Optimum Beamforming for CDMA Systems with Signal-Cancellation Despreading

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***Abstract*— Recent research in the area of beamforming for the uplink of DS – CDMA systems with base-station antenna arrays involves methods which rely on processing the correlation matrices of both the front-end received signal and of the signal obtained after *code*-matched-*filtering*. However, these beamforming methods were devised assuming an idealized received chip pulse waveform (CPW) and may not perform as well for other CPWs. Moreover, in actual systems, the quantization precision for the front-end signal is usually very low and, therefore, when this signal is directly used to compute beamforming weights, poor performance may result. The effective quantization precision is known to improve significantly by *despreading* the received signal. The new approach to beamforming proposed herein still relies on the signal despread by *code-filtering*, but uses the correlation matrix of the signal obtained from another despreading process, called *signal-cancellation*, instead of that of the front-end signal. This approach yields fast and accurate beamforming methods which are less dependent on the actual received CPW.**

I. Introduction

Smart antenna arrays can improve the performance in cellular CDMA systems [1] by beamforming (spatial filtering). However, recently proposed beamforming methods for CDMA uplink [1] [2] assume an idealized received chip pulse waveform (CPW) and proceed as follows: (1) the baseband received signal is despread employing the *code-filtering* technique [1]; (2) the correlation matrices of the despread and front-end signal vectors are then appropriately processed to produce the beamformer even though, in practice, the front-end received signal may have low quantization precision compared to the effective quantization precision of the despread signal [3].

Herein we try to relax the CPW assumption and to avoid using the correlation matrix of the front-end received signal vector. We consider *signal-cancellation* despreading [4], which cancels the intended-signal contribution, and show how it can be advantageously exploited to devise accurate and fast beamformers which satisfy the above-mentioned requirements.

Paper organization: Section II describes the *code-filtering* and *signal-cancellation* despreading techniques and Section III presents beamforming methods. Finally, Section IV contains the conclusions of this work.

II. Despreading methods

Consider an asynchronous DS – CDMA cellular system with a base station antenna array receiving signals from M mobile stations. The Processing Gain of the system (an integer) is defined as the ratio between the symbol and chip periods, i.e., $\text{PG} \triangleq T/T_c$. We used the signal model from [4].

Previously-proposed beamforming methods for CDMA uplink [1] [2], described in Section III, employed the correlation matrix of a symbol-spaced sample of the front-end received signal vector [1]

$$\mathbf{R}_{\mathbf{x}_n} \triangleq E\{\mathbf{x}_n \mathbf{x}_n^H\} = \sum_{m=1}^{M} P_m \mathbf{a}_m \mathbf{a}_m^H + \sigma^2 \mathbf{I}. \tag{1}$$

where P_m is the power of the signal received from the mth mobile, $\mathbf{a}_m$ is the corresponding array response vector [1] (ARV) and σ^2 is the AWGN variance. The intended mobile has index n. To cope with oversampling, CDMA modems minimize the word length for the quantization of the front-end received signal. Actually, most modems quantize the pre-despreading signal to only 4 bits [3]. We will not investigate performance-degrading effects of such low precision on beamforming, but we will propose approaches that avoid the problem altogether.

A. Despreading using the code-filtering approach

The *code-filtering* post-correlation signal vector $\mathbf{y}_n$ is obtained by correlating the received signal vector with the chip sequence of the intended mobile [1] [4]. Its (scaled) correlation matrix can be written as [1]

$$\begin{aligned}\mathbf{R}_{\mathbf{y}_n} &\triangleq \frac{1}{T_c} E\{\mathbf{y}_n \mathbf{y}_n^H\} \\ &= \text{PG} \cdot P_n \mathbf{a}_n \mathbf{a}_n^H + \xi \sum_{m \neq n}^{M} P_m \mathbf{a}_m \mathbf{a}_m^H + \zeta\, \sigma^2 \mathbf{I},\end{aligned} \tag{2}$$

where ξ and ζ are scalars determined by the received CPW [1]. Equation (2) can be recast as

$$\mathbf{R}_{\mathbf{y}_n} = \mathbf{R}_{\mathbf{s}_n} + \mathbf{R}_{\text{IN}}, \tag{3}$$

to show the intended signal contribution

$$\mathbf{R}_{\mathbf{s}_n} \triangleq \text{PG} \cdot P_n \mathbf{a}_n \mathbf{a}_n^H, \tag{4}$$

and the interference-plus-noise contribution

$$\mathbf{R}_{\text{IN}} \triangleq \xi \sum_{m \neq n}^{M} P_m \mathbf{a}_m \mathbf{a}_m^H + \zeta\, \sigma^2 \mathbf{I}. \tag{5}$$

Naguib [1] assumed that the chip sequences transmitted by the mobiles are made of square pulses and that the channel has ideal low-pass characteristics, with bandwidth $1/T_c$. In this case $\xi = \zeta = 1$ [1]. When the received pulse is rectangular $\xi = 2/3$ [1].

