Power Allocation for Dual Transmit Antenna Spatial Multiplexing Systems

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***Abstract*— In this paper, power allocation schemes for dual transmit multiple receive antenna communications is proposed and analyzed. Minimum error rate power allocation for a number of equalization schemes are proposed, including zero-forcing (ZF), successive interference cancellation (SIC) and ordered SIC (OSIC). It is shown both analytically and by numerical simulations that the proposed schemes improve system performance with the capability of eliminating error floors in ill-conditioned channels. Compared with the more general precoding methods, the proposed schemes significantly reduce processing complexity at both the transmitter and receiver as well as feedback overhead.**

I. Introduction and Motivation

The capacity of wireless communication systems can be increased substantially by using multiple transmit and receive antennas, provided that multipath scattering effects have been exploited appropriately [1]. Equalization for such multiple-input multiple-output (MIMO) spatial multiplexing (SM) systems can employ criteria such as zero-forcing (ZF), minimum mean-squared error (MMSE), successive interference cancellation (SIC), or maximum likelihood (ML). Recently, the Vertical Bell Laboratories Layered Space-Time (V-BLAST) architecture has been proposed as a simple spatial multiplexing scheme [1]. In V-BLAST, data streams are encoded independently and transmitted from each antenna, and detected at the receiver in an ordered SIC (OSIC) fashion.

In this paper, we study a MIMO system with two transmit antennas and at least two receive antennas (known also as two-input multiple output (TIMO) systems). The study of such a system can be motivated in a number of ways: 1) dual transmit MIMO systems are important in practical scenarios where there are limitations on cost and/or size to install more antennas; 2) it is easier to analyze TIMO systems than the more general MIMO systems, and some analysis results may offer insights into the more general MIMO systems.

When channel state information (CSI) is available at the transmitter, system performance can be improved. The availability of CSI at the transmitter is achievable in time-division duplex (TDD) systems due to the reciprocity of the uplink and downlink channels, or frequency-division duplex (FDD) systems with a feedback channel. Some efforts have been made to design optimum MIMO transmitter precoders using throughput or error rate criteria [2]. However, precoding schemes require complex processing at both transmit and receive side, as well as large feedback overhead. This motivates the following study of a simplified precoding scheme, which we refer to as power allocation for MIMO. Furthermore, optimizing error rate performance will be investigated, since minimization of error rate is aligned with the ultimate objective of communications system design.

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II. Dual Transmit Antenna (TIMO) Spatial Multiplexing

Consider a MIMO system with $N_t = 2$ and $N_r \geq 2$, i.e. a two-input multiple-output (TIMO) system. The received signal can be modelled as

$$\begin{aligned} \mathbf{r} &= \mathbf{H}\mathbf{s} + \boldsymbol{\eta} \\ &= s_1\mathbf{h}_1 + s_2\mathbf{h}_2 + \boldsymbol{\eta}, \end{aligned} \tag{1}$$

where $\mathbf{s} = [s_1 \ s_2]^T$ denotes the transmitted signal vector; $\mathbf{H} = [\mathbf{h}_1 \ \vdots \ \mathbf{h}_2]$ is the $N_r \times 2$ channel matrix; and $\boldsymbol{\eta}$ is the $N_r \times 1$ additive Gaussian noise vector. For simplification of analysis, we assume white noise and input, i.e., $\mathbb{E}[\mathbf{s}\mathbf{s}^{\mathcal{H}}] = E_s\mathbf{I}_2$ and $\mathbb{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathcal{H}}] = N_0\mathbf{I}_{N_r}$. We now briefly review several reception methods and their error rate performances. For simplicity of analysis and without loss of generality, we compare these schemes under BPSK modulation.

