

RAPTOR CODES FOR HYBRID ERROR-ERASURE CHANNELS WITH MEMORY

Yu Cao and Steven D. Blostein

Department of Electrical and Computer Engineering
Queen's University, Kingston, Ontario, Canada, K7L 3N6
Email: yu.cao@ece.queensu.ca, steven.blostein@queensu.ca

ABSTRACT

Raptor codes are a class of rateless codes which have been used as application layer forward error correction (FEC) codes for broadcasting over wireless channels. Raptor codes provide promising performance in both erasure channels and noisy channels. This paper investigates the transmission of Raptor codes over hybrid error-erasure channels with and without memory. Performance of Raptor codes over Binary Symmetric Channels with Erasures (BSCE) is analyzed. It has been found that the average overhead of Raptor codes is independent of the erasure rate of BSCE channels. Simulation results of BER performance also show similar behavior. When channel memory is considered, performance of Raptor codes over Gilbert-Elliott (GE) channels is investigated via simulation. The results show that memory degrades the performance of Raptor codes whether or not channel state information is available.

I. INTRODUCTION

In wireline internet communications, as packets can be lost due to network congestion, the channel can be easily modeled by a packet erasure channel. In wireless communications, packets can be both lost and corrupted. Traditionally, corrupted packets are discarded and not forwarded to the application layer. These dropped packets are either re-transmitted using an Automatic Retransmission reQuest (ARQ) protocol or are recovered using application layer forward error correction (FEC) codes. These schemes can result in large packet drops and hence very low throughput when the channel condition is poor. To mitigate the inefficiency of such schemes, newer cross-layer protocols allow corrupted packets to be relayed into the application layers. With these protocols, the application layer FEC sees both erasures and errors. Such channels can be modeled as hybrid error-erasure channels. Cross-layer hybrid error-erasure protocols have been proposed in different contexts, including wireless multimedia/video transmissions [3], as well as wireless Local Area Network (LAN) and Digital Video Broadcast for mobile Handheld (DVB-H) devices. These cross-layer protocols require the application layer forward error correction (FEC) to be efficient in correcting both errors and erasures. The performance of Reed Solomon

(RS) codes and Low Density Parity Check (LDPC) codes [8] over different hybrid erasure-error protocols (HEEPs) have been investigated via simulation studies of wireless video communications [3].

In this paper, we are interested in a recently introduced sparse graph code, known as a fountain code [5]. Fountain codes are a class of rateless codes whose rate is not predetermined before encoding but determined during decoding on-the-fly. The two most widely used fountain code implementations are Luby Transform (LT) codes [4] and Raptor codes [9]. Raptor codes provide nearly optimal performance for erasure channels with linear decoding complexity [9]. In addition, Raptor codes can be “universal”, meaning that the same codes can maintain near optimal performance regardless of the channel erasure probability. Because of these advantages, Raptor codes are particularly suited to broadcast channels where both the rateless and universality properties are important. Raptor codes have been applied to many applications in internet and wireless communications, such as multicasting, parallel downloading and peer-to-peer communications. For example, in third generation partnership program (3GPP) Multimedia Broadcast/Multimedia Services (MBMS), Raptor codes have been chosen as the FEC code in the application layer for file downloading services. The performance of Raptor codes other than over erasure channels has been only studied in a few papers [2] [7]. It is found in [2] that unlike Binary Erasure Channels (BECs), the optimal degree distribution for Raptor codes is no longer universal for Binary input Additive White Gaussian Noise (BIWGN) and Binary Symmetric Channels (BSCs), but depends on the noise level. Nevertheless, a Raptor code designed for BEC performs quite well in BSC and BIAWGN channels [2] [7]. Therefore, Raptor codes are good FEC candidates in correcting both erasures and errors. To date, however, the performance of Raptor codes for hybrid error-erasure channels has not been explored.