B. Despreading using the signal-cancellation approach

If the front-end signal is correlated with the chip sequence obtained by changing the polarity of every other chip in the original chip sequence for the intended mobile, the contribution from this mobile vanishes (for even PG) [4] and the (scaled) correlation matrix of the *signal-cancellation* post-correlation signal vector $\mathbf{z}_n$ is [4]

$$\mathbf{R}_{\mathbf{z}_n} \triangleq \frac{1}{T_c} E\{\mathbf{z}_n \mathbf{z}_n^H\} = \xi \sum_{m \neq n}^{M} P_m \mathbf{a}_m \mathbf{a}_m^H + \zeta \sigma^2 \mathbf{I}. \quad (6)$$

Notice, from (5) and (6), that regardless of the CPW

$$\mathbf{R}_{\text{IN}} \equiv \mathbf{R}_{\mathbf{z}_n}. \quad (7)$$

Despreading virtually increases the quantization precision of $\mathbf{y}_n$ and $\mathbf{z}_n$, e.g., if $\mathbf{x}_n$ is quantized with 4 bits and if $PG = 64$ then the effective precision of the despread signals is 7 bits [3, Ch. 7].

III. Maximum-SINR beamforming methods

An optimum beamforming approach for $\mathbf{y}_n$ is to maximize the signal to interference-plus-noise ratio (SINR) at the beamformer output, i.e.

$$\text{SINR} = \frac{\mathbf{w}^H \mathbf{R}_{\mathbf{s}_n} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\text{IN}} \mathbf{w}}. \quad (8)$$

The maximum-SINR beamformer can be shown to be the dominant eigenvector of the matrix pencil $(\mathbf{R}_{\mathbf{s}_n}, \mathbf{R}_{\text{IN}})$, i.e.

$$\mathbf{w}_{opt} = \mathbf{d}\left(\mathbf{R}_{\mathbf{s}_n}, \mathbf{R}_{\text{IN}}\right). \quad (9)$$

For the numerical results shown later in this work the dominant eigenvectors are computed using the Power Method (PM) [5].

Using (4) and (8), one can also write [1]

$$\mathbf{w}_{opt} = \mathbf{R}_{\text{IN}}^{-1} \mathbf{a}_n. \quad (10)$$

For the beamformers from (9) and (10) the SINR from (8) is

$$\text{SINR}_{max} = PG \cdot P_n \mathbf{a}_n^H \mathbf{R}_{\text{IN}}^{-1} \mathbf{a}_n. \quad (11)$$

A. Beamforming based on $\mathbf{w}_{opt} = \mathbf{R}_{IN}^{-1}\, \mathbf{a}_n$

1) Naguib's method based on code-filtering and $\mathbf{R}_{\mathbf{x}_n}$: Assuming an ideally-bandlimited received CPW ($\xi = \zeta = 1$), Naguib [1] showed that $\mathbf{R}_{\text{IN}}$ can be computed as

$$\mathbf{R}_{\text{IN},Naguib} = \mathbf{R}_{\mathbf{x}_n} - \frac{1}{\text{PG}} \mathbf{R}_{\mathbf{y}_n}, \quad (12)$$

using (1) and (2). Nevertheless, a different CPW will result in different ξ and ζ [1] and then $\mathbf{R}_{\text{IN},Naguib}$ calculated as above only approximates the true $\mathbf{R}_{\text{IN}}$ from (5).

TABLE I
RELATIVE ERRORS IN CALCULATED $\mathbf{a}_n$ AND $\mathbf{R}_{\text{IN}}$

	$\frac{\|\|\mathbf{a}_{n,calc} - \mathbf{a}_{n,true}\|\|}{\|\|\mathbf{a}_{n,true}\|\|}$		$\frac{\|\|\mathbf{R}_{IN,calc} - \mathbf{R}_{IN,true}\|\|}{\|\|\mathbf{R}_{IN,true}\|\|}$	
Method	*CPW in [1]*	*rectang. CPW*	*CPW in [1]*	*rectang. CPW*
Naguib	1.0840	1.0809	≈ 0	0.5105
SCMD	≈ 0	≈ 0	0	0

In [1] Naguib claims, without demonstration, that $\mathbf{a}_n = \mathbf{d}(\mathbf{R}_{\mathbf{y}_n}, \mathbf{R}_{\mathbf{x}_n})$. For details, see [1], Section 3.2.2, page 62, and Section 4.1, pages 84-91. However, we found that in fact

$$\mathbf{a}_n = \mathbf{R}_{\mathbf{x}_n} \mathbf{v}, \quad \text{with} \quad \mathbf{v} = \mathbf{d}(\mathbf{R}_{\mathbf{y}_n}, \mathbf{R}_{\mathbf{x}_n}). \quad (13)$$

Thus, Naguib's method for ARV calculation is not accurate unless $\mathbf{R}_{\mathbf{x}_n}$ is proportional to the identity matrix, $\mathbf{I}$. Note that this method to calculate the ARV was used for DOA-estimation in [6] and exhibited poor performance.

