

Outage performance of an energy-efficient relaying protocol over Nakagami fading channels

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Abstract—This paper proposes a new *Clipped Gain* (CG) relaying protocol that takes advantage of the first-hop channel state information available at the relay to efficiently use the transmit power. The CG allows the relay to transmit only when the first-hop signal-to-noise ratio (SNR) is above the outage threshold, otherwise, relay transmission is stopped as it leads in that case to an outage. Closed-form expressions of CG outage probability over Nakagami fading channels are derived. The outage performance of the CG protocol is compared to those of practical *Amplify-and-forward* protocols including *Fixed Gain* (FG) and *Variable Gain* (VG) protocols, and a *Decode-and-forward* (DF) protocol. Numerical results show that the CG outperforms the other practical AF protocols and even approaches DF outage performance at low SNR while maintaining low complexity of an AF protocol.

I. INTRODUCTION

In wireless systems, cooperative communication can exploit the benefit of spatial diversity and combat heavy path loss without requiring multiple antennas at the receivers and transmitters. For practical networks, the use of relays is motivated by the need for simple, inexpensive terminals with limited power and a single antenna.

Single-user and single-relay systems have been intensively investigated. The protocols proposed for those systems include those with fixed relaying methods as well as more involved protocols with adaptive relaying methods. In [4], [6], [7], the authors have, respectively, studied the outage probability performance of two-hop systems employing *Fixed Gain* (FG), *Unlimited Gain* (UG), and *Variable Gain* (VG) relaying protocols over a Rayleigh fading channel. In [10], [12], the performance of those same relaying protocols over a Nakagami fading channel have been investigated.

Moreover, recent studies have focussed on new relaying protocols that can reduce power consumption and improve performance. In [2], [3], power control schemes for the relay and the source with perfect and limited feedback are proposed. In the perfect feedback scheme, the channel states (CS) for the first-, second-, and direct links are assumed to be completely known for computing the optimal source and relay transmit power to avoid outage. The source and the relay then transmit with the required level of power subject to power constraints;

otherwise, transmission is stopped. Although this scheme yields promising results, perfect feedback is obviously not practical.

In [3], power control schemes with limited feedback are considered. A codebook for transmission power is designed based on the quantized space of possible channel states. To every region of the space is associated a [source,relay] power vector which minimizes (or cancels) the outage probability while maintaining an equal power allocation. Based on the CS information, the destination maps the measured channel state ([source,relay] power vector) to an element of a predesigned code book and then sends the index to the source and relay through a noiseless feedback link. In this scheme, the transmission is never stopped and outage occurs when the transmit power in the lowest signal-to-noise ratio (SNR) region cannot avoid outage.

In this paper, we introduce a new *Clipped Gain* protocol which exploits the channel state information (CSI) of the first hop available at the relay to transmit more efficiently. This protocol allows transmit power savings by stopping the transmission when the quality of the first hop leads to an outage. The CG protocol can be seen as a power control scheme in which the transmit power at the relay is not constant: the power level is either zero or P depending on the realized CSI of the source-relay link. In the case of CG, only the first-link SNR is measured at the relay. The relay compares the measured SNR to the outage threshold and then decides to transmit or to switch off transmission without any feedback from the destination. Extra energy saving can be obtained if we consider a feedback link from the relay to the source using one-bit acknowledgement if the SNR of the first hop is above the threshold or not.

A closed-form expression for CG outage performance over Nakagami fading channels is derived. CG outage performance is subsequently compared to those of existing practical *Amplify-and-forward* (AF) protocols including *Fixed Gain* (FG) and *Variable Gain* (VG) protocols, and a *Decode-and-forward* (DF) protocol. Numerical results show that the CG relay protocol improves outage performance with negligible added relay complexity. Therefore, the CG is well-suited for practical cellular relay networks.

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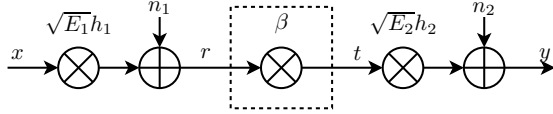


Fig. 1. Dual-hop communication system model

II. SYSTEM AND CHANNEL MODEL

We consider a two-hop communication channel in which a source is communicating with a destination through a relay terminal. The direct link is assumed to be in a deep fade and therefore is omitted. The relay amplifies its input signal by a factor β which is defined by the relaying protocol employed.

