Bounds on Timing Jitter Estimation in Cooperative Networks

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***Abstract*— Successful collaboration and resource sharing in cooperative networks require accurate estimation of multiple timing jitters. When combined with signal processing algorithms the estimated timing jitters can be applied to mitigate the resulting inter-symbol interference (ISI) and improve performance. In this paper we seek to derive the Cramer-Rao lower bounds (CRLB) for timing jitter estimation in distributed cooperative networks. In addition to being a benchmark for judging the performance of timing jitter estimators, the CRLBs can be applied to determine the effect of the choices of training sequence and cooperative protocol on timing jitter estimation.**

I. Introduction

SYNCHRONOUS cooperative communication systems have been shown to result in multiplexing and diversity gain through resource sharing amongst nodes within the network [1], [2]. However, effective cooperation requires synchronization parameters such as timing jitter and frequency offset to be accurately estimated [3]. In this paper we derive the bounds on timing jitter estimation in distributed *amplify-and-forward (AF)* and *decode-and-forward (DF)* relaying cooperative networks.

Cooperative communication systems are affected by multiple timing jitters, due to simultaneous transmissions from multiple nodes with different oscillators. The presence of timing jitter, results in *inter-symbol interference (ISI)* and *signal to noise ratio (SNR)* loss [4], [5]. In [5] the effect of timing synchronization errors on the performance of cooperative relay networks is analyzed, where it is demonstrated that timing jitters much smaller than the symbol interval can result in considerably higher pairwise error probabilities. Thus, timing jitter estimation accuracy plays an important role on the performance of cooperative networks.

The *Cramer-Rao lower bound (CRLB)* [6], is used as a quantitative benchmark for the performance of timing jitter estimators [7]–[9]. Moreover, the CRLB can be applied to determine the effect of network protocol and choice of training sequence on timing jitter estimation accuracy in cooperative systems.

The CRLB of timing jitter estimation for point-to-point *single-input-single-output (SISO)* and *multi-input-multi-output (MIMO)* systems is derived in [7] and [8], respectively, where it is demonstrated that the choice of the training sequence significantly impacts the estimation accuracy. In [9] the CRLB for timing jitter estimation in DF cooperative networks is derived. However, the results are limited to the case of Gaussian channels, the significance of the training sequence and network topology on the estimation performance is not investigated, and the analysis is limited to the case of DF. To the best of our knowledge CRLB for timing jitter estimation in AF cooperative networks has not been addressed to date.

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This paper first derives the CRLBs for timing jitter estimation for DF and AF multi-node cooperative systems for Rician fading and Gaussian channels, respectively. In the case of AF a new baseband receiver structure is proposed that enables accurate timing jitter estimation at the relays and destination. Next the CRLBs are used to determine the effect of training sequence design and number of relays on timing jitter estimation in distributed DF and AF cooperative networks.

This paper is organized as follows: Section II derives and analyzes the CRLBs for DF and AF cooperative networks and investigates the effect of different training sequences on the CRLB for timing jitter estimation. Section III presents numerical results that investigate the effect of number of relays on timing synchronization in cooperative networks.

Notation: italic letters (ϕ) are scalars, bold lower case letters ($\boldsymbol{\phi}$) are vectors, bold upper case letters ($\mathbf{\Phi}$) are matrices, $\mathbf{\Phi}_{k,m}$ represents the kth row and mth column element of $\mathbf{\Phi}$, $\odot$ stands for Schur (element-wise) product, and $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and Tr$(\cdot)$ denote conjugate, transpose, conjugate transpose (hermitian), and trace, respectively.

II. Cramer-Rao Lower Bound

In this section the CRLB for timing jitter estimation for half-duplex multi-node cooperative networks is derived.

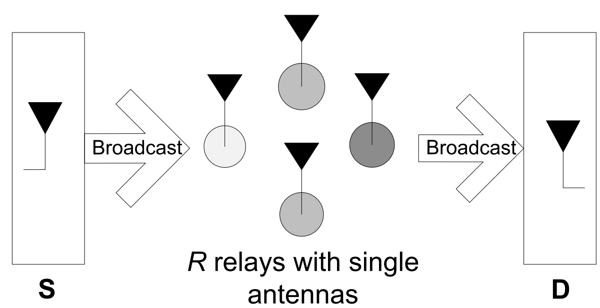

Fig. 1. The system model for the cooperative network.