2) Beamforming method based on signal-cancellation and code-filtering: Using the code-filtering and signal-cancellation covariance matrices from (2) and (6), the following matrix difference is taken

$$\mathbf{R}_{\mathbf{y}_n} - \mathbf{R}_{\mathbf{z}_n} = \text{PG} \cdot P_n \mathbf{a}_n \mathbf{a}_n^H, \quad (14)$$

which provides the ARV for the intended signal as

$$\mathbf{a}_n = \mathbf{d}(\mathbf{R}_{\mathbf{y}_n} - \mathbf{R}_{\mathbf{z}_n}). \quad (15)$$

Furthermore, (7) tells us that $\mathbf{R}_{\text{IN}} = \mathbf{R}_{\mathbf{z}_n}$. Thus, the optimum beamformer can be computed accurately using (10) without requiring the front-end signal or unrealistic assumptions on the CPW. Since this new method, denoted with SCMD from signal-cancellation matrix difference, and Naguib's method both require two correlation matrices, they have roughly the same computational complexity.

Error analysis results: consider a scenario with PG $= 128 \approx 21\,\text{dB}$, a 5-element uniform linear array with half-wavelength inter-element spacing (these settings are used for all the numerical results showed hereafter), the intended signal at $(90°; 0\,\text{dB})$ and two powerful interferers at $(85°; 10\,\text{dB})$ and $(95°; 10\,\text{dB})$ relative to (antenna endfire; noise variance). The correlation matrices $\mathbf{R}_{\mathbf{x}_n}$, $\mathbf{R}_{\mathbf{y}_n}$ and $\mathbf{R}_{\mathbf{z}_n}$ are computed using (1), (2) and (6), respectively. Table I shows the relative Frobenius norm [5] errors in $\mathbf{a}_n$ and $\mathbf{R}_{\text{IN}}$ when calculated with Naguib's method and the SCMD method proposed above, for the two extreme cases of CPW. Notice that while the latter method exactly determines $\mathbf{R}_{\text{IN}}$ and $\mathbf{a}_n$ in both cases $\mathbf{R}_{\text{IN}}$ computed with Naguib's method incurs a large error when the received CPW does not coincide with the CPW assumed in [1]. Moreover, Naguib's method for ARV-calculation method fails, as expected.

B. Beamforming based on dominant eigenvector

1) Choi's method based on code-filtering and $\mathbf{R}_{\mathbf{x}_n}$: for the ideally-bandlimited CPW [1], Choi [2] showed that maximizing the SINR in (8) amounts to maximizing

$$\frac{\mathbf{w}^H \mathbf{R}_{\mathbf{y}_n} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathbf{x}_n} \mathbf{w}}. \quad (16)$$

TABLE II

REQUIRED NUMBER OF PM ITERATIONS

Method	**Number of PM iterations**	
	CPW in [1]	*rectangular CPW*
Choi	11	10
SCB	2	2

The solution is obviously $\mathbf{w}_{opt} = \mathbf{d}(\mathbf{R}_{\mathbf{y}_n}, \mathbf{R}_{\mathbf{x}_n})$. Using (10) one can verify (13), supporting our claim that Naguib's method for ARV calculation is inaccurate.

2) Signal-Cancellation Beamforming (SCB): using (3) and (7), the SINR from (8) becomes

$$SINR = \frac{\mathbf{w}^H \left(\mathbf{R}_{\mathbf{y}_n} - \mathbf{R}_{\mathbf{z}_n}\right) \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathbf{z}_n} \mathbf{w}}, \qquad (17)$$

which is maximized by $\mathbf{w}_{opt} = \mathbf{d}(\mathbf{R}_{\mathbf{y}_n} - \mathbf{R}_{\mathbf{z}_n}, \mathbf{R}_{\mathbf{z}_n})$. This is the second newly-proposed beamforming approach.

PM Convergence Results: note that the PM, used to compute the dominant eigenvector, converges faster for a larger ratio of the dominant eigenvalue to the next-to-dominant eigenvalue [5]. Theoretically, $(\mathbf{R}_{\mathbf{y}_n} - \mathbf{R}_{\mathbf{z}_n})$ is a rank-one matrix while $\mathbf{R}_{\mathbf{y}_n}$ is full-rank. For the same scenario as in III-A.2, Table II shows the theoretical advantage in convergence speed of SCB over Choi's beamformer.