Real-world wireless fading channels are correlated, which results in burstiness of bit errors and erasures. A FEC usually performs best when each code symbol experiences independent fading. The traditional way to deal with channel memory is to interleave the encoded symbols prior to transmission. However, interleaving may introduce very large

delays and adds complexity, motivating the study of the performance of Raptor codes in channels with memory. To the authors' best knowledge, only [10] [11] have addressed this issue. In [10], the authors study the performance of fixed-rate Raptor codes over Rayleigh fading channels with memory via simulation. However, [10] considered Raptor codes as a pure physical layer code, while the conventional usage of Raptor codes are application-layer FEC codes. Also, hybrid error-erasure channels have not been considered in [10]. In [11], the author evaluates application layer Raptor codes over DVB-H where shadowing and effects of memory are taken into consideration. However, [11] considers traditional non-cross-layer protocols which drop all the corrupted packets and render pure erasure channels for the Raptor codes. Finally, [11] assumes that Raptor codes have a fixed two-percent overhead, which is not realistic

In this paper, we evaluate the performance of Raptor codes as application-layer FEC codes when cross-layer protocols are employed. In the following, we analyze and simulate the performance of some actual Raptor codes over Binary Symmetric Channels with Erasures (BSCE) and Gilbert-Elliott (GE) channels. These are fundamental channel models that are formed by different cross-layer and non cross-layer protocols. The main contribution of the paper is twofold: first, by using the rateless property of Raptor codes, we compare the overheads of Raptor codes over BSC and BSCE channels; second, by simulating the performance of actual Raptor codes over time-varying hybrid error-erasure channels using iterative decoding, we demonstrate the desirable performance of Raptor codes using different protocols. We also investigate the effect of channel memory in these situations.

II. RAPTOR CODES AND CHANNEL MODELS

II-A. Raptor codes

The first practical realization of fountain codes is the class of Luby Transform (LT) codes [4] that encode k information symbols (x_1, x_2, \dots, x_k) into a potentially infinite number of output symbols (z_1, z_2, z_3, \dots) . The encoding process is done as follows: first, randomly choose the degree of the encoded symbol, denoted by d , from a probability distribution Ω ; second, choose d distinct input symbols uniformly at random as neighbors of the output encoding symbol. The value of each output symbol is the modulo 2 bit-wise summation of its d chosen neighbors. The output bit stream is generated independently until the transmitter receives an acknowledgement (ACK) from the receiver of successful decoding or until a predesigned code rate is achieved. The degree distribution Ω is usually described by its generating polynomial $\Omega(x) = \sum_{i=1}^K \Omega_i x^i$, where Ω_i represents the probability that value i is chosen. Shokrollahi [9] extended the idea of LT codes to Raptor codes which reduces the decoding complexity to be linear for BEC. A Raptor code with parameters (k, C, Ω) is constructed by concatenating

a block code C with a LT code with degree distribution Ω . To encode a Raptor code, the precoder C first encodes k information symbols into \bar{k} intermediate symbol. The output symbol streams are then generated by applying the inner LT code on the \bar{k} intermediate symbols. In the following, we use the Raptor code in [9] for simulation and analysis. The pre-code of this Raptor code is a left regular and right Poisson LDPC code with rate 0.95, the variable nodes of this LDPC code have constant degree 4. The code dimension $k = 9500$ and the inner LT codes use the degree distribution,

$$\Omega(x) = 0.007969x + 0.493570x^2 + 0.166622x^3 + 0.072646x^4 + 0.082558x^5 + 0.056058x^8 + 0.037229x^9 + 0.055590x^{19} + 0.025023x^{65} + 0.003135x^{66}. \quad (1)$$

The decoding of the Raptor codes is normally done iteratively using the Belief Propagation (BP) algorithms over the graph. By considering the Tanner graph of LT codes and the Tanner graph of the LDPC pre-codes as one Tanner graph, the message passing rule of the BP decoding over the Tanner graph of Raptor codes is very similar to that of LDPC codes [7]. For a LT output bit z_i , the initial log-likelihood ratio (LLR), defined as $LLR(z_i) = \ln(P(z_i = 0|y_i)/P(z_i = 1|y_i))$ where y_i is the corresponding received signal, can be calculated based on the channel information [8]. For BEC,

$$LLR(z_i) = \begin{cases} +\infty, & y_i = 0 \\ -\infty, & y_i = 1 \\ 0, & y_i = \text{erasure}. \end{cases} \quad (2)$$

For BSC with crossover probability p ,

$$LLR(z_i) = (-1)^{y_i} \ln((1-p)/p). \quad (3)$$

The initial LLRs of the LT input bits can be set to zero. For BEC, the BP algorithm can be significantly simplified which allows for linear decoding complexity of Raptor codes [9].

