19 to 5.23, it can be seen that for all the systems the expected number of iterations to reach the optimal configuration is larger than the ES number of iterations on average, and this is attributed to the possibility of revisiting the same antenna configuration during the search for the optimal configuration. Similar to the (4:8,2:4) system, the expected number of iterations to reach the boundary states for all the system is significantly lower than performing ES. Table 5.24 summarizes the expected number of iterations

State	2 $TxAnt_{opt}$	1 $TxAnt_{opt}$	0 $TxAnt_{opt}$
3 $RxAnt_{opt}$	382.29 (385.97)	92.37 (94.43)	158.34 (154.63)
2 $RxAnt_{opt}$	40.14 (39.17)	7.31 (7.06)	15.16 (15.64)
1 $RxAnt_{opt}$	27.24 (25.93)	3.64 (3.67)	9.36 (9.58)
0 $RxAnt_{opt}$	110.76 (107.37)	26.12 (25.87)	45.82 (45.51)

Table 5.20: Expected number of iterations for each state of the (5:7,2:3) system. The total number of iterations for exhaustive search is 350. The simulated number of iterations are presented in brackets.

required to arrive at the state containing configurations that have all optimal transmit antennas and one non-optimal receive antenna for the different systems.

Table 5.24 shows that a near optimal set of antennas can be found using random swapping after much fewer iterations than exhaustive search. The average BER performance of the RAS-AS algorithm after performing the expected number of iterations required to reach the boundary state in Table 5.24 is examined in the next section.

The following presents the average number of computation per iteration required for calculating the matrix inverse via the fast antenna swapping and the Gauss-Jordan elimination method. The computation information from Table 5.11, 5.12, and 5.13 for $L_{tx} = 3,4,5$ is used.

For each of the Fast RAS-AS iteration with $p_{swap} = 1/2$, a transmit or receive antenna swap is equally likely to take place. Therefore, the average computation per iteration is

State	2 $TxAnt_{opt}$	1 $TxAnt_{opt}$	0 $TxAnt_{opt}$
4 $RxAnt_{opt}$	1328.60 (1470.17)	340.71 (347.26)	570.60 (592.38)
3 $RxAnt_{opt}$	86.85 (90.15)	21.21 (20.96)	36.67 (36.11)
2 $RxAnt_{opt}$	29.46 (30.90)	4.26 (4.17)	10.35 (9.67)
1 $RxAnt_{opt}$	44.83 (48.97)	9.03 (9.04)	17.59 (16.83)
0 $RxAnt_{opt}$	313.46 (316.70)	81.45 (84.13)	134.94 (136.82)

Table 5.21: Expected number of iterations for each state of the (5:9,2:4) system. The total number of iterations for exhaustive search is 1260. The simulated number of iterations are presented in brackets.

State	4 $TxAnt_{opt}$	3 $TxAnt_{opt}$	2 $TxAnt_{opt}$	1 $TxAnt_{opt}$	0 $TxAnt_{opt}$
4 $RxAnt_{opt}$	5035.58 (5273.15)	425.12 (428.14)	212.97 (211.90)	425.12 (435.90)	5035.58 (5153.34)
3 $RxAnt_{opt}$	425.12 (418.33)	35.21 (35.98)	15.40 (15.95)	35.21 (34.77)	425.12 (447.73)
2 $RxAnt_{opt}$	212.97 (217.27)	15.40 (15.96)	4.66 (5.25)	15.40 (16.18)	212.97 (215.99)
1 $RxAnt_{opt}$	425.12 (420.46)	35.21 (34.31)	15.40 (14.18)	35.21 (33.30)	425.12 (429.23)
0 $RxAnt_{opt}$	5035.58 (5292.31)	425.12 (410.03)	212.97 (205.94)	425.12 (409.74)	5035.58 (5035.74)

Table 5.22: Expected number of iterations for each state of the (8:8,4:4) system. The total number of iterations for exhaustive search is 4900. The simulated number of iterations are presented in brackets.

State	4 $TxAnt_{opt}$	3 $TxAnt_{opt}$	2 $TxAnt_{opt}$	1 $TxAnt_{opt}$	0 $TxAnt_{opt}$
4 $RxAnt_{opt}$	16052.85 (16034.36)	1064.30 (1128.10)	411.23 (406.72)	583.55 (546.53)	3584.88 (3670.36)
3 $RxAnt_{opt}$	1064.30 (1030.38)	73.07 (69.73)	26.13 (23.95)	38.90 (39.63)	252.31 (231.90)
2 $RxAnt_{opt}$	411.23 (430.05)	26.13 (25.77)	6.28 (7.22)	11.82 (12.62)	97.94 (97.73)
1 $RxAnt_{opt}$	583.55 (591.10)	38.90 (40.24)	11.82 (11.51)	19.30 (19.94)	139.14 (150.69)
0 $RxAnt_{opt}$	3584.88 (3794.92)	252.31 (215.47)	97.94 (93.52)	139.14 (149.30)	850.16 (872.29)

Table 5.23: Expected number of iterations for each state of the (9:9,4:4) system. The total number of iterations for exhaustive search is 15876. The simulated number of iterations are presented in brackets.

(N_{tx}:N_{rx},L_{tx}:L_{rx}) System	Expected Number of Iterations (Analysis)	Total Number of ES $\binom{N_{tx}}{L_{tx}}\binom{N_{rx}}{L_{rx}}$	ES Percentage
(5:7, 2:3)	40.14	350	11.47%
(6:6, 3:3)	64.90	400	16.23%
(4:8, 2:4)	36.09	420	8.59%
(5:9, 2:4)	86.85	1260	6.89%
(8:8, 4:4)	425.12	4900	8.68%
(9:9, 4:4)	1064.30	15876	6.70%

Table 5.24: Summary of the expected number of iterations to a boundary state for the different systems.

L_{tx}	Transmit Swap ($\mathscr{R}_m$)	Receive Swap ($\mathscr{R}_m$)	Average Computation per Iteration
3	107	163	$(107+163)/2 = 135$
4	173	263	$(173+263)/2 = 218$
5	255	387	$(255+387)/2 = 321$

Table 5.25: Average number of multiplications per iteration for the Fast RAS-AS algorithm.

L_{tx}	Transmit Swap ($\mathscr{R}_a$)	Receive Swap ($\mathscr{R}_a$)	Average Computation per Iteration
3	87	132	$(87+132)/2 = 109.5$
4	149	224	$(149+224)/2 = 186.5$
5	227	340	$(227+340)/2 = 283.5$

Table 5.26: Average number of additions per iteration for the Fast RAS-AS algorithm.

the average of the computation for the complexity reduced transmit antenna swapping and receive antenna swapping. The computation per iteration using the Gauss-Jordan method is the same for both transmit and receive antenna swapping. The average number of computations per iteration for the Fast RAS-AS algorithm are summarized in Table 5.25 and 5.26 in terms of multiplications and additions, respectively.