Throughout this paper it is assumed that in *phase I* the source broadcasts its *training sequence (TS)* to the relays and in *phase II* to efficiently estimate timing jitter, the relays transmit R distinct TSs simultaneously to the destination, Fig. 1. Frequency flat-fading channels are considered and the channels are assumed to not change over the length of the TS. Timing jitters, $\boldsymbol{\tau}$, are modeled as unknown non-random parameters with no assumptions on their distributions. Note that these assumption are in line with previous timing jitter estimation analyses performed for point-to-point systems in [7]–[9].

According to Fig. 2 the sampled baseband received training signal model, $r_k(iT_s)$, at the kth relay is given by

$$r_k(iT_s) = \sqrt{p^{[s]}} h_k \sum_{n=0}^{L-1} t^{[s]}(n) g(iT_s - nT - \tau_k^{[sr]} T) + v_k(iT_s), \quad (1)$$

where:

- L and T denote the length of the TS and the symbol duration, respectively, $g(t)$ is the transmitted pulse,
- T_s is the sampling time, where $T = NT_s$ and $N \geq 1$,
- $\mathbf{t}^{[\mathbf{s}]} \triangleq [t^{[s]}(0), \cdots, t^{[s]}(L-1)]^T$ is the **known** TS broadcast from the source to the relays, where without loss of generality, it is assumed that unit-amplitude TSs are transmitted ($|t^{[s]}(n)|^2 = 1 \ \forall n$),
- $\tau_k^{[sr]}$ is the normalized timing jitter from the source to the kth relay,
- h_k is the unknown channel gain from the source to the kth relay, $p^{[s]}$ is the transmitted, and
- $v_k(t)$ is the *additive Gaussian noise (AGN)* at the kth relay with mean zero and variance $\sigma_{v_k}^2$, denoted by $\mathcal{CN}(0, \sigma_{v_k}^2)$.

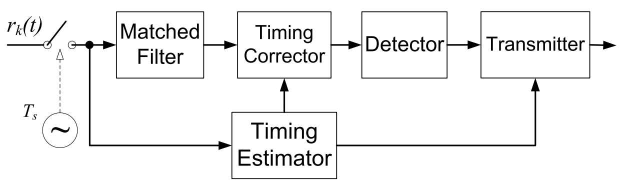

Fig. 2. Block diagram of a baseband receiver at the relays and destination.

Eq. (1) can be represented in matrix and vector form as

$$\mathbf{r_k} = \sqrt{p^{[s]}} h_k \mathbf{G_k^{[sr]}}(\tau_k^{[sr]}) \mathbf{t}^{[\mathbf{s}]} + \mathbf{v_k}, \quad (2)$$

where:

- $\mathbf{r_k} \triangleq [r_k(0), r_k(1), \cdots, r_k(NL-1)]^T$,
- $\mathbf{t}^{[\mathbf{s}]} \triangleq [t^{[s]}(0), t^{[s]}(1), \cdots, t^{[s]}(L-1)]^T$,
- $\mathbf{v_k} \triangleq [v_k(0), v_k(1), \cdots, v_k(NL-1)]^T$, and
- $\mathbf{G_k^{[sr]}}$ is an $NL \times N$ matrix, where $\left(\mathbf{G_k^{[sr]}}\right)_{m,l} \triangleq g((m-1)T_s - (l-1)T - \tau_k^{[sr]} T)$.