II-B. Hybrid error-erasure channels

The simplest form of hybrid error-erasure channel is the BSCE channel (Fig. 1). The BSCE channel is a discrete-

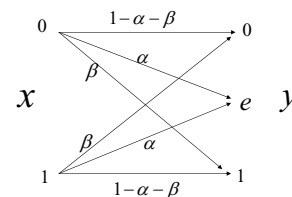


Fig. 1. BSCE channel

input, discrete-output channel with input alphabet $x \in (0, 1)$ and output alphabet $y \in (0, E, 1)$. The erasure probability $\alpha = Pr(y = E|x = 0) = Pr(y = E|x = 1)$, the error

probability $\beta = Pr(y = 1|x = 0) = Pr(y = 0|x = 1)$. Denote the probability of bit error occurs conditioned on the case that the information bit is not erased as p . Then $p = \frac{\beta}{1-\alpha}$ and the information capacity of such a hybrid channel can be easily calculated as [1],

$$C = (1 - \alpha)(1 - h_b(p)), \quad (4)$$

where $h_b(p) = -p \log p - (1 - p) \log(1 - p)$ is the binary entropy function. In the following, we use $BSC E(\alpha, p)$ to represent a BSCE channel with erasure probability α and conditional error probability p .

II-C. Gilbert-Elliott (GE) channels

GE channels are time varying binary symmetric channels containing a good state G and a bad state B (Fig. 2). The crossover probability, determined by the current state,

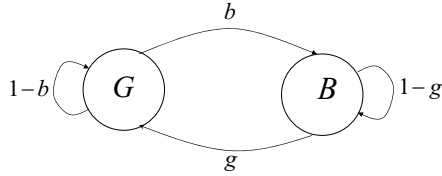


Fig. 2. Structure of the Gilbert Elliott channel

is usually small for the bad state and large for the good state. The transition between the two states form a binary Markov process. Let P_G and P_B represent the crossover probabilities in good and bad states respectively, where g and b represents the transition probabilities between state G and state B , $\{s_l\}_{l=0}^{\infty}$ represent the states at time l , $s_l \in \{G, B\}$. Then the stationary distribution can be obtained as $[\pi_g, \pi_b] = \left[\frac{g}{b+g}, \frac{b}{b+g} \right]$. A good-to-bad ratio ρ is defined as $\rho = P(s_l = G)/P(s_l = B) = g/b$. By induction on l , it can be easily verified that [6], for $\xi \in \{G, B\}$, $P(s_l = \xi | s_0 = \xi) - P(s_l = \xi | s_0 \neq \xi) = (1 - g - b)^l$, Hence parameter

$$\mu = 1 - g - b \quad (5)$$

is defined as the channel memory ($\mu \in [0, 1]$). When $\mu = 0$, the channel is memoryless, i.e., the current state is independent of previous states.

The capacity of GECs depends on the availability of channel state information (CSI)¹ in the receiver [6]. Let C_{CSI} represents the channel capacity when CSI is available at the receiver, i.e., the receiver knows the current state. C_{CSI} is not affected by channel memory. C_{NM} and C_{μ} represent the channel capacity when channel has no memory, channel memory is μ , respectively. For a fixed P_G , P_B and ρ , it is shown in [6] that $C_{NM} \leq C_{\mu} \leq C_{CSI}$.

¹Throughout this paper, CSI only refers to the information of current channel state. The value P_G and P_B is assumed to be known by the receiver in all the cases.

III. RAPTOR CODES OVER BSCE CHANNELS

As a rateless code, the performance of Raptor codes over BSCE channels can be measured by the average overhead by which all the information symbols are successfully decoded. To decode Raptor codes, the receiver collects output symbols and records the reliability of the symbol as a measure of the amount of information received. Once the total amount of information received slightly exceeds that of the source, the receiver starts to decode. If decoding fails, the receiver waits for a certain number of bits and attempts again to decode. For a Raptor code with k information symbols, let n represent the number of generated coded symbols required for a successive decoding. The overhead of Raptor codes is defined as,

$$\varepsilon = \frac{n - (k/C)}{(k/C)} = \frac{C}{R} - 1, \quad (6)$$

where C is the channel capacity and $R = k/n$ is the realized rate. Then the following property is satisfied:

