Outage Probability Comparisons for Diversity Systems with Cochannel Interference in Rayleigh Fading

Yi Song, Steven D. Blostein, and Julian Cheng

***Abstract*—Space diversity is an effective method to combat fading and cochannel interference (CCI) in wireless systems. In this work, outage performances of several diversity schemes, including a practical variation of maximal-ratio combining (MRC) that does not require signal-to-noise ratios at different antennas, equal-gain combining (EGC) and selection combining (SC), are compared analytically for an interference-limited environment in a Rayleigh fading channel. Our analysis provides insight into performance of diversity schemes in the presence of CCI, as well as assesses the impact of cochannel interferer power distributions.**

I. Introduction

In space diversity, the received signals at antenna branches are combined to combat fading and cochannel interference (CCI). The common diversity schemes are maximal-ratio combining (MRC), equal gain combining (EGC), and selection combining (SC) [1]. A number of papers have studied outage performance of these diversity systems in fading and CCI [2]-[8]. However, to the authors' knowledge, a comparative analysis of relative outage performance for these combining schemes in fading and CCI has not been attempted. Such knowledge can be useful to better understand the design tradeoffs in practical cellular systems. The outage comparison for MRC, EGC and SC with fading and additive white Gaussian noise was treated in Brennan's classical paper [1]. In this work, we provide a comparison study, both analytically and numerically, on the outage probability of diversity systems with CCI and flat Rayleigh fading. Our analysis considers an arbitrary number of interferers, as well as arbitrary interferer power distributions.

We assume that CCI is the dominant source of system degradation. Therefore, for simplicity, we ignore thermal noise in our analysis and consider an interference-limited environment [3],[7]-[9]. The outage is defined as the event when the signal-to-interference ratio (SIR) at the combiner output drops below a threshold β, i.e., $P_{\text{OUT}}(\beta) = \Pr\{\text{SIR} < \beta\}$. In an interference-limited environment, MRC, which maximizes output signal-to-noise ratio (SNR) and whose weights depend on noise powers on antenna branches [1], becomes invalid. Therefore, we consider a variation of MRC, we denote as channel-matched combining (CMC), whose weights are given as the desired user's channel response vector[1]. In practical systems where diversity branches are usually assumed to have the same noise powers, MRC is reduced to CMC [10].

Yi Song and Steven D. Blostein are with the Department of Electrical and Computer Engineering, Queen's University, Kingston, Ontario, Canada K7L 3N6, E-mail: {songy,sdb}@ee.queensu.ca; Julian Cheng is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4, E-mail: jcheng@ee.ualberta.ca.

[1] In [7], the combining scheme the authors called MRC is really CMC since an interference-limited environment was assumed.

II. System Model

We consider a system where the desired signal is corrupted by L interfering signals, all transmitting data at rate $1/T$. Assuming perfect synchronization for the desired user and sampling the output of the receiver matched filter at time $t = nT$, we obtain the baseband signal vector at an M-element receiver as [7]

$$\boldsymbol{r}[n] = \sqrt{P_s}\boldsymbol{c}_s a_s[n] + \sum_{i=1}^{L} \sqrt{P_i}\boldsymbol{c}_i \underbrace{\left(\sum_{m=-\infty}^{\infty} a_i[m] h(nT - mT - \tau_i) \right)}_{z_i[n]} \quad (1)$$