A. Decode-and-Forward Cooperative Networks

Based on the above and the fact that the TS transmitted from the relays are known, the sampled received **training** signal model at the destination, $\mathbf{y} = [y(0), y(2), \cdots, y(NL-1)]^T$, for a DF cooperative network consisting of R relay nodes is given by

$$\mathbf{y} = \sum_{k=1}^{R} \left(\sqrt{p_k^{[r]}} f_k \mathbf{G_k^{[rd]}}(\tau_k^{[rd]}) \mathbf{t_k^{[r]}} \right) + \mathbf{w}, \quad (3)$$

where:

- $\left(\mathbf{G_k^{[rd]}}\right)_{m,l} \triangleq g((m-1)T_s - (l-1)T - \tau_k^{[rd]} T)$ and $\tau_k^{[rd]}$ is the timing jitter from the kth relay to the destination,
- $\mathbf{t_k^{[r]}} \triangleq [t_k^{[r]}(0), \cdots, t_k^{[r]}(L-1)]^T$ is the kth relay's TS,
- f_k represents the unknown channel gain from the kth relay to the destination, $p_k^{[r]}$ is the transmitted power, and
- $\mathbf{w} \triangleq [w(0), w(2), \cdots, w(NL-1)]^T$ is the AGN at the destination with $w(n)$ distributed as $\mathcal{CN}(0, \sigma_w^2)$.

According to (1) and (3), for DF networks two sets of timing jitters, $\boldsymbol{\tau}^{[\mathbf{sr}]} \triangleq \{\tau_1^{[sr]}, \cdots, \tau_R^{[sr]}\}$ and $\boldsymbol{\tau}^{[\mathbf{rd}]} \triangleq \{\tau_1^{[rd]}, \cdots, \tau_R^{[rd]}\}$ need to be estimated. Moreover, since the DF protocol requires the signals at the relays to be decoded, $\boldsymbol{\tau}^{[\mathbf{sr}]}$ needs to be estimated and equalized at the relays, where $\mathbf{t}^{[\mathbf{s}]}$ received in *phase I* is used for timing jitter estimation similar to that of a point-to-point system.

According to the received signal model in (3), the timing jitters, $\boldsymbol{\tau}^{[\mathbf{rd}]}$ need to be jointly estimated at the destination. For notational clarity, $(\cdot)^{[\mathbf{DF}]}$ is used instead of $(\cdot)^{[\mathbf{rd}]}$ below.

Derivation of the CRLB for the joint estimation of $\boldsymbol{\tau}^{[DF]}$: The CRLB is derived for the general case of zero-mean additive Gaussian noise, $\mathbf{w}$ with $\mathcal{CN}(0, \boldsymbol{\Sigma}_\mathbf{w})$ and Rician frequency-flat fading channels, where f_k is a Gaussian random channel gain with $\mathcal{CN}(\mu_{f_k}, \sigma_{f_k}^2)$, respectively. For notational convenience we introduce the following variables:

- $\boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]} \triangleq \mathbf{G}_\mathbf{k}^{[\mathbf{DF}]} \mathbf{t}_\mathbf{k}^{[\mathbf{r}]}$, $\boldsymbol{\delta}_\mathbf{k}^{[\mathbf{DF}]} \triangleq \partial \boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]} / \partial \tau_k^{[\mathrm{DF}]} = \frac{\partial \mathbf{G}_\mathbf{k}^{[\mathbf{DF}]}}{\partial \tau_k^{[\mathrm{DF}]}} \mathbf{t}_\mathbf{k}^{[\mathbf{r}]}$,
- $\mathbf{D_f} \triangleq \mathrm{diag}(\sqrt{p_1^{[r]}} \mu_{f_1}, \sqrt{p_2^{[r]}} \mu_{f_2}, \cdots, \sqrt{p_R^{[r]}} \mu_{f_R})$ is an $R \times R$ matrix,
- $\boldsymbol{\Delta}^{[\mathbf{DF}]} \triangleq [\boldsymbol{\delta}_\mathbf{1}^{[\mathbf{DF}]}, \boldsymbol{\delta}_\mathbf{2}^{[\mathbf{DF}]}, \cdots, \boldsymbol{\delta}_\mathbf{R}^{[\mathbf{DF}]}]$ is an $NL \times R$ matrix, and
- $\boldsymbol{\Xi}_\mathbf{k}^{[\mathbf{DF}]} \triangleq \boldsymbol{\delta}_\mathbf{k}^{[\mathbf{DF}]} \left(\boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]}\right)^H + \boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]} \left(\boldsymbol{\delta}_\mathbf{k}^{[\mathbf{DF}]}\right)^H$ is an $NL \times NL$ matrix.