The subscripts $_1$ and $_2$ are used to denote parameters, statistics, and random variables associated with the first- and second-hop link of a two-hop communication model as depicted in Figure 1. $\varepsilon[\cdot]$ represents expectation operator. The source transmits a signal $x(t)$ that is affected by path loss, shadowing and flat fading of the first hop link before reaching the relay terminal. Additionally, white Gaussian noise with variance σ_1^2 is added to the received signal at the relay. $x(t)$ is assumed to have unit average power. The received signal at the relay terminal R can be written as

$$r(t) = \sqrt{E_1}h_1x(t) + n_1, \quad (1)$$

where E_1 is the constant average channel energy factor and h_1 is a complex Gaussian random variable. In our model, E_1 represents the effect of path loss and shadowing and h_1 models the effect of flat fading. The first-hop link coefficient is then $\sqrt{E_1}h_1$.

The signal is amplified at the relay R by a gain factor β before being forwarded to the destination. Thus, the received signal at the destination can then be written as

$$y(t) = \sqrt{E_2}h_2\beta \cdot r(t) + n_2, \quad (2)$$

where $\sqrt{E_2}h_2$ is the second-hop channel coefficient and n_2 is the additional noise at the destination with variance σ_2^2 .

Combining (1) and (2), the equivalent end-to-end SNR can be written as

$$\gamma_{eq} = \frac{E_1E_2 |h_1 \cdot \beta h_2|^2}{E_2 |\beta h_2|^2 \sigma_1^2 + \sigma_2^2}. \quad (3)$$

The average SNR of the first and second hop can be defined as: $\gamma_1 = \bar{\gamma}_1|h_1|^2$ and $\gamma_2 = \bar{\gamma}_2|h_2|^2$ where $\bar{\gamma}_1 = \frac{E_1}{\sigma_1^2}$ and $\bar{\gamma}_2 = \frac{E_2}{\sigma_2^2}$ are, respectively, the first and second link average SNR.

III. AMPLIFY-AND-FORWARD RELAYING PROTOCOL

In this section, we describe the existing AF protocols (VG, UG, and FG) and provide the references for the closed-form outage probability expressions of those AF protocols.

A. Variable Gain protocol

The VG protocol is a practical relaying protocol that equalizes the fluctuations of the backward (or first-hop) channel

based on the knowledge of the first-hop instantaneous fading amplitude. By limiting the transmit power at the relay to unity, the VG protocol is [8], [11]:

$$\beta_{VG} = \frac{1}{\sqrt{E_1 |h_1|^2 + \sigma_1^2}}. \quad (4)$$

The closed-form expressions of VG outage probability over Rayleigh fading channel and Nakagami fading channels have been derived in [7] and [12] respectively.

B. Unlimited Gain protocol

The UG relaying protocol is a benchmark protocol able to invert the channel regardless of its magnitude:

$$\beta_{UG} = \frac{1}{\sqrt{E_1 |h_1|^2}}. \quad (5)$$

Indeed, (4) is a tight upper bound of (5) in which the additive noise power at the relay has been neglected. As a result, when the fading coefficient value of the first hop drops close to zero as encountered under Rayleigh fading conditions, infinite relay gain and therefore infinite transmit power is required. UG protocol is not applicable in practice; however, as argued in [6], [11], its performance may serve as benchmark for practical protocols.

The closed-form expressions of UG outage probability over Rayleigh fading channels and Nakagami fading channels have been derived in [6] and [10] respectively.

C. Fixed Gain protocol

The FG relaying protocol uses the first-hop average channel energy instead of the fading amplitude. By limiting the transmit power at the relay to unity, the FG protocol is [11]:

$$\beta_{FG} = \frac{1}{\sqrt{E_1 + \sigma_1^2}}. \quad (6)$$

The closed-form expressions of FG outage probability over Rayleigh fading channels and Nakagami fading channels have been derived in [7] and [12], respectively.

IV. CLIPPED GAIN RELAYING PROTOCOL

Assuming that the SNR of the first-hop channel can be accurately measured at the relay, we propose a new Clipped Gain protocol which takes advantage of this knowledge at the relay to improve the outage performance.