From Table 5.25 and 5.26, it can be seen that for $L_{tx} = 3$, performing the matrix inversion via the fast antenna swapping is almost as efficient as the 108 multiplications and 105 additions required for Gauss-Jordan elimination from Table 5.13.

From Table 5.25 and 5.26, it can be seen that for $L_{tx} = 4$, updating the matrix inverse via the fast antenna swapping is more efficient than the 256 multiplications and 264 additions required when performing direct matrix inversion using the Gauss-Jordan method from Table 5.13. Similarly for $L_{tx} = 5$, the fast antenna swapping is more efficient than the 500

L_{tx}	Average Multiplication Reduction per Iteration	Average Addition Reduction per Iteration
4	$(256-218)/256 = 0.1484$ (14.84%)	$(264-186.5)/264 = 0.2936$ (29.36%)
5	$(500-321)/500 = 0.3580$ (35.80%)	$(535-283.5)/535 = 0.4701$ (47.01%)

Table 5.27: Average reduction in the number of multiplications and additions per iteration.

multiplications and 535 additions required for the Gauss-Jordan method from Table 5.13. The computational reduction compared to the Gauss-Jordan elimination for these two cases are presented in Table 5.27.

From Table 5.27, it can be seen that for $L_{tx} = 4$, the fast antenna swapping provides a 14.84% and 29.36% reduction in the number of multiplications and additions over the Gauss-Jordan method per iteration on average. For $L_{tx} = 5$, the amount of computational reduction is 35.80% and 47.01% for the number of multiplications and additions, respectively.

Therefore, for systems with $L_{tx} \geq 4$ number of RF chains, the antenna swapping of the Fast RAS-AS algorithm would provide computational savings in each iteration on average over performing direct matrix inversion using the Gauss-Jordan method. For $L_{tx} = 3$, the antenna swapping of the Fast RAS-AS algorithm has slightly more multiplications and additions than the Gauss-Jordan method in each iteration on average.

Using the expected number of iterations from the analysis in Table 5.24, which shows the expected number of iterations to reach the boundary state with all optimal transmit antennas and near optimal receive antennas, the computational savings compared to the full complexity ES is examined. The overall computational savings for the different systems are presented in Table 5.28.

(N_{tx}:N_{rx},L_{tx}:L_{rx}) System	Overall Savings of Multiplications	Overall Savings of Additions
(5:7, 2:3)	88.53%	88.53%
(6:6, 3:3)	83.77%	83.77%
(4:8, 2:4)	91.41%	91.41%
(5:9, 2:4)	93.11%	93.11%
(8:8, 4:4)	92.61%	93.87%
(9:9, 4:4)	94.29%	95.27%

Table 5.28: Summary of the overall computational savings of the different systems.

For the case of $L_{tx} = 2$ and $L_{tx} = 3$, it is more efficient to use the Gauss-Jordan elimination method for matrix inversion. For these two cases, the computational savings only comes from the reduced number of iterations, which ranges from 6.89% to 16.23% of the ES iterations for the first four systems in Table 5.24. Therefore, the computational savings is about 83.77% to 93.11% for these systems, and the results are summarized in Table 5.28. For the (8:8,4:4) and (9:9,4:4) systems with $L_{tx} \geq 4$, other than the computational savings from the performing much fewer iterations than the ES, the matrix inversion via the fast antenna swapping also provides computational savings in each iteration. From Table 5.27, for the case of $L_{tx} = 4$, each iteration using the fast antenna swapping provides 14.84% saving in the amount of multiplications and 29.36% saving in the amount of additions on average. Therefore, for the (8:8,4:4) system, there would be $8.68\% \times 14.84\% = 1.29\%$ and $8.68\% \times 29.36\% = 2.55\%$ reduction in the amount of multiplications and additions for the expected 8.68% of ES iterations performed. The overall computational savings would come from performing fewer iterations and from the reduction of multiplication and addition operations per iteration using fast antenna swapping. The expected overall amount of multiplication and addition computational savings for the (8:8,4:4) system would be $91.32\% + 1.29\% = 92.61\%$ and $91.32\% + 2.55\% = 93.87\%$, respectively from using the Fast RAS-AS algorithm over the full complexity ES. The results for systems with $L_{tx} = 4$ are summarized in Table 5.28. The effect on the average BER performance from the overall computational savings is small for all of the systems, as simulation results in the next section show that close to optimal average BER performance can be achieved for all of the systems.

5.7.5 Simulation Results

This section presents the simulation results for the RAS-AS algorithm with a random swapping sequence. The pseudocode of the algorithm in Table 4.3 is implemented and its average BER performance is evaluated using Monte Carlo simulation. The performances of the RAS-AS and Fast RAS-AS algorithm are identical, as both are computationally equivalent. The Fast RAS-AS algorithm has a lower computational complexity than that of the RAS-AS algorithm making it suitable for implementation purposes. All the simulations are performed with $p_{swap} = \frac{1}{2}$, and the results are averaged over 1000 channel realizations. The simulation results for the greedy version of the RAS-AS algorithm in Table 5.2 are also presented. The results for a (4:8,2:4) system are first presented, followed by the results for other system configurations at the end of the chapter. For a (4:8,2:4) system, the simulation results for different percentages of the total number of exhaustive search iterations are presented in Figure 5.5.

The number of ES iterations for a (4:8,2:4) system is 420, and from the analysis, the expected number of iterations from the previous section to reach a near optimal set of antennas is 36.09 or 8.59% of the ES iterations. It is observed from Figure 5.5 that after performing 8.59% of the ES iterations, the algorithm can find a set of antennas that can

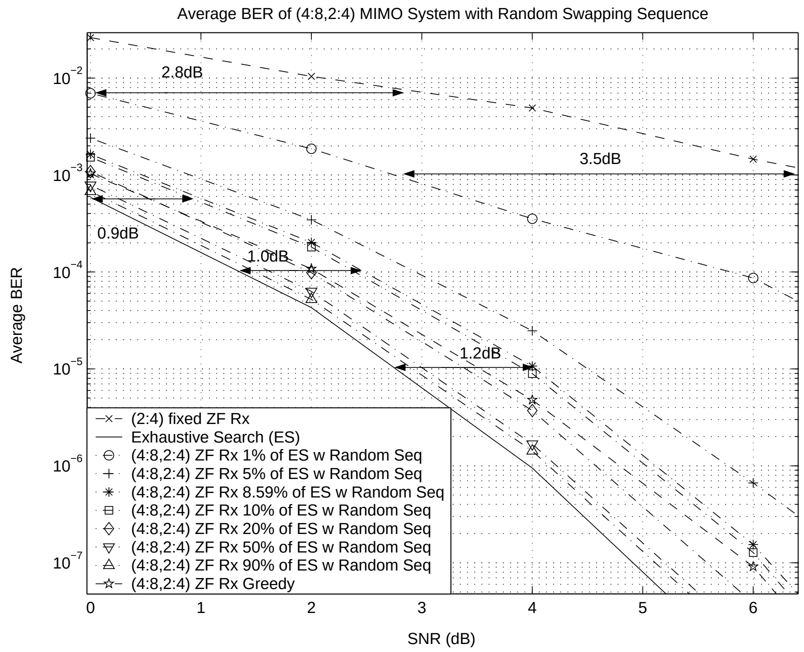

Figure 5.5: Average BER of (4:8,2:4) MIMO system with random swapping sequences.