1) ZF Receiver: The estimate of $\mathbf{s}$ is given by

$$\hat{\mathbf{s}} = \mathbf{H}^\dagger\mathbf{r} = \mathbf{s} + \mathbf{H}^\dagger\boldsymbol{\eta}, \tag{2}$$

which is unbiased. The decision-point signal-to-noise ratio (SNR) of the k-th signal stream, $k = 1, 2$, is given by

$$\gamma_{Z,k} = \frac{E_s}{N_0[(\mathbf{H}^{\mathcal{H}}\mathbf{H})^{-1}]_{k,k}} \stackrel{\text{def}}{=} \gamma_s g_{Z,k}^2, \tag{3}$$

where $g_{Z,k}^2 \stackrel{\text{def}}{=} [(\mathbf{H}^{\mathcal{H}}\mathbf{H})^{-1}]_{k,k}^{-1}$ denotes the power gain of k-th stream. The power gains can be calculated as

$$g_{Z,1}^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_2\|^2}, \quad g_{Z,2}^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_1\|^2}, \tag{4}$$

where $\Delta_{\mathbf{H}} \stackrel{\text{def}}{=} \|\mathbf{h}_1\|^2\|\mathbf{h}_2\|^2 - |\mathbf{h}_2^{\mathcal{H}}\mathbf{h}_1|^2$.

2) SIC Receiver: Without loss of generality, we assume detecting stream $k=1$ first. Assuming ZF equalization is employed, the power gain in detecting s_1 is the same as that of the ZF receiver, i.e., $g_{S,1}^2 = g_{Z,1}^2$. We assume that $\hat{s}_1 = s_1$, and perform interference cancellation, i.e.,

$$\mathbf{r}' = \mathbf{r} - \hat{s}_1\mathbf{h}_1 + \boldsymbol{\eta} = s_2\mathbf{h}_2 + \boldsymbol{\eta}. \quad (5)$$

The detection of s_2 in (5) with ZF equalization is equivalent to maximum ratio combining (MRC), with power gain $g_{S,2}^2 = \|\mathbf{h}_2\|^2$.

3) OSIC Receiver: The SNR-based ordering scheme detects the stream with largest decision-point SNR first, or, equivalently, detects the stream with largest power gain first. From (4), this is equivalent to choosing the stream corresponding to the maximum channel norm, i.e.,

$$k_1 = \arg\max_l \gamma_{Z,l} = \arg\max_l g_{Z,l}^2 = \arg\max_l \|\mathbf{h}_l\|^2.$$

In other words, SNR-based ordering is equivalent to norm-based ordering in TIMO systems. Therefore, we obtain the power gains as

$$g_{O,1}^2 = \frac{\Delta_{\mathbf{H}}}{\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\}}, \quad g_{O,2}^2 = \min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\}.$$

The average BER of the above receivers can be calculated as

$$\bar{P}(\gamma_s; g_1^2, g_2^2) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2}\right), \quad (6)$$

where the power gains g_1^2 and g_2^2 depend on the receiver structure; $\mathcal{Q}(x) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-y^2/2}dy$. We note that for SIC and OSIC receivers, (6) is only a lower bound due to the neglecting of error propagation. However, at moderate-to-high SNR regimes, this lower bound closely approximates the average BER since error propagation is minimal.

III. Power Allocation in TIMO

Denote the power allocated to the k-th stream as p_k^2 $(k=1,2)$. The received signal can be written as

$$\mathbf{r} = \mathbf{HPs} + \boldsymbol{\eta}, \quad (7)$$

where $\mathbf{P} \stackrel{\text{def}}{=} \text{diag}\{p_1, p_2\}$. We assume that the total transmit power is normalized as

$$p_1^2 + p_2^2 = 2. \quad (8)$$

Note that the power allocation scheme is a special case of more general precoding schemes that employ a precoder $\mathbf{P}$ that is not constrained in structure. The structural constraint introduced in power allocation significantly reduces both computational complexity and feedback overhead.

Power allocation can employ a number of optimization criteria, e.g., throughput, SNR or mean squared-error (MSE) [2]. In this paper, we will consider the minimization of the bit error rate (MBER) as the optimization criterion, MBER being critical to communications system design. The average BER of TIMO with power allocation can be obtained by generalizing (6) to

$$\bar{P}(\gamma_s; \{g_k^2\}; \{p_k^2\}) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2 p_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2 p_2^2}\right). \quad (9)$$

To minimize the average BER in (9) under transmit power constraint (8), no closed-form solution exists. While an iterative algorithm is given in [3], it is also shown in [3] that by approximating the objective function, a simpler closed-form solution can be found as

$$p_k^2 = \left(\frac{\ln g_k^2 + \nu}{\gamma_s g_k^2}\right)_+, \quad (k=1,2), \quad (10)$$

where $(x)_+ \stackrel{\text{def}}{=} \max\{0, x\}$, and ν is chosen to satisfy power constraint (8). It is shown in [3] that the approximate solution has performance very close to that of the MBER solution.