where P_s and P_i are, respectively, the transmitting powers of the desired and the ith interfering signals. Data symbols $a_s[n]$ and $a_i[m]$ are mutually independent with zero-mean and unit variance. The delay of the ith interfering signal relative to the desired signal, τ_i, is assumed to be uniformly distributed over the interval $[0, T)$. The combined transmitter and receiver impulse response, $h(t)$, is a Nyquist pulse with a raised cosine spectrum and roll-off factor ρ, where $0 \le \rho \le 1$. The channel vectors of the desired and the interfering signals, $\boldsymbol{c}_s$ and $\boldsymbol{c}_i$'s, are mutually independent. All channel vectors are assumed to be quasi-static (constant over a time frame [7]) and to have uncorrelated realizations in different frames. We further assume independent Rayleigh fading among diversity branches, i.e., the elements of $\boldsymbol{c}_s$ and $\boldsymbol{c}_i$ are independent identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables (RV's) with zero-mean and unit variance. In (1), $z_i[n]$ denotes the signal inter-symbol interference from the ith interferer. It can be shown [11] that $\mathbb{E}\left[z_i[n]\right] = 0$, $\mathbb{E}\left[|z_i[n]|^2\right] = 1 - \rho/4$, and $\mathbb{E}\left[z_i[n] z_j^*[n]\right] = 0$ for $i \neq j$. The channel vectors of the desired user and the ith interferer can be expressed, component-wise, as $\boldsymbol{c}_s = \left[\alpha_{s,1} e^{j\theta_{s,1}} \cdots \alpha_{s,M} e^{j\theta_{s,M}}\right]^T$ and $\boldsymbol{c}_i = \left[\alpha_{i,1} e^{j\theta_{i,1}} \cdots \alpha_{i,M} e^{j\theta_{i,M}}\right]^T$, respectively, where $\theta_{s,j}$ and $\theta_{i,j}$ are uniformly distributed over $[0, 2\pi)$. The fading amplitudes $\alpha_{s,j}$ and $\alpha_{i,j}$ are Rayleigh distributed with PDF $f_\alpha(\alpha) = 2\alpha e^{-\alpha^2}, \alpha \ge 0$.

III. Outage Probabilities under CCI

A. CMC

Using weight vector $\boldsymbol{w}_{\text{CMC}} = \boldsymbol{c}_s$, the output of CMC becomes

$$\boldsymbol{w}_{\text{CMC}}^H \boldsymbol{r}[n] = \sqrt{P_s}(\boldsymbol{c}_s^H \boldsymbol{c}_s) a_s[n] + \sum_{i=1}^{L} \sqrt{P_i}(\boldsymbol{c}_s^H \boldsymbol{c}_i) z_i[n].$$

The output SIR can be written as

$$\begin{aligned}\mathrm{SIR}_{\text{CMC}} &= \frac{P_s \left|\boldsymbol{c}_s^H \boldsymbol{c}_s\right|^2}{(1-\rho/4)\sum_{i=1}^{L} P_i \left|\boldsymbol{c}_s^H \boldsymbol{c}_i\right|^2} \\ &= \frac{\left|\boldsymbol{c}_s\right|^2}{(1-\rho/4)\sum_{i=1}^{L} \frac{1}{\Lambda_i}\frac{\left|\boldsymbol{c}_s^H \boldsymbol{c}_i\right|^2}{\left|\boldsymbol{c}_s\right|^2}} = \frac{\sum_{j=1}^{M}\alpha_{s,j}^2}{(1-\rho/4)\sum_{i=1}^{L}\eta_i/\Lambda_i}\end{aligned} \tag{2}$$

where $\Lambda_i \triangleq P_s/P_i$ is the power ratio of the desired signal to the ith interfering signal, and $\eta_i \triangleq \left|\boldsymbol{c}_s^H \boldsymbol{c}_i\right|^2 / \left|\boldsymbol{c}_s\right|^2$. It has been shown in [7] that $\boldsymbol{c}_s^H \boldsymbol{c}_i / \left|\boldsymbol{c}_s\right|$ is a circularly symmetric complex Gaussian RV with zero-mean and unit variance, and is independent of $\boldsymbol{c}_s$. Hence, η_i is exponentially distributed with unit mean.

It can be shown [12] that the outage probability of CMC for both equal ($\Lambda_1 = \cdots = \Lambda_L = \Lambda$) and distinct ($\Lambda_i \neq \Lambda_j$ for $i \neq j$) interferer powers can be expressed as

$$P_{\text{OUT, CMC}}(\beta) = \begin{cases} \left(\frac{\beta_0}{\beta_0+\Lambda}\right)^M \sum_{k=0}^{L-1} \frac{(k+M-1)!}{k!(M-1)!}\left(\frac{\Lambda}{\beta_0+\Lambda}\right)^k & \text{equal powers} \\ \sum_{i=1}^{L} \pi_i \left(\frac{\beta_0}{\beta_0+\Lambda_i}\right)^M & \text{distinct powers} \end{cases} \tag{3}$$

where $\beta_0 = (1-\rho/4)\,\beta$ and $\pi_k = \prod_{\substack{i=1 \\ i\neq k}}^{L} \frac{\Lambda_i}{\Lambda_i - \Lambda_k}$. It can be verified that (3) is numerically equivalent to the outage expressions in [6, (13)-(14)] and [7, (43)]. However, as shown in Section IV, (3) are more suitable for analytical comparison.