achieve close to optimal average BER performance that is about 0.9dB, 1.0dB, and 1.2dB away from the ES performance, at average BERs of 10^{-3}, 10^{-4}, and 10^{-5} respectively. This shows that the computational savings in Table 5.28 for the (4:8,2:4) system can be realized with only little average BER performance loss. It is observed that the average BER performance achieved by the greedy version of the algorithm is better than the performance achieved by the algorithm after the expected number of iterations, and it is within 1dB of the optimal performance for average BERs of 10^{-3} to 10^{-5}. This can be attributed to the higher average number of iterations for the (4:8,2:4) system using the greedy algorithm, which is 56.93 or 13.55% of ES iterations found from simulation.

Similar to the deterministic swapping sequence case, a large average BER improvement over the case of using a fixed subset of antennas is observed by performing only 1% of the RAS-AS iterations. The performance gain at an average BER of 10^{-2} and 10^{-3} is 2.8dB and 3.5dB respectively. This illustrates the benefit of performing a few RAS-AS iterations

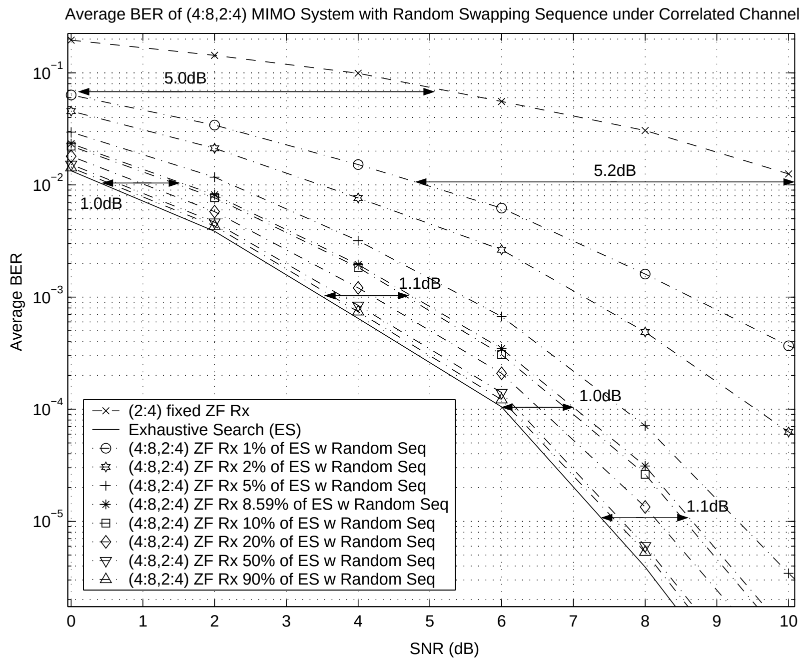

Figure 5.6: Average BER of (4:8,2:4) MIMO system with random swapping sequence under spatially correlated channels.

over using a fixed subset of antennas without antenna selection.

The correlation matrix in (2.10) is used in the simulation of the performance of the RAS-AS algorithm under transmit antenna correlated channel condition. Figure 5.6 presents the RAS-AS average BER performance for a (4:8,2:4) system using a random swapping sequence with different percentages of ES iterations.

From Figure 5.6, it can be seen that under spatial transmit antenna correlation, the system exhibits a higher average BER across all SNRs when compared to the performance under an uncorrelated channel in Figure 5.5. Similar to the deterministic swapping sequence case, the slopes of the average BER curves are less steep, as the diversity order is reduced due to correlation among the antennas. It is observed from Figure 5.6 that after performing 8.59% of the ES iterations, the algorithm can find a set of antennas that can achieve an average BER performance that is about 1.0dB, 1.1dB, 1.0dB, and 1.1dB away from the ES performance, at average BERs of 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} respectively. Similar average BER performance gaps from the optimal performance curve are observed under uncorrelated channel condition.

Similar to the uncorrelated case, a large average BER performance gain over using a fixed subset of antennas is observed after performing only 1% of the RAS-AS iterations. The observed gain in the correlated case is 5.0dB around an average BER of 10^{-1} and 5.2dB around an average BER of 10^{-2}.

Other system configurations are also simulated, and these include the (5:7,2:3), (6:6,3:3), (5:9,2:4), (8:8,4:4), and (9:9,4:4) systems. The expected number of iterations for finding a near optimal set of antennas using the RAS-AS algorithm with a random swapping sequence is analyzed for these systems, and the results are summarized in Table 5.29. Monte Carlo simulations over 1000 uncorrelated MIMO channel realizations are used to evaluate the average BER performance of these systems with the RAS-AS algorithm, and these are presented in Figures 5.7 to 5.11.

After performing the expected number of iterations to reach the boundary state with a near optimal set of receive antennas and all optimal transmit antennas, the resulting average BER performance curve is compared with the average BER performance curve obtained using ES. Table 5.29 summarizes the performance results for the different systems. The performance gaps marked on the graphs are tabulated in the last column of Table 5.29. The performance gap refers to the SNR distance between the average BER performance curve achieved using the expected number of iterations to the boundary state, and the average BER performance curve achieved using the globally optimal configuration. The second column of Table 5.29 shows the expected number of iterations to the boundary state found using the analysis from the earlier sections. The number of exhaustive search iterations and

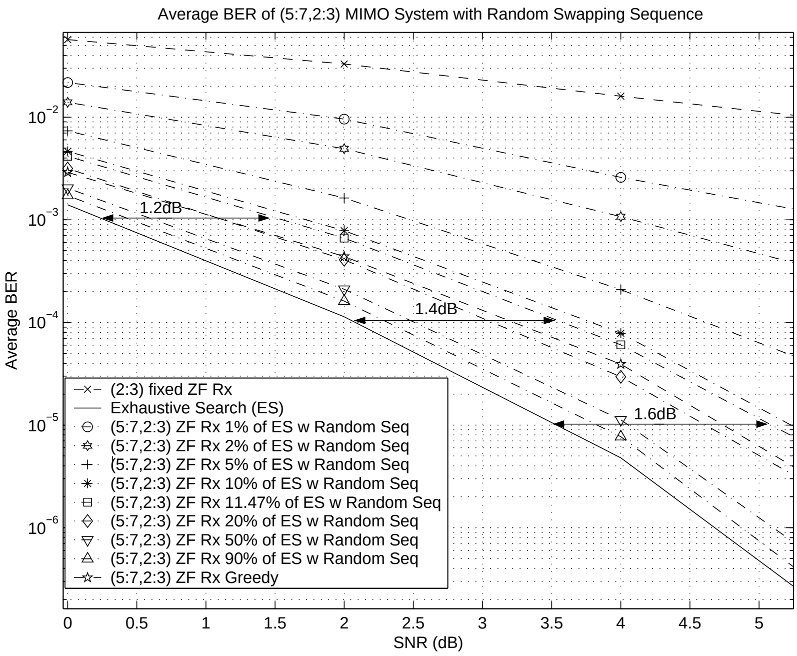

Figure 5.7: Average BER of (5:7,2:3) MIMO system with random swapping sequence under an uncorrelated channel.