C. Numerical Simulation Results

The following results are for rectangular CPW and estimated correlation matrices. The intended signal arrives at $(90°; 0\,\mathrm{dB})$ and the interfering signals at $(90° \pm 5°, \pm 15°, \pm 25°, \pm 35°; 10\,\mathrm{dB})$. The simulated front-end received signal was quantized at computer precision, which is extremely high compared to that in actual CDMA modems). Thus, our results show a lower bound on the possible performance improvement attainable with signal-cancellation despreading.

In Fig. 1 note the poor performance of Naguib's method (even maximum-SNR beamforming, where $\mathbf{w}_{opt} = \mathbf{a}_n$, is better!). SCB yields SINR very close to that of theoretical maximum SINR beamforming. Choi's method has similar performance (curve not shown). However, Fig. 2 shows that, unlike for SCB, the number of PM iterations required for convergence in Choi's method is large and increases significantly when powerful interferers are added to the system, thus slowing the beamforming operation.

These results show that, even when the quantization effect is not considered, the proposed signal-cancellation-based beamforming methods are more accurate and faster than Naguib's [1] and Choi's [2] methods, respectively.

IV. CONCLUSION

In this work, for the uplink of a CDMA system setting, we showed that *signal-cancellation* despreading, which eliminates the intended signal, may be employed together with *code-filtering* despreading, which enhances the intended signal, to devise beamformers that outperform in terms of accuracy and speed those computed using the correlation matrix of the front-end received signal vector. Future work will consider other, more realistic, received chip waveforms.

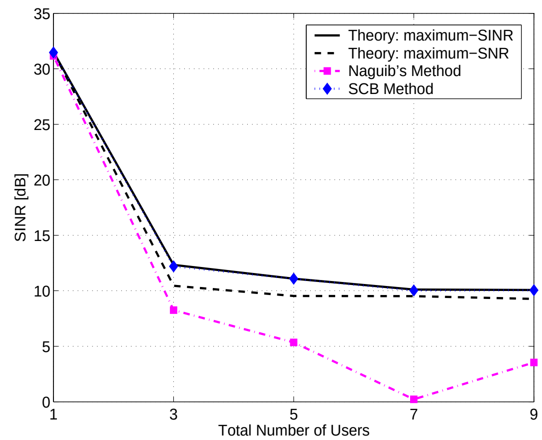

Fig. 1. SINR vs. the number of signal sources

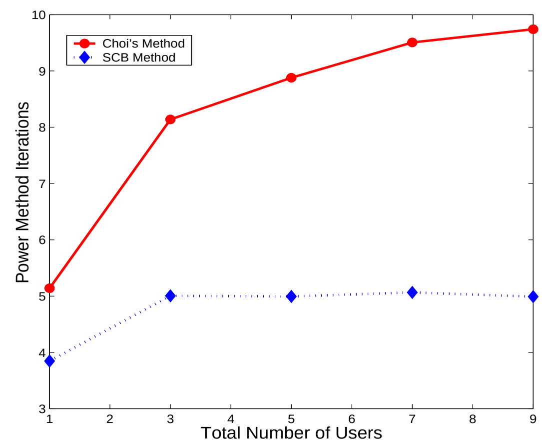

Fig. 2. Number of PM iterations vs. the number of signal sources

REFERENCES

[1] A. Naguib, "Adaptive antennas for CDMA wireless networks," Ph.D. dissertation, Stanford University, Stanford, CA, Aug. 1996.

[2] S. Choi, J. Choi, H.-J. Im, and B. Choi, "A novel adaptive beamforming algorithm for antenna array CDMA systems with strong interferers," *IEEE Trans. Veh. Technol.*, vol. 51, no. 5, pp. 808–816, Sept. 2002.

[3] S. Sheng and R. Brodersen, *Low Power CMOS Wireless Communications. A wideband CDMA system design*. Boston/Dordrecht/London: Kluwer Academic Publishers, 1998.

[4] T. Luo and S. D. Blostein, "Using signal cancellation for optimum beamforming in a cellular CDMA system," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing ICASSP'98*, vol. 4, May 1998, pp. 2493–2496. [Online]. Available: http://ipcl.ee.queensu.ca/

[5] G. H. Golub and C. F. V. Loan, *Matrix Computations*. Baltimore, Maryland: John Hopkins, 1985.

[6] Y. S. Song, H. M. Kwon, and B. J. Min, "Computationally efficient smart antennas for CDMA wireless communications," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1613–1628, Nov. 2001.