B. EGC

Using combining weight vector $\boldsymbol{w}_{\text{EGC}} = \left[e^{j\theta_{s,1}} \cdots e^{j\theta_{s,M}}\right]^T$, the EGC output is

$$\begin{aligned}\boldsymbol{w}_{\text{EGC}}^H \boldsymbol{r}[n] &= \sqrt{P_s}\left(\sum_{j=1}^{M} \alpha_{s,j}\right) a_s[n] \\ &\quad + \sum_{i=1}^{L} \sqrt{P_i}\left(\sum_{j=1}^{M} \underbrace{\alpha_{i,j} e^{j(\theta_{i,j}-\theta_{s,j})}}_{g_{i,j}}\right) z_i[n]\end{aligned}$$

where $g_{i,j}$ is a circularly symmetric complex Gaussian RV with zero-mean and unit variance. The SIR can be expressed as

$$\begin{aligned}\mathrm{SIR}_{\text{EGC}} &= \frac{P_s \left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2}{(1-\rho/4)\sum_{i=1}^{L} P_i \left|\sum_{j=1}^{M} g_{i,j}\right|^2} \\ &= \frac{\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2}{(1-\rho/4)\sum_{i=1}^{L}\mu_i/\Lambda_i}\end{aligned} \tag{4}$$

where $\mu_i \triangleq \left|\sum_{j=1}^{M} g_{i,j}\right|^2$ is exponentially distributed with mean M. An exact outage analysis for EGC in Rayleigh fading and CCI can be found in [8, (4)-(7)].

C. SC

The outage event occurs when the branch with maximum SIR value drops below a pre-defined threshold. That is, $P_{\text{OUT,SC}}(\beta) = \Pr\left\{\mathrm{SIR}_{\text{SC},1} < \beta, \cdots, \mathrm{SIR}_{\text{SC},M} < \beta\right\}$, where $\mathrm{SIR}_{\text{SC},i}$ is the SIR at the ith receiving antenna. The outage probability expressions of SC can be obtained from [2]-[4] as

$$P_{\text{OUT,SC}}(\beta) = \begin{cases} \left[1 - \left(\frac{\Lambda}{\beta_0+\Lambda}\right)^L\right]^M & \text{equal powers} \\ \left[\sum_{k=1}^{L} \pi_k \frac{\beta_0}{\beta_0+\Lambda_k}\right]^M & \text{distinct powers.} \end{cases} \tag{5}$$

IV. Analytical Outage Probability Comparisons

A. Outage Probability Comparison for CMC and EGC

We rewrite the output SIR expression of CMC in (2) as

$$\mathrm{SIR}_{\text{CMC}} = \frac{M\sum_{j=1}^{M}\alpha_{s,j}^2}{(1-\rho/4)\sum_{i=1}^{L} M\eta_i/\Lambda_i} = \frac{M\sum_{j=1}^{M}\alpha_{s,j}^2}{(1-\rho/4)\sum_{i=1}^{L}\nu_i/\Lambda_i} \tag{6}$$

where $\nu_i \triangleq M\eta_i$ is exponentially distributed with mean M. Comparing (6) with (4), we recognize that $\xi_1 \triangleq \sum_{i=1}^{L}\nu_i/\Lambda_i$ in (6) and $\xi_2 \triangleq \sum_{i=1}^{L}\mu_i/\Lambda_i$ in (4) have the same distribution. In (4) and (6), the denominator is independent of the numerator. Thus, we have

$$\begin{aligned}P_{\text{OUT,CMC}}(\beta) &= \Pr\left\{M\sum_{j=1}^{M}\alpha_{s,j}^2/\xi_1 < \beta_0\right\} \\ &= \int \Pr\left\{M\sum_{j=1}^{M}\alpha_{s,j}^2 < \beta_0\xi\right\} f_{\xi_1}(\xi)d\xi\end{aligned}$$

and

$$\begin{aligned}P_{\text{OUT,EGC}}(\beta) &= \Pr\left\{\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2 \Big/ \xi_2 < \beta_0\right\} \\ &= \int \Pr\left\{\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2 < \beta_0\xi\right\} f_{\xi_2}(\xi)d\xi.\end{aligned}$$