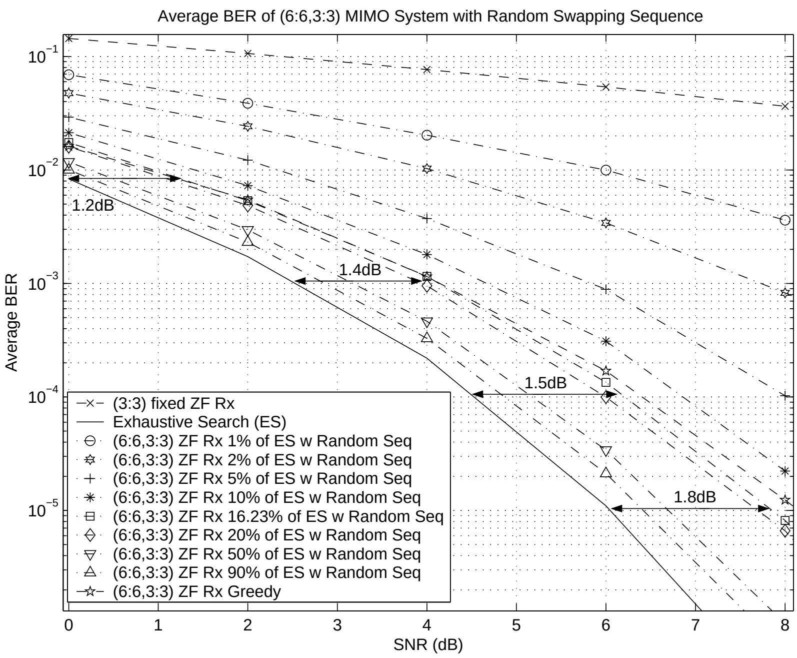

Figure 5.8: Average BER of (6:6,3:3) MIMO system with random swapping sequence under an uncorrelated channel.

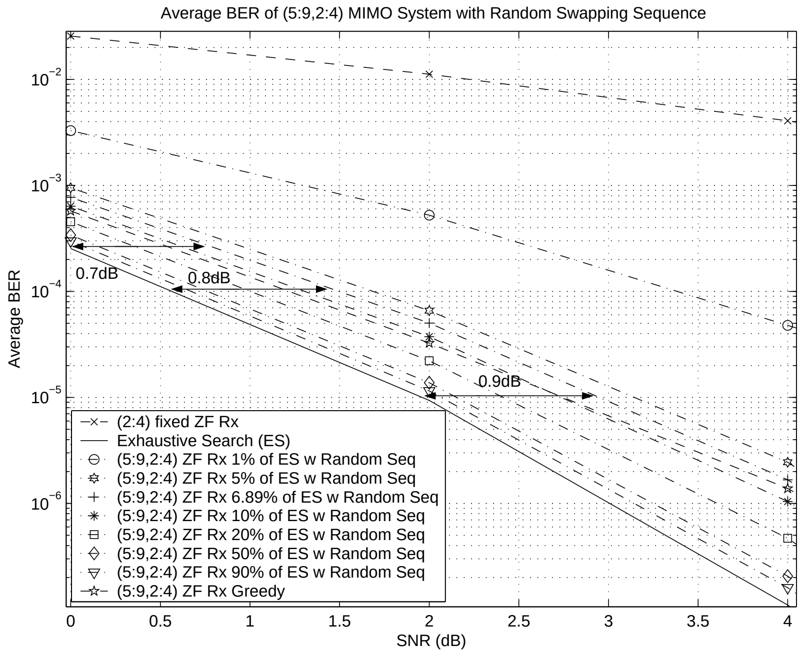

Figure 5.9: Average BER of (5:9,2:4) MIMO system with random swapping sequence under an uncorrelated channel.

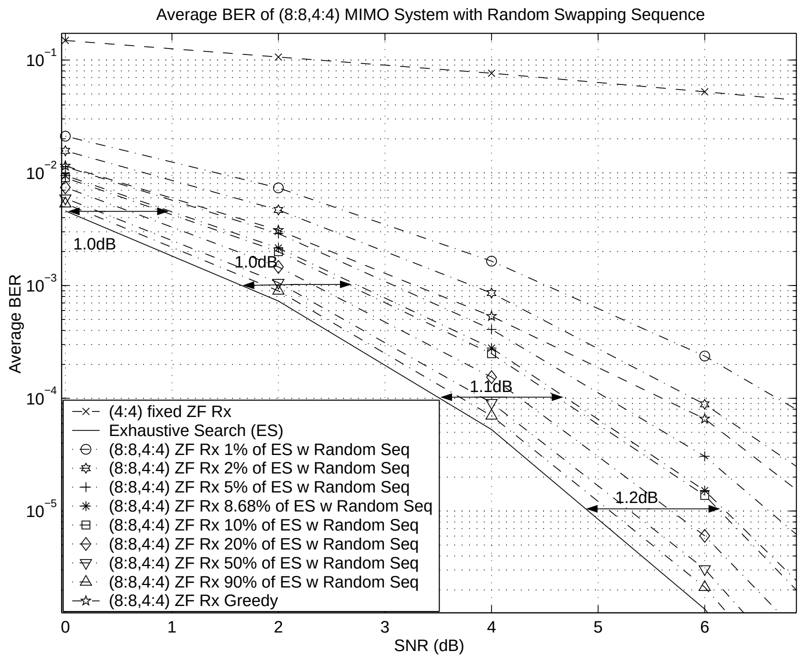

Figure 5.10: Average BER of (8:8,4:4) MIMO system with random swapping sequence under an uncorrelated channel.