For OSIC, we need to examine the effect of power allocation on ordering. Without loss of generality, we assume $\|\mathbf{h}_1\| \ge \|\mathbf{h}_2\|$, and by norm-based ordering, s_1 is detected first. Denote the corresponding power gains as α_1^2 and α_2^2. Consider the opposite detection ordering. Denote the resulting power gains as β_1^2 and β_2^2. From the previous analysis, we have $\alpha_1^2\alpha_2^2 = \beta_1^2\beta_2^2 = \Delta_{\mathbf{H}}$. By assumption ($\|\mathbf{h}_1\| \ge \|\mathbf{h}_2\|$), we have $\beta_1^2 \le \alpha_1^2 \le \beta_2^2$, and $\beta_1^2 \le \alpha_2^2 \le \beta_2^2$. At moderate-to-high SNR ($\gamma_s \gg 1$), it can be shown that

$$\bar{P}(\gamma_s; \alpha_1^2, \alpha_2^2) \le \bar{P}(\gamma_s; \beta_1^2, \beta_2^2),$$

i.e., for TIMO at moderate-to-high SNR, the norm-based ordering (or, equivalently, SNR-based ordering), which is optimal for TIMO without power allocation, is also optimal for the power allocation scheme, in the sense of minimizing average BER.

Overhead and Complexity Issues: For a TIMO system using a general precoding method, the channel matrix is fed back to the transmitter, requiring $N_t \times N_r = 2N_r$ complex numbers, or equivalently, $4N_r$ real numbers. On the other hand, if the proposed power allocation scheme is employed, only $N_t = 2$ real numbers are required at the transmitter, which is $\frac{1}{2N_r}$ of that of precoding. Usually precoding schemes also require diagonalization of a channel matrix, which involves matrix transformation and/or singular value decomposition, while using power allocation, operations performed at the transmitter are trivial. Therefore, the proposed power allocation scheme has less overhead and complexity compared to that of general precoding schemes.

IV. Ill-Conditioned TIMO Channels

Ill-conditioned channel refers to a channel matrix that is ill-conditioned, i.e., $\frac{\lambda_{\max}(\mathbf{H}^{\mathcal{H}}\mathbf{H})}{\lambda_{\min}(\mathbf{H}^{\mathcal{H}}\mathbf{H})} \gg 1$, where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote maximum and minimum eigenvalues of a matrix. For TIMO channels, without loss of generality, we can assume $\mathbf{h}_2 \approx a \cdot \mathbf{h}_1$ with $a \in \mathbb{C}$, and the channel matrix $\mathbf{H} \approx \mathbf{h}_1[1 \;\; a]$. The least-squares (LS) estimate of a is given by

$$\hat{a}_{LS} = \mathbf{h}_1^{\dagger}\mathbf{h}_2 = \frac{\mathbf{h}_1^{\mathcal{H}}\mathbf{h}_2}{\|\mathbf{h}_1\|^2}. \quad (11)$$

Due to the ill-conditioned channel matrix, average error rate performance of spatial multiplexing experiences error floors. This motivates our consideration of general precoding for ill-conditioned channels. An approximate MBER precoder under a transmit power constraint can be found to be

$$\mathbf{P} = \sqrt{\frac{2}{5(1+|a|^2)}} \begin{bmatrix} 1 \\ a^* \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}. \quad (12)$$

The derivation is omitted due to lack of space. We note that the precoder (12) is designed for OSIC receiver. Also note that (12) has rank one, and can be viewed as power allocation (vector $[2, 1]$) followed by transmit beamformer (vector $[1, a^*]^T$) pointing to the approximate MBER direction. We will refer to this scheme as OSIC with beamforming. It can be shown that the average BER for precoded OSIC can be approximated as

$$\bar{P}(\gamma_s; \mathbf{h}_1, a) \approx \frac{3}{20} \exp\left\{-\frac{4}{5}\gamma_s\|\mathbf{h}_1\|^2(1+|a|^2)\right\}, \quad (13)$$

which does not experience error floor effects.