Since PDF $f_{\xi_1}(\xi) = f_{\xi_2}(\xi)$ and $\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2 \leq M\sum_{j=1}^{M}\alpha_{s,j}^2$ due to the Cauchy-Schwarz inequality, we have $P_{\text{OUT,CMC}}(\beta) \leq P_{\text{OUT,EGC}}(\beta)$, where equality is achieved when $M = 1$ (single antenna). When $M > 1$, the outage probability for CMC is strictly lower than that for EGC. The above conclusion holds for arbitrary interferer power distributions.

B. Outage Probability Comparison for CMC and SC

For the case of one interferer, by setting $L = 1$ in (3) and (5), it is easy to see that the outage probabilities for CMC and SC are identical for $L = 1$. For $L > 1$, it can be proved that [12], for the special case of equal interferer powers, the outage probabilities for CMC are smaller than those of SC. For distinct interferer powers and $L > 1$, our numerical results presented in Section V suggest that CMC still outperforms SC.

C. Outage Probability Comparison for EGC and SC

As shown in Section V, the relative outage performance of EGC and SC depends on factors such as the number of interferers and the interferer power distribution. More interestingly, SC can have better outage performance than EGC in the presence of one dominant interferer. An exact analytical outage comparison for EGC and SC in CCI is difficult.

V. Numerical Results

In obtaining the numerical results, we set $\rho = 0$. Fig. 1 plots the outage probabilities versus the outage threshold β with four diversity branches and equal interferer powers ($\Lambda = 10$ dB) for $L = 1, 2$, and 6 interferers. Fig. 1 confirms that the outage probabilities for CMC are smaller than those of EGC in all cases considered. Fig. 1 also indicates that SC and CMC have the same outage performance when the system has one interferer.

Fig. 2 studies the effect of interferer power distribution on the outage probabilities. We first define the ratio of the desired signal power to average interference power as $\Lambda_{avg}(\text{dB}) = 10\log_{10}\frac{P_s}{(1/L)\sum_{i=1}^{L}P_i}$. Denoting the normalized interference power vector by $\boldsymbol{q} = [q_1, q_2, \ldots, q_L]$, where $\sum_{i=1}^{L} q_i = 1$, we can calculate the power ratio $\Lambda_i(\text{dB}) = P_s/P_i(\text{dB}) = \Lambda_{avg}(\text{dB}) - 10\log_{10}(Lq_i)$. With four diversity branches, Fig. 2 compares the outage probabilities for two interferers with a highly unbalanced interference power vector $[0.1, 0.9]$ and for six interferers with a more evenly-distributed interference power vector $[0.05, 0.1, 0.15, 0.22, 0.23, 0.25]$. In both cases, as expected, CMC outperforms both EGC and SC. The relative performance for EGC and SC, however, depends on the interferer power distribution. With six interferers, Fig. 2 shows that EGC outperforms SC in a scenario which approximates the equal interferer power case studied in Fig. 1. For two interferers, however, EGC is inferior to SC. This can be explained by noting that the interference power vector $[0.1, 0.9]$ represents the case of a strong dominant interferer, a scenario where the outage performance of SC is almost equivalent to that of CMC.

In the presence of noise, we used Monte-Carlo simulation to evaluate the outage probabilities. Assuming all antennas have the same noise power, for the case of four antennas and $\Lambda = 10$ dB, our results show that for one interferer, SC outperforms EGC at high SNRs such as 20dB and is inferior to EGC at lower SNRs such as 10dB.

VI. Conclusion

We have analytically compared the outage performance of CMC, EGC, and SC for an interference-limited environment in flat Rayleigh fading. We have shown that CMC has a lower outage probability than that of EGC, and that CMC has no greater outage probability than that of SC. The relative outage performance between EGC and SC, however, depends on the number of interferers and interferer power distribution. For finite SNRs, the simulation results show that the relative performance between EGC and SC is SNR-dependent.