For a two-hop communication channel with a general AF relay gain β , if the first-hop SNR falls below the outage threshold, the system will be in outage regardless of the relay-destination SNR [9, Lemma 1]. Using this result, we introduce a *Clipped Gain* protocol that transmits only during good source-relay channel conditions.

Let us assume that the relay knows the threshold value γ_{th} of the receiver. This is a reasonable assumption because outage threshold is a system parameter. The gain could then be set to zero when $\gamma_1 \leq \gamma_{th}$, avoiding loss in performance whenever outage is certain. The resulting *Clipped Gain* can therefore

help saving transmit power and reducing interference in the network:

$$\beta_{CG}^2 = \begin{cases} 0, & \gamma_1 < \gamma_{th} \\ \frac{G}{E_1|h_1|^2}, & \gamma_1 \geq \gamma_{th}. \end{cases} \quad (7)$$

The equivalent end-to-end SNR of the two-hop communication channel using the relay gain β_{CG}^2 (7) can be written as:

$$\gamma_{eq}^{CG} = \begin{cases} 0, & \gamma_1 < \gamma_{th} \\ \frac{G\gamma_1\gamma_2}{\gamma_1 + G\gamma_2}, & \gamma_1 \geq \gamma_{th} \end{cases} \quad (8)$$

where G is the normalization factor that sets the average transmit power to unity, i.e., $\varepsilon[|\beta r(t)|^2] = \varepsilon[E_1|h_1\beta|^2] + \varepsilon[\sigma_1^2\beta^2] = 1$. The factor G can be evaluated for Nakagami fading channels to yield

$$G_N = \frac{\Gamma[m_1]}{\frac{m_1}{\bar{\gamma}_1}\Gamma\left[(m_1-1), \frac{\gamma_{th}m_1}{\bar{\gamma}_1}\right] + \Gamma\left[m_1, \frac{\gamma_{th}m_1}{\bar{\gamma}_1}\right]} \quad (9)$$

where $\Gamma[\cdot]$ and $\Gamma[\cdot, \cdot]$ are, respectively, the Gamma and incomplete Gamma function defined in [5, Eq. 8.310.1] and in [1, Eq. 6.5.3]. The derivation of (9) is given in Appendix A.

In the case of a Rayleigh fading channel, $m_1 = 1$ and G_N reduces to

$$G_R = \frac{1}{e^{-\frac{\gamma_{th}}{\bar{\gamma}_1}} + \frac{1}{\bar{\gamma}_1}E_1\left[\frac{\gamma_{th}}{\bar{\gamma}_1}\right]} \quad (10)$$

where $E_1[\cdot]$ is the exponential integral function defined in [1, Eq. 5.1.1].

The normalization factor allows a constant average relay transmit power and a finite relay gain yielding an applicable protocol for the relay channel. Moreover, the relay transmit power is more efficiently employed. Namely, the CG relay is saving power when the condition of the first link is poor, i.e., the first link SNR is below γ_{th} , and subsequently uses this power to transmit when the SNR of the first link is above γ_{th} .

V. PERFORMANCE ANALYSIS

In this section, we derive closed-form expressions of CG outage probability over Nakagami fading channels. From those results, a more compact expression for outage probability over a Rayleigh fading channel is also provided. We relate the CG outage probability expressions to the previously derived result for UG protocol. The new outage performance results for CG protocol can also be shown to match the simulations.

A. Outage probability for Clipped Gain relay

Given that the *Clipped Gain* protocol prevents transmission in the cases where the first hop SNR drops below the outage threshold γ_{th} and that those cases always lead to an outage event independently of the relay protocol employed, the outage probability for Nakagami fading channels can be calculated from (8) as

$$\begin{aligned} P_{out,N}^{CG}(\gamma_{th}) &= P\left[\frac{G_N\gamma_1\gamma_2}{\gamma_1 + G_N\gamma_2} < \gamma_{th}\right] \\ &= \int_0^\infty P\left[\frac{G_N\gamma_1\gamma_2}{\gamma_1 + G_N\gamma_2} < \gamma_{th} | \gamma_2\right] p_{\gamma_2}(\gamma_2) d\gamma_2 \end{aligned} \quad (11)$$

where $p_{\gamma_2}(\gamma_2)$ is the gamma probability density function of the second hop SNR.