According to (3), $\mathbf{y}$ is distributed as $\mathbf{y} \sim \mathcal{CN}(\boldsymbol{\mu}_\mathbf{y}, \boldsymbol{\Sigma}_\mathbf{y})$, where

$$\boldsymbol{\mu}_\mathbf{y} = E[\mathbf{y}] = \sum_{k=1}^{R} \sqrt{p_k^{[r]}} \mu_{f_k} \boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]}, \quad (4a)$$

$$\boldsymbol{\Sigma}_\mathbf{y} = E[(\mathbf{y} - \boldsymbol{\mu}_\mathbf{y})(\mathbf{y} - \boldsymbol{\mu}_\mathbf{y})^H] = \sum_{k=1}^{R} p_k^{[r]} \sigma_{f_k}^2 \boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]} \left(\boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]}\right)^H + \boldsymbol{\Sigma}_\mathbf{w}. \quad (4b)$$

To determine the CRLB, the $R \times R$ *Fisher's Information Matrix (FIM)* needs to be determined. In the case of parameter estimation in a complex Gaussian observation sequence, the entries of FIM are given by [6]

$$\mathbf{FIM}(\boldsymbol{\lambda})_{k,m} = 2\mathrm{Re}\left[\frac{\partial \boldsymbol{\mu}_\mathbf{y}^H}{\partial \lambda_k} \boldsymbol{\Sigma}_\mathbf{y}^{-1} \frac{\partial \boldsymbol{\mu}_\mathbf{y}}{\partial \lambda_m}\right] + \mathrm{Tr}\left[\boldsymbol{\Sigma}_\mathbf{y}^{-1} \frac{\boldsymbol{\Sigma}_\mathbf{y}}{\lambda_k} \boldsymbol{\Sigma}_\mathbf{y}^{-1} \frac{\boldsymbol{\Sigma}_\mathbf{y}}{\lambda_m}\right], \quad (5)$$

where $\boldsymbol{\lambda} = \left\{\tau_1^{[\mathrm{DF}]}, \tau_2^{[\mathrm{DF}]}, \cdots, \tau_R^{[\mathrm{DF}]}\right\}$. Using (5), the CRLB for the estimation of $\boldsymbol{\tau}^{[\mathbf{DF}]}$, which is given by the **diagonal** elements of the inverse of **FIM** can be calculated as

$$\mathbf{CRLB_R}(\boldsymbol{\tau}^{[\mathbf{DF}]}) = \Big(\underbrace{2\mathrm{Re}\left\{\mathbf{D_f}^H \left(\boldsymbol{\Delta}^{[\mathbf{DF}]}\right)^H \boldsymbol{\Sigma}_\mathbf{y}^{-1} \boldsymbol{\Delta}^{[\mathbf{DF}]} \mathbf{D_f}\right\}}_{\textbf{Part I}} + \underbrace{\boldsymbol{\Upsilon}^{[\mathbf{DF}]}\Big)^{-1}}_{\textbf{Part II}}, \quad (6)$$

where the elements of the $R \times R$ matrix $\boldsymbol{\Upsilon}^{[\mathbf{DF}]}$ are given by

$$\boldsymbol{\Upsilon}^{[\mathbf{DF}]}(\cdot)_{k,m} = \mathrm{Tr}\left[p_k^{[r]} p_m^{[r]} \sigma_{f_k}^2 \sigma_{f_m}^2 \boldsymbol{\Sigma}_\mathbf{y}^{-1} \boldsymbol{\Xi}_\mathbf{k}^{[\mathbf{DF}]} \boldsymbol{\Sigma}_\mathbf{y}^{-1} \boldsymbol{\Xi}_\mathbf{m}^{[\mathbf{DF}]}\right]. \quad (7)$$

Based on (6) the following remarks are in order:

1) In the case of AWGN and Gaussian channels, where $\sigma_{f_k}^2 = 0$ the covariance matrix of the observation vector, $\mathbf{y}$, $\boldsymbol{\Sigma}_\mathbf{y} = \sigma_w^2 \mathbf{I}$. Therefore, the CRLB in (6) simplifies to

$$\mathbf{CRLB_G}(\boldsymbol{\tau}^{[\mathbf{DF}]}) = \frac{\sigma_w^2}{2} \left(\mathrm{Re}\left\{\mathbf{D_f}^H \left(\boldsymbol{\Delta}^{[\mathbf{DF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{DF}]} \mathbf{D_f}\right\}\right)^{-1}, \quad (8)$$

which is similar to the CRLB in [9].