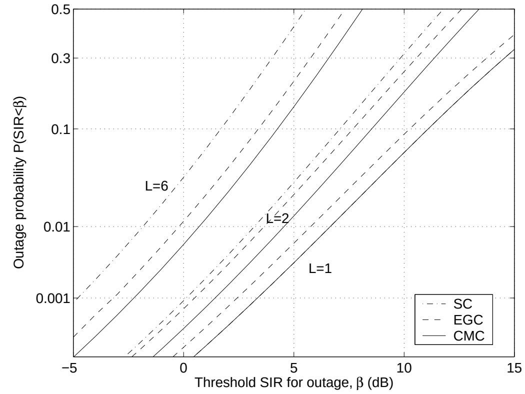

Fig. 1. Analytical outage probability for an interference-limited environment: equal interferer powers ($\Lambda = 10$ dB), $M = 4$ antennas, and $L = 1, 2$, and 6 interferers.

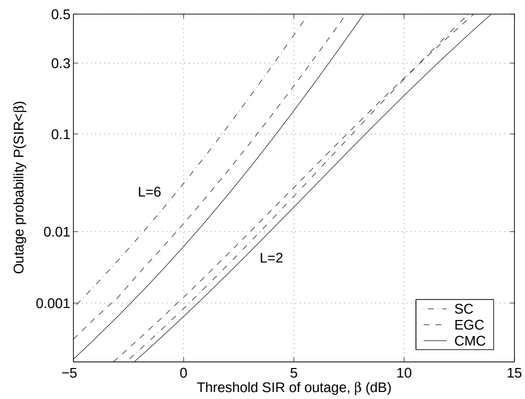

Fig. 2. Analytical outage probability for an interference-limited environment: distinct interferer powers ($\Lambda_{avg} = 10$ dB), $M = 4$ antennas, and $L = 2$ and 6 interferers. The interference power vectors for $L = 2$ and $L = 6$ are, respectively, $[0.1, 0.9]$ and $[0.05, 0.1, 0.15, 0.22, 0.23, 0.25]$.

References

[1] D. G. Brennan, "Linear diversity combining techniques," *Proc. IRE*, vol. 47, pp. 1075-1102, June 1959.

[2] K. W. Sowerby and A. G. Williamson, "Selection diversity in multiple interferer mobile radio systems," *Electron. Lett.*, vol. 24, no. 24, pp. 1511-1513, Nov. 1988.

[3] A. A. Abu-Dayya and N. C. Beaulieu, "Outage probabilities of diversity cellular systems with cochannel interference in Nakagami fading," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 343- 355, Nov. 1992.

[4] Y. Yao and A. U. H. Sheikh, "Investigations into cochannel interference in microcellular mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 114-123, May 1992.

[5] J. Cui and A. Sheikh, "Outage probability of cellular radio systems using maximal ratio combining in the presence of multiple interferers," *IEEE Trans. Commun.*, vol. 47, pp. 1121-1124, Aug. 1999.

[6] V. A. Aalo and C. Chayawan, "Systems using maximal ratio combining in Rayleigh fading channel with multiple interferers," *Electron. Lett.* , vol. 36, no. 15, pp. 1314-1315, July 2000.

[7] A. Shah and A. M. Haimovich, "Performance analysis of maximal ratio combining and comparison with optimum combining for mobile radio communications with cochannel interference," *IEEE Trans. Veh. Technol.*, vol. 49, pp. 1454-1463, July 2000.

[8] Y. Song, S. D. Blostein, and J. Cheng, "Exact outage probability for equal gain combining with cochannel interference in Rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 865-870, Sept. 2003.

[9] A. J. Paulraj, *The Mobile Communications Handbook*, J. D. Gibson, Ed. 2nd ed. New York: CRC/IEEE, 1999.

[10] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-CDMA for next-generation mobile communication systems," *IEEE Commun. Mag.*, vol. 36, pp. 56-69, Sept. 1998.

[11] N. C. Beaulieu and A. A. Abu-Dayya, "Bandwidth efficient QPSK in cochannel interference and fading," *IEEE Trans. Commun.*, vol. 43, pp.2464–2474, Sept. 1995.

[12] Y. Song, *Multiple-Input Multiple-Output Wireless Communication Systems with Cochannel Interference*, Ph.D. thesis, Queen's University, 2003.