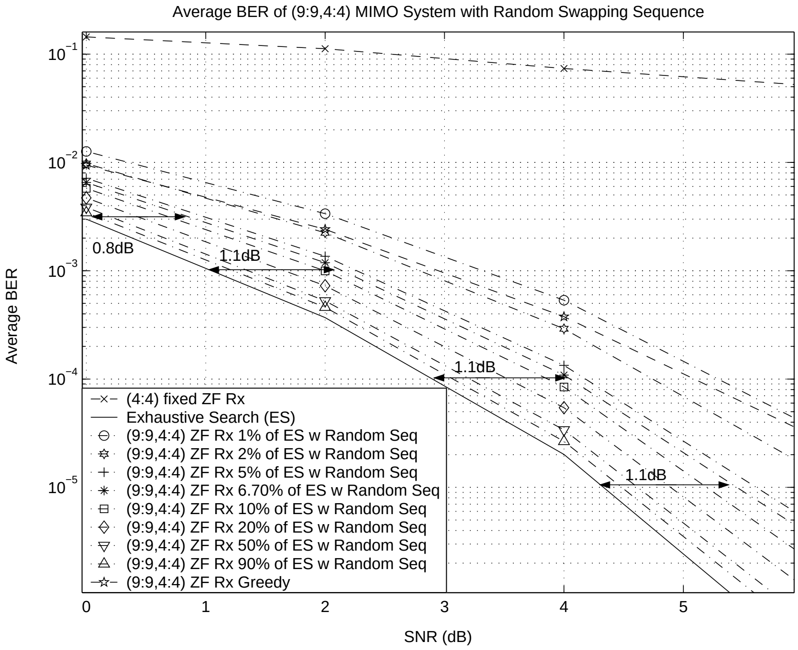

Figure 5.11: Average BER of (9:9,4:4) MIMO system with random swapping sequence under an uncorrelated channel.

(N_{tx}:N_{rx},L_{tx}:L_{rx}) System	Expected Number of Iterations (Analysis)	Total Number of ES $\binom{N_{tx}}{L_{tx}}\binom{N_{rx}}{L_{rx}}$	ES Percentage	Average BER Performance Gap
(5:7 , 2:3)	40.14	350	11.47%	1.2dB to 1.6dB
(6:6 , 3:3)	64.90	400	16.23%	1.2dB to 1.8dB
(4:8 , 2:4)	36.09	420	8.59%	0.9dB to 1.2dB
(5:9 , 2:4)	86.85	1260	6.89%	0.7dB to 0.9dB
(8:8 , 4:4)	425.12	4900	8.68%	1.0dB to 1.2dB
(9:9 , 4:4)	1064.30	15876	6.70%	0.8dB to 1.1dB

Table 5.29: Summary of the average BER performance of the different systems after expected number of iterations required to obtain a near optimal set of antennas.

(N_{tx}:N_{rx},L_{tx}:L_{rx}) System	Average Number of Iterations (Simulation)	Total Number of ES $\binom{N_{tx}}{L_{tx}}\binom{N_{rx}}{L_{rx}}$	ES Percentage
(5:7 , 2:3)	47.74	350	13.64%
(6:6 , 3:3)	48.51	400	12.13%
(4:8 , 2:4)	56.93	420	13.55%
(5:9 , 2:4)	78.87	1260	6.26%
(8:8 , 4:4)	101.16	4900	2.06%
(9:9 , 4:4)	132.72	15876	0.84%

Table 5.30: Summary of the average number of iterations of the different systems using the greedy algorithm.

the percentage of ES iterations are shown in the next two columns in Table 5.29. For all of the systems listed, it can be seen that after performing the expected number of iterations to the boundary state, the RAS-AS algorithm is able to find an antenna configuration that comes within 2dB of the ES performance, for average BERs around 10^{-2} to 10^{-5} and for SNRs greater or equal to 0dB. The result is significant as this shows that the 83% to 95% computational savings in Table 5.28 for all of the systems can be realized with only little average BER performance loss.

The complexity of the greedy algorithm can be characterized by the average number of iterations, and these are found from simulations and are summarized in Table 5.30 for the different systems. Using the average number of iterations from Table 5.30, it can be seen from Figures 5.5, 5.7 to 5.11 that the performances achieved using the greedy algorithm are better than the performances achieved with the Fast RAS-AS algorithm using a random swapping sequence after similar percentage of ES iterations. The performance gain is within 1dB for the different systems. For example, from Figure 5.11, the performance achieved for the (9:9,4:4) system using the greedy algorithm after 0.84% of ES iterations is slightly better than the performance achieved using the Fast RAS-AS algorithm after 1.0% of the ES iterations. The two performance curves are within 1dB of each other. In general, the greedy algorithm can achieve better average BER performance than the Fast RAS-AS algorithm after the same number of iterations for the different systems. However, the Fast RAS-AS algorithm permits a large range of controllable performance and complexity tradeoffs and the greedy algorithm does not offer this flexibility. The performance and computational complexity tradeoff can be observed from the different percentage curves in Figures 5.7 to 5.11. The figures can provide useful information when deciding on the number of RAS-AS iterations to use in order to meet a target performance level, or when given a computational constraint the expected performance level can be predicted.

It is also observed in Figures 5.7 to 5.11 that there is a diminishing return in the average BER performance gain as the number of RAS-AS iterations increases. The diminishing return is also observed for the (4:8,2:4) system. Therefore, Figures 5.7 to 5.11 can help identify the point of diminishing return in the average BER performance, and help decide on the number of RAS-AS iterations to use.

Simulations are performed to confirm the variance obtained from the analysis of the state with near optimal numbers of antennas derived in equations 5.60 and 5.61. During the simulations, the RAS-AS algorithm is executed over a fixed channel realization and SNR. The empirical number of iterations used to find the best antenna configuration is recorded over 1000 different RAS-AS executions. The variance of the number of iterations from the simulation is presented in Table 5.31 for the different systems. It can be seen from Table 5.31 that the simulated variances match up with the variances found from the analysis.

$(N_{tx}$:N_{rx},L_{tx}:$L_{rx})$ System	Simulated Variance	Expected Variance (Analysis)
(5:7 , 2:3)	1645.86	1639.33
(6:6 , 3:3)	4506.60	4264.19
(4:8 , 2:4)	1507.65	1334.44
(5:9 , 2:4)	8191.20	7612.13
(8:8 , 4:4)	182212.28	180841.18
(9:9 , 4:4)	1094990.18	1135183.44

Table 5.31: Summary of the simulated and expected variance of the different systems.

Chapter 6

Conclusions and Future Work

This chapter summarizes the conclusions and contributions in this thesis. Suggestions for future work are provided at the end of the chapter.

6.1 Conclusions

This thesis proposes a novel joint transmit and receive antenna selection algorithm based on the concept of random antenna selection (RAS). The proposed algorithm has the advantage of requiring reduced channel training and estimation at startup corresponding to the RF resources, and additional training and estimation is spread over time and performed when new antennas are swapped in. The proposed algorithm can converge to the globally optimal antenna configuration, i.e., the subset of transmit and receive antennas with minimum BER as number of iterations increase. The main computation in the RAS algorithm is the computation of the matrix inverse for the power gain. The merits of RAS is justified through the ABER outage probability analysis in Chapter 3. It is found that the RAS algorithm can find a non-outage set of antennas with small number of iterations. At an SNR of 0dB and for an ABER outage threshold of 10^{-3}, it is found that after only 5 RAS iterations a non-outage set of antennas can be found.