The resulting closed-form expression for *Clipped Gain* outage probability is:

$$\begin{aligned} P_{out,N}^{CG}(\gamma_{th}) &= 1 - \frac{(m_1-1)!e^{-\gamma_{th}\left(\frac{m_1}{\bar{\gamma}_1} + \frac{m_2}{\bar{\gamma}_2 G_N}\right)}}{\left(\frac{\bar{\gamma}_2}{m_2}\right)^{m_2} \Gamma(m_2)\Gamma(m_1)} \\ &\sum_{k=0}^{m_1-1} \sum_{l=0}^k \sum_{r=0}^{m_2-1} \frac{1}{k!} \binom{k}{l} \binom{m_2-1}{r} \left(\frac{m_1\gamma_{th}}{\bar{\gamma}_1}\right)^k \left(\frac{\gamma_{th}}{G_N}\right)^{\frac{(-1-r+l)}{2} + m_2} \\ &2 \cdot \left(\frac{m_1\bar{\gamma}_2\gamma_{th}}{\bar{\gamma}_1 m_2}\right)^{\frac{(r-l+1)}{2}} K_{l-r-1}\left(2\sqrt{\frac{m_1 m_2 \gamma_{th}^2}{\bar{\gamma}_1 \bar{\gamma}_2 G_N}}\right) \end{aligned} \quad (12)$$

where $\binom{\cdot}{\cdot}$ denotes the binomial coefficient and $K_\nu(\cdot)$ is the ν th order modified Bessel function of the second kind defined in [5, Eq. 8.432]. The detailed derivation of (12) is provided in Appendix B.

Taking $m_1 = m_2 = 1$ and using simple algebraic manipulations and properties of modified Bessel functions [1, Eq. 9.6.6], (12) reduces to the following compact closed-form expression of the outage probability of *Clipped Gain* over the Rayleigh fading channel:

$$P_{out,R}^{CG}(\gamma_{th}) = 1 - e^{-\gamma_{th}\left(\frac{1}{\bar{\gamma}_1} + \frac{1}{G_R\bar{\gamma}_2}\right)} \cdot \frac{2\gamma_{th}}{\sqrt{G_R\bar{\gamma}_1\bar{\gamma}_2}} K_1\left(\frac{2\gamma_{th}}{\sqrt{G_R\bar{\gamma}_1\bar{\gamma}_2}}\right), \quad (13)$$

where $K(\cdot)$ is the first-order modified Bessel function of the second kind defined in [5, Eq. 9.6.22].

Furthermore, for $G_R = 1$ and $m_1 = m_2 = 1$, (12) reduces to the expression of outage probability for UG protocol over the Rayleigh fading channel found in [6].

From (12), a closed-form expression for outage probability for UG protocol over Nakagami fading channels can be derived:

$$P_{out,N}^{UG} = P_{out,N}^{CG} \Big|_{G_N=1}. \quad (14)$$

The thereby obtained expression (14) is less compact than the elegant closed-form expression derived in [10] but applies for a more general channel model in which the first and second links do not have the same average SNR.

B. Outage probability for Decode-and-forward over Nakagami fading channels

Using similar arguments as in [6] for computing the outage probability of a DF relay system over a Rayleigh fading channel, the outage probability of DF over Nakagami fading channels can be obtained assuming that an outage in a DF relay system occurs if either one of the two links is in outage. Assuming independent outage events,

$$P_{out,N}^{DF}(\gamma_{th}) = 1 - \frac{\Gamma\left[m_1, \frac{m_1\gamma_{th}}{\bar{\gamma}_1}\right]}{\Gamma[m_1]} \cdot \frac{\Gamma\left[m_2, \frac{m_2\gamma_{th}}{\bar{\gamma}_2}\right]}{\Gamma[m_2]}. \quad (15)$$

C. Outage performance comparison

In this section, we compare outage performance of the proposed CG relay protocol to those of FG, VG, UG, and DF