V. Numerical Results and Discussions

We compare the BER's of several transmit schemes over fading channels.

Uncorrelated Rayleigh Fading: Fig. 1 is a plot of the average BER of a variety of transceivers in an uncorrelated Rayleigh fading channel. We observe that at a BER of 10^{-3}, power allocation scheme offers 0.8, 1.3 and 0.9 dB gains for ZF, SIC and OSIC receivers, respectively. We can also see that OSIC with beamforming, though designed for ill-conditioned channels, outperforms OSIC without power allocation, as well as outperforms SIC and ZF schemes at SNR's larger than 6 dB.

Correlated Ricean Fading: Correlated fading channels were simulated using the transmit correlation matrix

$$\mathbf{R}_t = \begin{bmatrix} 1 & j0.9 \\ -j0.9 & 1 \end{bmatrix},$$

which corresponds to scenarios with high spatial antenna correlation. Fig. 2 illustrates average BER's of all schemes studied in correlated Ricean fading channels with Ricean factor $K = 10$ dB. OSIC with beamforming outperforms all other schemes shown in the figure. At a BER of 10^{-3}, the SNR gain offered by OSIC with beamforming over OSIC with power allocation is around 4 dB. From Fig. 2, we can also see that SIC with power allocation outperforms OSIC without power allocation.

VI. Conclusions

Power allocation for dual transmit antenna spatial multiplexing is proposed in this paper. Minimizing error rate performance has been investigated. Transmit optimization for ill-conditioned channels is also studied. Simulation results show that the proposed methods offer superior performance over traditional spatial multiplexing without transmit optimization. Compared with more general precoding schemes, the proposed schemes reduce complexity and feedback overhead. As future work, performance degradation using power allocation versus precoding will be considered.

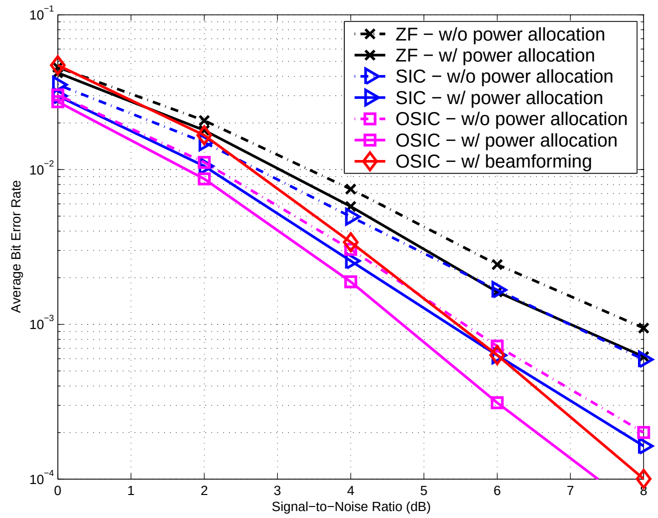

Fig. 1. Average BER performance in uncorrelated Rayleigh (well-conditioned) fading channel.

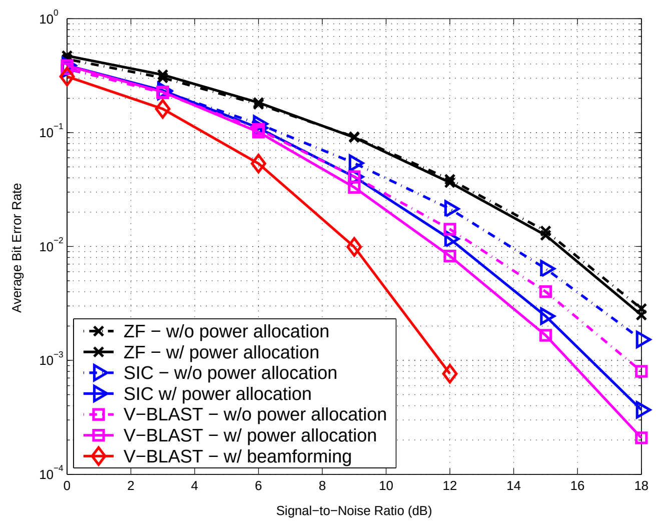

Fig. 2. Average BER performance in correlated Ricean (ill-conditioned) fading channel (K = 10 dB).

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