A computationally efficient realization of the RAS algorithm is possible through the concept of swapping antennas in each iteration. The development of this work started by establishing a relationship between swapping a pair of antennas and performing a rank-2 matrix modification in Chapter 4. Through this relationship, the Woodbury formula for matrix inversion update of a modified matrix is applied to update the matrix inverse required in the power gain calculation. This results in the RAS-AS algorithm described by the pseudocode at the end of Chapter 4. We note that the BER performance of the RAS-AS algorithm can be further improved by applying the AMBER power allocation scheme [31].

A fast implementation of the RAS-AS algorithm is introduced in Chapter 5, which is made possible by the rank-2 matrix modification from swapping a pair of antennas. Simplifications of the algorithm are presented and computational complexity is analyzed in Chapter 5. Each iteration of the Fast RAS-AS algorithm performs a transmit or receive antenna swapping operation, and each iteration is found to have $O(L_{tx}^2)$ complex multiplications and additions for performing the swapping operation and evaluating the average BER selection criterion. This chapter also presents a greedy version of the Fast RAS-AS algorithm.

The average number of multiplications and additions per iteration for the Fast RAS-AS algorithm for matrix inversion update is also computed and compared to the computation required when the Gauss-Jordan elimination method is used for the matrix inversion in each iteration. It is found that for $L_{tx} = 3$, the inversion update using the fast antenna swapping is almost as efficient as using the Gauss-Jordan elimination method in each iteration. For $L_{tx} \geq 4$, the matrix inversion update via the fast antenna swapping provides computational savings over using the Gauss-Jordan method in each iteration on average. The computational reduction per iteration on average using the Fast RAS-AS algorithm grows as L_{tx} increases.

The expected number of iterations of the RAS-AS algorithm is analyzed for both deterministic and random swapping sequences. For deterministic swapping sequences, it is found that on average, half of the number of exhaustive search iterations would be required in order to find the optimal antenna configuration.

A (4:8,2:4) MIMO system is simulated and it is observed that a significant BER performance gain over using a fixed subset of antennas is possible after using the RAS-AS algorithm with only 1% of the ES iterations. This shows the benefit of using a few RAS-AS iterations over performing no antenna selection. Simulation results show that for a deterministic swapping sequence, after performing the expected number of iterations found by analysis, the RAS-AS algorithm can achieve an average BER performance that is about 0.6dB and 1.3dB away from the optimal performance found using exhaustive search, for uncorrelated channels and channels with transmit antenna correlation, respectively.

For a random swapping sequence, the behavior of the RAS-AS algorithm can be modeled by a random walk on a finite state Markov chain. The first passage probability into the different states in the Markov chain model is used to find the expected number of iterations for the RAS-AS algorithm using a random swapping sequence. It is found that the expected number of iterations to reach the optimal state using a random swapping sequence is on average larger than the number of ES iterations, and this is expected as the random swapping sequence has the possibility of revisiting the same configuration. On the other hand, the boundary states surrounding the optimal state with near optimal set of transmit and receive antennas require significantly fewer iterations to reach on average.

For the (4:8,2:4) system, it is found by analysis that when a random swapping sequence is used, on average 36.09 RAS-AS iterations would be required to find an antenna configuration in a boundary state with all optimal transmit antennas and one non-optimal receive antenna. This represents about 8.59% of the total number of possible antenna configurations. After performing the expected number of iterations to reach the boundary state, it is found through simulation that the RAS-AS algorithm can achieve an average BER performance that is about 0.9dB to 1.2dB from the optimal performance, for uncorrelated channels and channels with transmit antenna correlation. The RAS-AS algorithm behaves similarly under transmit antenna correlated channel, with worse average BER performance than the uncorrelated channel case due to reduced diversity in the system. The computational savings from the expected number of iterations amount to 91.41% reduction in the number of multiplications and additions for the (4:8,2:4) system when compared to the full complexity ES. The computational saving is realized with little average BER performance loss which further highlights the significance of the result.

At the conclusion of Chapter 5, systems with different numbers of antennas on the transmit and receive sides are analyzed to determine their expected number of iterations to reach the boundary state in the Markov chain model, and their average BER performances are simulated for uncorrelated channels. Simulation results show that after performing the expected number of iterations for the different systems, the RAS-AS algorithm is able to find a set of antennas that are about 0.7dB to 1.8dB away from the optimal performance on average for the different systems. This shows that the computational savings from the reduced number of iterations and the reduction in the number of multiplications and additions per iteration from fast antenna swapping is realizable with little average BER performance loss. For example, the Fast RAS-AS algorithm allows the (8:8,4:4) system to reduce the number of multiplications and additions by 92.61% and 93.87%, respectively, while achieving close to optimal average BER performance that is 1.0dB to 1.2dB away from the ES performance. The computational savings for the other systems are similarly realized while achieving close to optimal performance.

By comparing the performance achieved by the greedy algorithm and the Fast RAS-AS algorithm after the same number of iterations, it is found that the greedy algorithm performs better, and the performance is within 1dB of the performance achieved using the Fast RAS-AS algorithm with a random swapping sequence. However, the greedy algorithm does not offer the same performance and complexity tradeoff flexibility as the Fast RAS-AS algorithm.

The performance and computational complexity tradeoffs can be observed from the simulation graphs. This allows a system designer to determine the number of iterations to use given an average BER performance constraint, or the average BER performance can be predicted given a computational constraint. The expected number of iterations and variances obtained from the analysis are also verified through simulation, and the results from the analysis match closely with the results from the simulation.

The algorithms proposed in [16] [26] [27] are not comparable to the RAS-AS algorithm as they make selections based on different criteria or information. In [16] a capacity maximization selection criterion is used, and the algorithm assumes that the full complexity MIMO channel is estimated, which uses more channel information and have different training periods than the proposed algorithm. In [26], the antenna selection algorithm is developed for space-time coded system, and in [27], only second-order channel statistics are used.

From the results presented in this thesis, it is shown that the proposed Fast RAS-AS algorithm provides efficient joint transmit and receive antenna selection. The algorithm requires a minimal amount of channel estimation at startup. Further channel estimation is performed as needed, making the Fast RAS-AS algorithm suitable for systems with large numbers of antennas. The concept of incremental estimation and training of the MIMO channel can also be applied to other existing antenna selection algorithms. The algorithm can be used for transmit antenna only or receive antenna only selection as special cases, and it is also applicable to both uncorrelated and correlated MIMO channels. The application of the Fast RAS-AS algorithm in temporally correlated channels is an interesting area for future work, as the Fast RAS-AS algorithm can provide the flexibility of performing small updates to maintain or improve performance over time, using the best antenna configuration found in the previous time slot as the starting point. With the many advantages and low computational complexity, the Fast RAS-AS algorithm is a candidate for solving the problem of finding an efficient joint transmit and receive antenna selection algorithm.