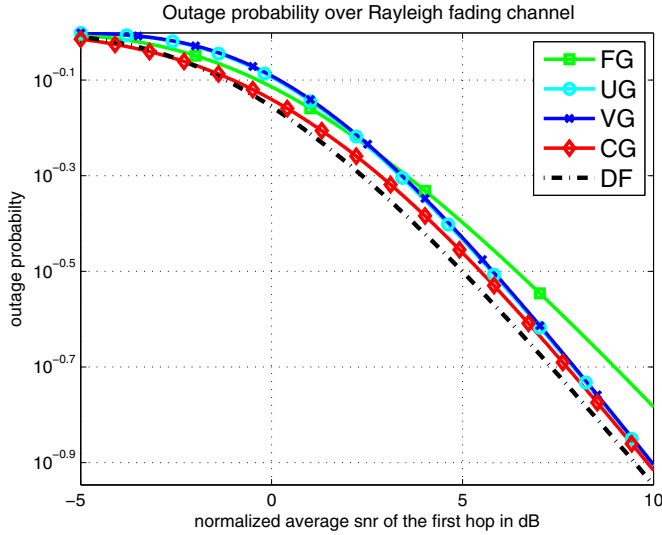


Fig. 2. Outage performance vs average SNR of FG, UG, VG, and CG over a Rayleigh fading channel for $\gamma_2 = 5 * \gamma_1$ and outage threshold $\gamma_{th} = 10dB$

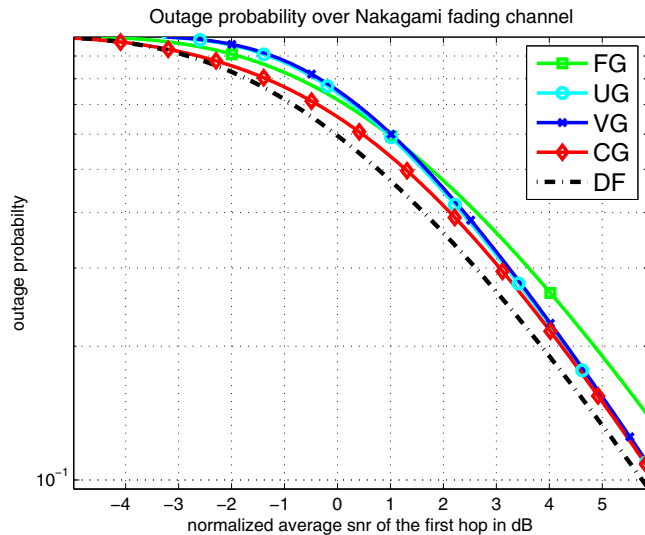


Fig. 3. Outage performance vs average SNR of FG, UG, VG, and CG over Nakagami fading channel for $\gamma_2 = 5 * \gamma_1$ and outage threshold $\gamma_{th} = 10dB$

relay protocols. Figures 2 and 3 show the outage probabilities curves over Rayleigh fading and Nakagami fading channels, respectively. The outage probability curves are obtained by numerically computing the closed-form expressions (12), (13), (14), given above and the previously published closed-form expressions in [4], [6], [7], [10], [12].

The outage threshold, γ_{th} is taken to be 10 dB and the relay-destination (or second-hop) average SNR is taken to be five times the source-relay (or first-hop) average SNR: $\gamma_2 = 5 * \gamma_1$. For plotting Figure 3, Nakagami parameters m_1 and m_2 are taken to be equal to 2 and 5, respectively.

Figures 2 and 3 show that the CG relay protocol outperforms

the FG, VG, and UG protocols for low to mid-range SNR and has similar performance to VG and UG for high SNR. Indeed, the CG protocol saves transmission power during non-favorable events (i.e., when the realized first-hop SNR is below the outage threshold leading the system to outage) and its efficiency is thereby optimized for low average SNR regions where those non-favorable events are more probable. Therefore, for low to mid-range SNR, the CG protocol yields better performance than other AF protocols.

Moreover, numerical results show that the outage performance of the CG protocol is similar to that of the DF protocol for low SNR. For low to mid-range SNR, even though the DF protocol outperforms all the AF protocols, the CG still yields better outage performance than those of VG and FG. At high average SNR, all the protocols including CG and DF yield the same diversity gain.

Thus, the CG protocol represents a good tradeoff between the VG and DF protocols. Indeed, the CG protocol, with only negligible additional complexity compared to the VG protocol to calculate the SNR of the first-hop, has performance close to that of the DF relay protocol at low to mid-range SNR regions.