2) According to (8) to ensure that the CRLB for the estimation of $\boldsymbol{\tau}^{[\mathbf{DF}]}$ is bounded, the TS transmitted from each relay needs to be distinct, $\mathbf{t}_{\mathbf{1}}^{[\mathbf{r}]} \neq \mathbf{t}_{\mathbf{2}}^{[\mathbf{r}]} \neq \cdots \neq \mathbf{t}_{\mathbf{R}}^{[\mathbf{r}]}$, to ensure accurate timing jitter estimation (Fig. 3).
3) Close inspection of (8) also reveals that the application of orthogonal training sequences improves estimation performance and lowers the CRLB by reducing the off-diagonal elements of **FIM** to zero (Fig. 3).
4) In the case of $R = 1$ and AWGN, the CRLBs in (6) and (8) simplify to

$$\mathrm{CRLB}_{\mathrm{R}}(\tau^{[\mathrm{DF}]}) = \frac{\sigma_w^2}{2p^{[r]}|\mu_f|^2} \times \left(\mathrm{Re}\left\{\left(\boldsymbol{\delta}^{[\mathbf{DF}]}\right)^H \boldsymbol{\Sigma}_{\mathbf{y}}^{-1}\boldsymbol{\delta}^{[\mathbf{DF}]}\right\}.\right)^{-1} \quad (9)$$

5) Note that (9) also represents the CRLB for the estimation of $\tau_k^{[sr]}$ at the kth relay, where the parameters corresponding to the source to relay link need to be used instead.
6) Finally, the MLE for the simultaneous estimation of multiple timing jitters is presented in [10], where it is demonstrated that the CRLB can be reached at mid-to-high SNR.

Fig. 3 illustrates that for non-orthogonal TSs (*dotted lines*) the CRLB significantly increases when the timing jitter values are close to one another. However, in the case of orthogonal TSs (*solid lines*) the CRLB is independent of the value of $\boldsymbol{\tau}^{[\mathbf{DF}]}$, resulting in accurate joint timing jitter estimation for both close and far apart timing jitter values.

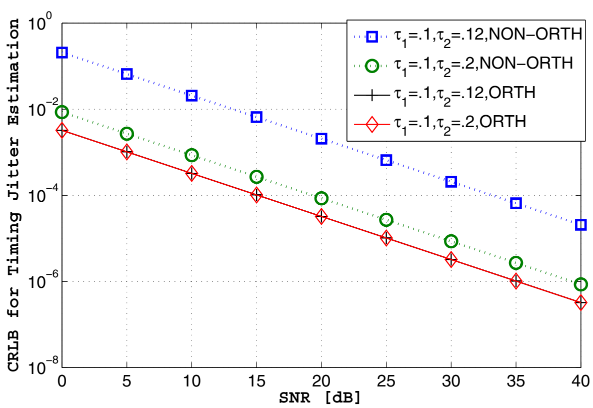

Fig. 3. Comparison of the CRLB in (8) for orthogonal TSs and non-orthogonal TSs when $\tau_1^{[\mathrm{DF}]} \simeq \tau_2^{[\mathrm{DF}]}$ and $\tau_1^{[\mathrm{DF}]} \neq \tau_2^{[\mathrm{DF}]}$ ($R = 2$, $L = 64$, and $N = 2$).

B. Amplify-and-Forward Cooperative Networks

Similar to DF, in AF networks the relays need to estimate timing jitters from the source to the relays, $\boldsymbol{\tau}^{[\mathbf{sr}]}$, to ensure synchronous transmission and successful cooperation in *phase II*. Furthermore, by estimating $\boldsymbol{\tau}^{[\mathbf{sr}]}$, the TS at the kth relay, $\mathbf{t}_{\mathbf{k}}^{[\mathbf{r}]}$ can be used to modulate the received training signal in *phase I*. This ensures that the kth relay has a distinct TS, which is required for accurate timing jitter estimation as described previously and shown again below. Fig. 4 represents the proposed block diagram of the baseband processing in AF relaying cooperative networks, which is necessary to achieve timing synchronization throughout the network.