6.2 Future Work

The following are suggestions for future research:

- The proposed RAS-AS algorithm is developed for MIMO ZF receivers. It is of interest to investigate and adapt the RAS-AS algorithm for other types of MIMO receivers.
- In this thesis, deterministic and completely random swapping sequences are proposed. It would be interesting to investigate the performance of the algorithm using other types of swapping sequences. A possible sequence is one that changes or adapts based on determining the worst antenna to swap out in each iteration.
- The parameter p_{swap} is arbitrary chosen to be $\frac{1}{2}$ in this thesis, resulting in a completely random choice between performing a transmit or receive antenna swapping operation in each iteration of the algorithm. It would be interesting to investigate the impact of this parameter on the performance and expected number of iterations of the algorithm. There may exist an optimal value for p_{swap} depending on the number of transmit and receive antennas. The parameter may be adaptively adjusted based on the past performance gain from performing transmit or receive antenna swapping.
- In this thesis, a temporally uncorrelated MIMO channel is considered, and the RAS-AS algorithm chooses a completely uninformed starting configuration by randomly selecting a set of antennas at the beginning of the algorithm. The case of temporally correlated MIMO channels is an interesting area for future work, where the RAS-AS algorithm can use the best antenna configuration found in the previous time slot as the starting configuration and perform small updates to maintain or improve performance over time.

Appendix A

Chi-Square Statistics

The following presents the PDF and CDF of a Chi-Square random variable x with n DOF.

The PDF of a Chi-Square random variable with n DOF is as follows

$$f(x) = \begin{cases} \frac{e^{\frac{x}{2}} x^{\frac{n}{2}-1}}{2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)} & , \quad \text{for } x \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases} . \tag{A.1}$$

Integrating (A.1) results in the Chi-square CDF, and it is given as

$$F(x) = \frac{\Upsilon\left(\frac{n}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \quad \text{for } x \geq 0 \tag{A.2}$$

where $\Upsilon(. \ , .)$ is the lower incomplete Gamma function given as

$$\Upsilon(\alpha, x) = \int_0^x t^{\alpha-1} e^{-t} dt \tag{A.3}$$

and $\Gamma(.)$ is the complete Gamma function given below

$$\Gamma(\beta) = \int_0^\infty x^{\beta-1} e^{-x} dx \tag{A.4}$$

$\Gamma(.)$ can also be expressed as $\Upsilon(. \ , \infty)$, in terms of the lower incomplete Gamma function.

The moment generating function of a Chi-square random variable x is as follows

$$m(t) = E\left[e^{tx}\right] = (1-2t)^{-\frac{n}{2}} \quad -\infty < t < -\frac{1}{2} \tag{A.5}$$

where $E[.]$ is the expectation operator.

Appendix B

Weighted-Chi-Square Statistics

Let x be Chi-square distributed, and let $y = wx$ be a weighted Chi-square random variable. Using the following transformation on the Chi-square PDF

$$f(y) = \frac{1}{|w|} f_X\left(\frac{x}{w}\right) \tag{B.1}$$

the PDF of y for $w > 0$ can be found to be

$$f(y) = \begin{cases} \frac{e^{-\frac{y}{2w}} y^{\frac{n}{2}-1}}{(2w)^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)} & , \quad \text{for } y \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases} . \tag{B.2}$$

Integrating (B.2) results in the weighted-Chi-square CDF, and it is found to be

$$F(y) = \frac{\Upsilon\left(\frac{n}{2}, \frac{y}{2w}\right)}{\Gamma\left(\frac{n}{2}\right)} \quad \text{for } y \geq 0 \tag{B.3}$$

where $\Upsilon(.\ ,.)$ and $\Gamma(.)$ are defined in (A.3) and (A.4), respectively. For a w-weighted Chi-square random variable with n DOF, the MGF is

$$m_w(t) = E\left[e^{ty}\right] = E\left[e^{twx}\right] = (1-2tw)^{-\frac{n}{2}} \quad -\infty < t < -\frac{1}{2w} \tag{B.4}$$

where $E[.]$ is the expectation operator.

Appendix C

(4:8,2:4) Transition Probability Matrix

$$
\left(\begin{matrix}
0 & 0.5000 & 0 & 0.5000 & 0 & 0 & 0 & 0 & 0 & \\
0.1250 & 0.2500 & 0.1250 & 0 & 0.5000 & 0 & 0 & 0 & 0 & \\
0 & 0.5000 & 0 & 0 & 0 & 0.5000 & 0 & 0 & 0 & \\
0.0313 & 0 & 0 & 0.1875 & 0.5000 & 0 & 0.2813 & 0 & 0 & \\
0 & 0.0313 & 0 & 0.1250 & 0.4375 & 0.1250 & 0 & 0.2813 & 0 & \\
0 & 0 & 0.0313 & 0 & 0.5000 & 0.1875 & 0 & 0 & 0.2813 & \\
0 & 0 & 0 & 0.1250 & 0 & 0 & 0.2500 & 0.5000 & 0 & \\
0 & 0 & 0 & 0 & 0.1250 & 0 & 0.1250 & 0.5000 & 0.1250 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0.1250 & 0 & 0.5000 & 0.2500 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0.2813 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2813 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2813 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 &
\end{matrix}\right.
$$

$$
\cdots \begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0.1250 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.1250 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.1250 & 0 & 0 & 0 \\
0.1875 & 0.5000 & 0 & 0.0313 & 0 & 0 \\
0.1250 & 0.4375 & 0.1250 & 0 & 0.0313 & 0 \\
0 & 0.5000 & 0.1875 & 0 & 0 & 0.0313 \\
0.5000 & 0 & 0 & 0 & 0.5000 & 0 \\
0 & 0.5000 & 0 & 0.1250 & 0.2500 & 0.1250 \\
0 & 0 & 0.5000 & 0 & 0.5000 & 0
\end{array} \right)
$$

Bibliography

[1] E. Telatar, “Capacity of multi-antenna Gaussian channels,” *Eur. Trans. Telecommun.*, vol. 10, pp. 585 – 595, Nov 1999.

[2] V. Tarokh, N. Seshadri, and A. Calderbank, “Space-time codes for high data rate wireless communication: performance criterion and code construction,” *IEEE Transactions on Information Theory*, vol. 44, pp. 744 – 765, Mar 1998.

[3] A. Gorokhov, D. Gore, and A. Paulraj, “Receive antenna selection for MIMO spatial multiplexing: theory and algorithms,” *IEEE Transactions on Signal Processing*, vol. 51, pp. 2796 – 2807, Nov 2003.

[4] M. Win and J. Winters, “Analysis of hybrid selection/maximal-ratio combining in Rayleigh fading,” *IEEE Transactions on Communications*, vol. 47, pp. 1773 – 1776, Dec 1999.