VI. CONCLUSION

The *Clipped Gain* protocol takes advantage of the first-hop CSI available at the relay to save transmit power in the low SNR region. As a result, the CG protocol needs only negligible additional relay complexity compared to the existing AF protocols to yield outage probability performance close to that of a DF relay. The CG protocol, therefore, can be viewed as an attractive power efficient protocol.

APPENDIX A NORMALIZATION FACTOR

The relay transmit power is set to unity. Thus,

$$\varepsilon[|t|^2] = \varepsilon[E_1 | h_1 \beta |^2] + \varepsilon[\sigma_1^2 \beta^2] = 1. \quad (16)$$

For $\gamma_1 > \gamma_{th}$, using (7) one can write: $E_1 | h_1 \beta |^2 = G_N$ and $\sigma_1^2 \beta^2 = \frac{G_N}{\gamma}$.

Then,

$$\varepsilon[\sigma_1^2 \beta^2] = \varepsilon[f_{\gamma_1}(\gamma)] = \int_{\gamma_{th}}^{\infty} \frac{G_N}{\gamma} p_{\gamma_1}(\gamma) d\gamma \quad (17)$$

with $p_{\gamma_1}(\gamma) = \frac{\gamma^{m_1-1} e^{-\frac{\gamma}{\gamma_1}}}{\left(\frac{\gamma_1}{m_1}\right)^{m_1} \Gamma(m_1)}$ as the γ_1 is gamma distributed.

By changing the integration variable, $t = \frac{\gamma}{\gamma_{th}}$, (17) can be written in terms of $E_n(z) = \int_1^{\infty} \frac{e^{-zt}}{t^n} dt$ [1, Eq. 5.1.4]. Then, using the formula [1, Eq. 5.1.45], (17) becomes:

$$\varepsilon[\sigma_1^2 \beta^2] = \frac{G_N}{\Gamma[m_1]} \frac{m_1}{\gamma_1} \Gamma \left[m_1 - 1, \frac{m_1 \gamma_{th}}{\gamma_1} \right]. \quad (18)$$

Substituting $t = \frac{\gamma}{\gamma_1}$, the second part of the expression (16) can be written as:

$$\varepsilon[E_1 | h_1 \beta |^2] = \int_{\gamma_{th}}^{\infty} G_N p_{\gamma_1}(\gamma) d\gamma = \frac{G_N}{\Gamma[m_1]} \Gamma \left[m_1, \frac{m_1 \gamma_{th}}{\gamma_1} \right]. \quad (19)$$

Combining (16), (17), and (19) along with simple algebraic manipulations gives 9.

APPENDIX B OUTAGE PROBABILITY

Similarly to the derivation steps for VG and FG outage probability expressions in [12], (11) can be written as the sum of two integrals I_1 and I_2 :

$$P_{out,N}^{CG} = \int_0^{\frac{\gamma_{th}}{G_N}} P \left[\gamma_1 > \frac{G_N \gamma_2 \gamma_{th}}{G_N \gamma_2 - \gamma_{th}} \middle| \gamma_2 \right] p_{\gamma_2}(\gamma_2) d\gamma_2 + \int_{\frac{\gamma_{th}}{G_N}}^{\infty} P \left[\gamma_1 \leq \frac{G_N \gamma_2 \gamma_{th}}{G_N \gamma_2 - \gamma_{th}} \middle| \gamma_2 \right] p_{\gamma_2}(\gamma_2) d\gamma_2 = I_1 + I_2. \quad (20)$$

Then, I_1 and I_2 can be expressed as

$$I_1 = \int_0^{\frac{\gamma_{th}}{G_N}} 1 \cdot p_{\gamma_2}(\gamma_2) d\gamma_2 = 1 - \frac{\Gamma \left[m_2, \frac{m_2 \gamma_{th}}{\bar{\gamma}_2 G_N} \right]}{\Gamma[m_2]} \quad (21)$$

$$I_2 = \int_{\frac{\gamma_{th}}{G_N}}^{\infty} \left[1 - \frac{\Gamma \left[m_1, \frac{m_1 \gamma_{th} G_N \gamma_2}{\bar{\gamma}_1 (\gamma_2 G_N - \gamma_{th})} \right]}{\Gamma[m_1]} \right] \cdot p_{\gamma_2}(\gamma_2) d\gamma_2 \quad (22)$$