Note that the CRLB for the estimation of $\boldsymbol{\tau}^{[\mathbf{sr}]}$ for AF networks is similar to that of DF and is given by (9).

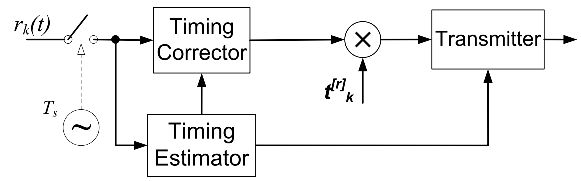

Fig. 4. Block diagram of the proposed baseband processing at the relays for AF cooperative networks.

The sampled received training signal model at the destination is given by

$$y(iT_s) = \underbrace{\sum_{k=1}^{R}\sum_{n=0}^{L-1} p_k^{[sd]}\zeta_k f_k h_k t^{[s]}(n) t_k^{[r]}(i) g(iT_s - nT - \tau_k^{[rd]}T)}_{\textbf{desired signal}} + \underbrace{\sum_{k=1}^{R}\zeta_k\sqrt{p_k^{[r]}} f_k \tilde{v}_k(iT_s) + w(iT_s)}_{\textbf{overall noise}}, \quad (10)$$

where:

- $\tilde{v}_k(iT_s) \triangleq v_k(iT_s) t_k^{[r]}(i)$ and $t_k^{[r]}(i)$ is the ith symbol of the kth relay's TS,
- $\zeta_k \triangleq 1/\sqrt{p^{[s]}|h_k|^2 + \sigma_{v_k}^2}$ satisfies the kth relay's power constraint, and $p_k^{[sd]} \triangleq \sqrt{p_k^{[r]} p^{[s]}}$.

Eq. (10) can be represented in matrix and vector form as

$$\mathbf{y} = \sum_{k=1}^{R}\zeta_k p_k^{[sd]} f_k h_k \left(\mathbf{G}_{\mathbf{k}}^{[\mathbf{rd}]}\mathbf{t}^{[\mathbf{s}]}\right) \odot \mathbf{t}_{\mathbf{k}}^{[\mathbf{r}]} + \sum_{k=1}^{R}\zeta_k\sqrt{p_k^{[r]}} f_k \tilde{\mathbf{v}}_{\mathbf{k}} + \mathbf{w}. \quad (11)$$

According to (11), R timing jitters, $\boldsymbol{\tau}^{[\mathbf{rd}]}$, need to be jointly estimated at the destination. For notational clarity, $(\cdot)^{[\mathbf{AF}]}$ is used instead of $(\cdot)^{[\mathbf{rd}]}$ below.

Derivation of the CRLB for the joint estimation of $\boldsymbol{\tau}^{[AF]}$: For AF relaying cooperative networks under the consideration of Rician or Rayleigh frequency-flat fading channels there does not exist an explicit CRLB for the estimation of $\boldsymbol{\tau}^{[\mathbf{AF}]}$, due to the presence of the term $f_k h_k$ in (10) and since the product of two Gaussian random variables is not a Gaussian random variable and its *probability distribution function (PDF)* is very difficult to calculate, [11]. Thus, in this paper the CRLB is derived for the case of **Gaussian** frequency-flat fading channels, where h_k and f_k are modeled as constants, corresponding to the case of slow fading with $\sigma_{h_k}^2 = \sigma_{f_k}^2 = 0$. Moreover, $\tilde{\mathbf{v}}_{\mathbf{k}}$, $\tilde{\mathbf{v}}_{\mathbf{m}}$, and $\mathbf{w}$ are assumed to be mutually independent for $k \neq m$ and $\forall k$, respectively.