[5] S. Sanayei and A. Nosratinia, “Antenna selection in MIMO systems,” *IEEE Communications Magazine*, vol. 42, pp. 68 – 73, Oct 2004.

[6] G. J. Foschini, “Layered space-time architecture for wireless communication in fading environments when using multi-element antennas,” *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 41 – 59, 1996.

[7] A. Gorokhov, “Antenna selection algorithms for MEA transmission systems,” *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 3, pp. III–2857 – III–2860, May 2002.

[8] M. Gharavi-Alkhansari and A. Gershman, “Fast antenna subset selection in MIMO systems,” *IEEE Transactions on Signal Processing*, vol. 52, pp. 339 – 347, Feb 2004.

[9] Y.-S. Choi, A. Molisch, M. Win, and J. Winters, “Fast algorithms for antenna selection in MIMO systems,” *IEEE 58th Vehicular Technology Conference*, vol. 3, pp. 1733 – 1737, Oct 2003.

[10] J.-S. Park and D.-J. Park, “A new antenna selection algorithm with low complexity for MIMO wireless systems,” *IEEE International Conference on Communications*, May (in press) 2005.

[11] Z. Zhou, Y. Dong, X. Zhang, W. Wang, and Y. Zhang, “A novel antenna selection scheme in MIMO systems,” *International Conference on Communications, Circuits and Systems*, vol. 1, pp. 190 – 194, Jun 2004.

[12] A. Gorokhov, D. Gore, and A. Paulraj, “Performance bounds for antenna selection in MIMO systems,” *IEEE International Conference on Communications*, vol. 5, pp. 3021 – 3025, May 2003.

[13] H. Zhang and H. Dai, “Fast transmit antenna selection algorithms for MIMO systems with fading correlation,” *IEEE 60th Vehicular Technology Conference*, vol. 3, pp. 1638 – 1642, Sept 2004.

[14] S. Sandhu, R. Nabar, D. Gore, and A. Paulraj, “Near-optimal selection of transmit antennas for a MIMO channel based on Shannon capacity,” *Conference Record of the*

Thirty-Fourth Asilomar Conference on Signals, Systems and Computers, vol. 1, pp. 567 – 571, Oct 29 - Nov 1 2000.

[15] D. Gore, R. Nabar, and A. Paulraj, “Selecting an optimal set of transmit antennas for a low rank matrix channel,” *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 5, pp. 2785 – 2788, Jun 2000.

[16] A. Gorokhov, M. Collados, D. Gore, and A. Paulraj, “Transmit/receive MIMO antenna subset selection,” *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 2, pp. 13–16, May 2004.

[17] S. Sanayei and A. Nosratinia, “Capacity maximizing algorithms for joint transmit-receive antenna selection,” *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 1773 – 1776, Nov 2004.

[18] M. Jensen and M. Morris, “Efficient capacity-based antenna selection for MIMO systems,” *IEEE Transactions on Vehicular Technology*, vol. 54, no. 1, pp. 110 – 116, Jan 2005.

[19] D. Brennan, “Linear diversity combining techniques,” *Proceedings of the IEEE*, vol. 91, pp. 331 – 356, Feb 2003.

[20] M.-S. Alouini and M. Simon, “An MGF-based performance analysis of generalized selection combining over Rayleigh fading channels,” *IEEE Transactions on Communications*, vol. 48, pp. 401 – 415, Mar 2000.

[21] S. W. Kim and E. Y. Kim, “Optimum receive antenna selection minimizing error probability,” *IEEE Wireless Communications and Networking*, vol. 1, pp. 441 – 447, Mar 2003.

[22] X. N. Zeng and A. Ghrayeb, “Receive antenna selection for space-time block codes,” *Canadian Conference on Electrical and Computer Engineering*, vol. 3, pp. 1361 – 1365, May 2004.

[23] B. Badic, P. Fuxjaeger, and H. Weinrichter, “Performance of quasi-orthogonal space-time code with antenna selection,” *Electronics Letters*, vol. 40, no. 20, pp. 1282 – 1284, Sept 2004.

[24] A. Molisch, M. Win, and J. Winters, “Reduced-complexity transmit/receive-diversity systems,” *IEEE Transactions on Signal Processing*, vol. 51, pp. 2729 – 2738, Nov 2003.

[25] R. J. Heath and A. Paulraj, “Antenna selection for spatial multiplexing systems based on minimum error rate,” *IEEE International Conference on Communications*, vol. 7, pp. 2276 – 2280, Jun 2001.

[26] D. Gore and A. Paulraj, “MIMO antenna subset selection with space-time coding,” *IEEE Transactions on Signal Processing*, vol. 50, pp. 2580 – 2588, Oct 2002.

[27] D. Gore, R. Heath, and A. Paulraj, “Statistical antenna selection for spatial multiplexing systems,” *IEEE International Conference on Communications*, vol. 1, pp. 450 – 454, April 28 - May 2 2002.

[28] ——, “Transmit selection in spatial multiplexing systems,” *IEEE Communications Letters*, vol. 6, pp. 491 – 493, Nov 2002.

[29] D.-S. Shiu, G. Foschini, M. Gans, and J. Kahn, “Fading correlation and its effect on the capacity of multielement antenna systems,” *IEEE Transactions on Communications*, vol. 48, pp. 502 – 513, Mar 2000.

[30] L. Schumacher, K. Pedersen, and P. Mogensen, "From antenna spacings to theoretical capacities - guidelines for simulating MIMO systems," *The 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 2, pp. 587 – 592, Sept 2002.

[31] N. Wang and S. Blostein, "Minimum BER power allocation for MIMO spatial multiplexing systems," *IEEE International Conference on Communications*, May (in press) 2005.

[32] G. Zielke, "Inversion of modified symmetric matrices," *Journal of the ACM*, vol. 15, pp. 402 – 408, July 1968.

[33] S. Grossman, *Elementary Linear Algebra*, 5th ed. Orlando, FL: Saunders College Publishing, 1994.

[34] A. Papoulis and S. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. New York, NY: McGraw-Hill, 2002.

Vita

Edmund Chun Yue Tam

EDUCATION

M.Sc.(Eng.) Candidate	Queen's University	Sept. 2003 – Present
	Electrical and Computer Engineering	
Honors B.A.Sc. Computer Engineering	University of Waterloo	Sept. 1998 – May 2003
	Electrical and Computer Engineering	

EXPERIENCE

Research Assistant	Sept. 2003 – Present
	Electrical and Computer Engineering, Queen's University
Teaching Assistant	Sept. 2003 – Dec. 2003
	Electrical and Computer Engineering, Queen's University

AWARDS

Ontario Graduate Scholarship, Queen's University, 2004-2005.

McLaughlin Fellowship, Queen's University, 2003-2004.

Queen's Graduate Awards, Queen's University, 2003-2005.