$$= \frac{\Gamma \left[m_2, \frac{m_2 \gamma_{th}}{\bar{\gamma}_2 G_N} \right]}{\Gamma[m_2]} - J$$

where $\Gamma(\cdot)$ and $\Gamma(\cdot, \cdot)$ are, respectively, the Gamma and incomplete Gamma functions defined in [5, Eq. 8.310.1] and in [1, Eq. 6.5.3], and $J = \int_{\frac{\gamma_{th}}{G_N}}^{\infty} \frac{\Gamma \left[m_1, \frac{m_1 \gamma_{th} G_N \gamma_2}{\bar{\gamma}_1 (\gamma_2 G_N - \gamma_{th})} \right]}{\Gamma[m_1]} \cdot p_{\gamma_2}(\gamma_2) d\gamma_2$. With the following notation: $y = \frac{\gamma_{th}}{G_N}$, $b = \frac{m_2}{\bar{\gamma}_2}$, $d = \frac{m_1 \gamma_{th}}{\bar{\gamma}_1}$, $a = m_2$, $c = m_1$, $e = 0$, J can be written in a more general form as:

$$J = \int_y^{\infty} x^{a-1} e^{-bx} \Gamma \left[c, d \left(\frac{x+e}{x-y} \right) \right] dx. \quad (23)$$

As d and e are real numbers and restricting c to be a positive integer, the incomplete gamma function in Equation (23) can be written as follows using [5, Eq. 8.352.2, Eq. 1.111] :

$$\Gamma \left[c, d \left(\frac{x+e}{x-y} \right) \right] = (c-1)! e^{-d} e^{-d \frac{y+e}{x-y}} \sum_{k=0}^{c-1} \sum_{l=0}^k \frac{d^k}{k!} \binom{k}{l} \left(\frac{y+e}{x-y} \right)^l. \quad (24)$$

Therefore, after transformation of the integration variable, J becomes:

$$J = (c-1)! e^{-(d+by)} \sum_{k=0}^{c-1} \sum_{l=0}^k \sum_{r=0}^{a-1} \frac{d^k}{k!} \binom{k}{l} \binom{a-1}{r} (y+e)^l y^{a-r-1} \int_0^{\infty} x^{r-l} e^{-bx} e^{-d \frac{y+e}{x}} dx. \quad (25)$$

The integral in Equation (25) can be solved using [5, Eq. 3.471.9]. Thus, J can be expressed with the initial variables (replacing the notation $y = \frac{\gamma_{th}}{G_N}$, $b = \frac{m_2}{\bar{\gamma}_2}$, $d = \frac{m_1 \gamma_{th}}{\bar{\gamma}_1}$, $a =$

m_2 , $c = m_1$, $e = 0$ in its expression) as:

$$J = \frac{(m_1 - 1)! e^{-\gamma_{th} \left(\frac{m_1}{\bar{\gamma}_1} + \frac{m_2}{\bar{\gamma}_2 G_N} \right)}}{\left(\frac{\bar{\gamma}_2}{m_2} \right)^{m_2} \Gamma(m_2) \Gamma(m_1)} \sum_{k=0}^{m_1-1} \sum_{l=0}^k \sum_{r=0}^{m_2-1} \frac{1}{k!} \binom{k}{l} \binom{m_2-1}{r} \left(\frac{m_1 \gamma_{th}}{\bar{\gamma}_1} \right)^k \left(\frac{\gamma_{th}}{G_N} \right)^{\frac{(-1-r+l)}{2} + m_2} \cdot 2 \cdot \left(\frac{m_1 \bar{\gamma}_2 \gamma_{th}}{\bar{\gamma}_1 m_2} \right)^{\frac{(r-l+1)}{2}} K_{l-r-1} \left(2 \sqrt{\frac{m_1 m_2 \gamma_{th}^2}{\bar{\gamma}_1 \bar{\gamma}_2 G_N}} \right). \quad (26)$$

Combining the expressions (20), (26), (21), and (22) gives the outage probability closed-form expression over Nakagami fading channels (12).

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