For notational convenience the following variables are defined

- $\mathbf{D}_{\boldsymbol{\alpha}} \triangleq \mathrm{diag}(\alpha_1, \cdots, \alpha_R)$ is an $R \times R$ matrix,
- $\alpha_k \triangleq \zeta_k p_k^{[sd]} f_k h_k$, $\beta_k \triangleq \zeta_k\sqrt{p_k^{[r]}} f_k$,
- $\boldsymbol{\Delta}^{[\mathbf{AF}]} \triangleq [\boldsymbol{\delta}_{\mathbf{1}}^{[\mathbf{AF}]}, \boldsymbol{\delta}_{\mathbf{2}}^{[\mathbf{AF}]}, \cdots, \boldsymbol{\delta}_{\mathbf{R}}^{[\mathbf{AF}]}]$ is an $NL \times R$ matrix,

- $\boldsymbol{\delta}_{\mathbf{k}}^{[\mathbf{AF}]} \triangleq \partial \boldsymbol{\xi}_{\mathbf{k}}^{[\mathbf{AF}]} / \partial \tau_k^{[\mathrm{AF}]} = \left(\frac{\partial \mathbf{G}_{\mathbf{k}}^{[\mathbf{AF}]}}{\partial \tau_k^{[\mathrm{AF}]}} \mathbf{t}^{[\mathbf{s}]} \right) \odot \mathbf{t}_{\mathbf{k}}^{[\mathbf{r}]}$, and
- $\boldsymbol{\xi}_{\mathbf{k}}^{[\mathbf{AF}]} \triangleq \left(\mathbf{G}_{\mathbf{k}}^{[\mathbf{AF}]} \mathbf{t}^{[\mathbf{s}]} \right) \odot \mathbf{t}_{\mathbf{k}}^{[\mathbf{r}]}$.

Based on the above definitions, (11) can be rewritten as

$$\mathbf{y} = \sum_{k=1}^{R} \alpha_k \boldsymbol{\xi}_{\mathbf{k}}^{[\mathbf{AF}]} + \sum_{k=1}^{R} \beta_k \tilde{\mathbf{v}}_{\mathbf{k}} + \mathbf{w}. \tag{12}$$

According to (12), $\mathbf{y}$, is distributed as $\mathbf{y} \sim \mathcal{CN}(\boldsymbol{\mu}_{\mathbf{y}}, \boldsymbol{\Sigma}_{\mathbf{y}})$, where

$$\boldsymbol{\mu}_{\mathbf{y}} = \sum_{k=1}^{R} \alpha_k \boldsymbol{\xi}_{\mathbf{k}}^{[\mathbf{AF}]} \tag{13a}$$

$$\boldsymbol{\Sigma}_{\mathbf{y}} = \sum_{k=1}^{R} |\beta_k|^2 \boldsymbol{\Sigma}_{\mathbf{v_k}} + \boldsymbol{\Sigma}_{\mathbf{w}}. \tag{13b}$$

To determine the CRLB, the $R \times R$ FIM according to (5) needs to be determined where $\boldsymbol{\lambda} = \left\{ \tau_1^{[\mathrm{AF}]}, \tau_2^{[\mathrm{AF}]}, \cdots, \tau_R^{[\mathrm{AF}]} \right\}$. Using (5) the CRLB for the estimation of $\boldsymbol{\tau}^{[\mathbf{AF}]}$, which is given by the **diagonal** elements of the inverse of **FIM**, can be calculated as

$$\mathbf{CRLB_R}(\boldsymbol{\tau}^{[\mathbf{AF}]}) = \left(2\mathrm{Re} \left\{ \mathbf{D}_{\boldsymbol{\alpha}}^H \left(\boldsymbol{\Delta}^{[\mathbf{AF}]} \right)^H \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \boldsymbol{\Delta}^{[\mathbf{AF}]} \mathbf{D}_{\boldsymbol{\alpha}} \right\} \right)^{-1}. \tag{14}$$

Based on (14) the following remarks are in order:

1) In the case of **white** Gaussian noise, where $\mathbf{v_k}$ and $\mathbf{w}$ are distributed according to $\mathcal{CN}(0, \sigma_{v_k}^2 \mathbf{I})$ and $\mathcal{CN}(0, \sigma_w^2 \mathbf{I})$, respectively, the CRLB simplifies to

$$\mathbf{CRLB_G}(\boldsymbol{\tau}^{[\mathbf{AF}]}) = \frac{\sum_{k=1}^{R} \left(|\beta_k|^2 \sigma_{v_k}^2 \right) + \sigma_w^2}{2} \times \left(2\mathrm{Re} \left\{ \mathbf{D}_{\boldsymbol{\alpha}}^H \left(\boldsymbol{\Delta}^{[\mathbf{AF}]} \right)^H \boldsymbol{\Delta}^{[\mathbf{AF}]} \mathbf{D}_{\boldsymbol{\alpha}} \right\} \right)^{-1}. \tag{15}$$

2) Note that due to the term $\left(\sum_{k=1}^{R} \left(|\beta_k|^2 \sigma_{v_k}^2 \right) + \sigma_w^2 \right)$ in (15) as the number of relays in an AF relaying cooperative network increases the CRLB for the estimation of $\boldsymbol{\tau}^{[\mathbf{AF}]}$ also increases.
3) Based on (15), similar to DF relaying, to accurately estimate the timing jitters for each relay node (nonsingular **FIM**), the transmitted TSs need to be distinct, $\mathbf{c_1} \neq \mathbf{c_2} \neq \cdots \neq \mathbf{c_R}$.
4) In the case of AF networks by combining the noise terms in (12) the training signal model at the destination can be represented as

$$\mathbf{y} = \sum_{k=1}^{R} \alpha_k \mathbf{G}_{\mathbf{k}}^{[\mathbf{rd}]} (\tau_k^{[\mathrm{AF}]}) \mathbf{c_k} + \mathbf{z_c}, \tag{16}$$

where $\mathbf{z_c} \triangleq \sum_{k=1}^{R} \beta_k \mathbf{v_k} + w(n)$. Note that the signal model in (16) is similar to that of DF networks in (3). Therefore, the MLE in [8] can be applied to estimate $\boldsymbol{\tau}^{[\mathbf{AF}]}$ in the case of AF networks.

III. Numerical Results and Discussions

Throughout this section the propagation loss is modeled as $\beta = (d/d_0)^{-m}$, [4], where d is the distance between the transmitter and receiver, d_0 is the reference distance, and m is the path loss exponent. The following results are based on $d_0 = 1$km and $m = 2.7$, which corresponds to urban area cellular networks. The relays' distances from the source and destination, $d^{[sr]} = d^{[rd]} = 1$km. Finally, the roll-off factor is set to .22.

Fig. 5 presents a comparison between the CRLB for timing jitter estimation in DF and AF relaying cooperative networks as the number of relays increases. The CRLBs in (8) and (15) are plotted for $R = 2$ and $R = 4$ relays. $\boldsymbol{\tau}^{[\mathbf{DF}]} = \boldsymbol{\tau}^{[\mathbf{AF}]} = \{.1, .2, .3, .4\}$ and the TSs are selected from Walsh-Hadamard codes. Fig. 5 demonstrates that compared to DF, an AF relaying cooperative network requires the SNR from the relays to the destination to be a minimum of 5dB higher in order to reach the same timing jitter estimation accuracy. Fig. 5 also shows that in both cases of DF and AF relaying cooperative networks, as the number of relays within the network increases, timing jitter estimation accuracy decreases, with AF showing the most significant estimation performance loss.

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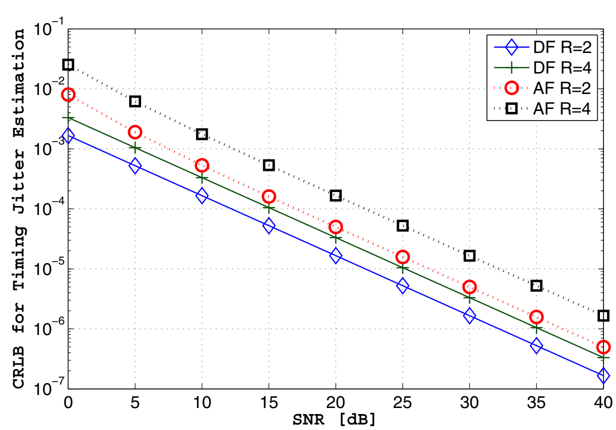

Fig. 5. Comparison of the CRLB in (8) and (15) for different numbers of relays in the case of DF and AF relaying cooperative networks, respectively ($L = 64$ and $N = 2$).