Property: The average overhead of a Raptor code (or a LT code) required for successful decoding over a $BSC E(\alpha, p)$ channel is the same as that over a $BSC(p)$ channel. Intuitively, the property can be interpreted as a compensation for erasure loss: the Raptor codes need to generate more code symbols by a factor corresponding to the erasure rate for $BSC E(\alpha, p)$ channels compared to that of $BSC(p)$ channels. However, the numbers of symbols discarded, successfully received as well as the ratio between them vary for each realization. Although the numbers and the patterns of symbols discarded are irrelevant to the code performance, they are important in practice as they contribute to the time that the receiver needs to wait for a successful decoding. Therefore, a simple formal proof for this property is provided:

Proof: Consider a rateless Raptor code transmitted over a $BSC E(\alpha, p)$ channel. Denote S and X as random variables representing the number of coded symbols received and the number of coded symbols that are erased on successful Raptor decoding, respectively. The average number of coded symbols generated is then $\bar{n} = E\{S + X\}$. By taking the average conditioned on S , we have $\bar{n} = E\{S + X\} = E_S\{E\{S + X|S\}\} = E_S\{S + E\{X|S\}\}$. If an un-erased symbol is considered as a successful trial and an erased symbol is considered as a failed trial with failure probability equal to α , then conditional on the number of successful trials $S = s$, X follows a negative binomial distribution which gives the probability of $s - 1$ successes and x failures in $x + s - 1$ trials, and success on the $(x + s)$ -th trial, i.e.,

$$P_{X|S}(x|s) = \binom{x + s - 1}{s - 1} (1 - \alpha)^s \alpha^x. \quad (7)$$

The average of the negative binomial distribution is easily computed as $E\{X|S\} = \frac{\alpha}{1-\alpha} S$, hence $E\{S + X\} = \frac{1}{1-\alpha} E\{S\}$. By substituting (4) into (6), the average overhead

is obtained as $\bar{\varepsilon} = \frac{\bar{n} - (k/C)}{(k/C)} = E\{S\}(1 - h_b(p))k^{-1} - 1$. Since the erased symbol does not contribute to the code performance, and each coded symbol is generated independently, $E\{S\}$ is the same for the $BSC E(\alpha, p)$ channel and $BSC(p)$ channel. Hence $\bar{\varepsilon}$ is also independent of erasure rate α . \square

As a special case of the property, a Raptor code has the same overhead for all BEC channels with different erasure rates. This also confirms the “universality” of Raptor codes over BEC channels, i.e, if a Raptor code is optimal for one BEC channel, it is also optimal for BEC channels with all possible erasure rates.

IV. SIMULATION RESULTS AND ANALYSIS

Due to the decoding complexity, the performance in terms of realized rate is not simulated. As an alternative, the performance of Raptor codes over $BSC E(\alpha, p)$ is measured by bit error rate (BER) versus the inverse of code rate R^{-1} , which is proportional to the number of coded symbols generated. The decoding of Raptor codes over BSCE channels is performed by using initial LLRs according to (2) and (3) and performing BP decoding the same way as BSC channels. Fig. 3 shows the performance of Raptor codes over $BSC E(\alpha, p)$ channels with different parameters α and p . The figure includes both the cases of BEC ($p = 0$) and BSC ($\alpha = 0$). The capacity of BSCE channels is evaluated by Eq. (4). From Fig. 3, it can be seen that to achieve an average BER of 10^{-2} , the Raptor codes require approximately 9%, 10% and 11% overhead for BEC , $BSC(p = 0.05)$, and $BSC(p = 0.11)$, respectively, compared to their own information capacity bound. It can also be found that a $BSC E(\alpha, p)$ channel requires approximately the same overhead as a $BSC(p)$ channel with the same parameter p . However, the BER curve of the $BSC E(\alpha, p)$ is not an exact horizontal shift of the BER curve of the $BSC(p)$ channel. This is because for a fixed rate Raptor code over $BSC E(\alpha, p)$ where $\alpha \neq 0$, the number of non-erased symbols varies with each implementation, while this value does not change for $BSC(p)$. The simulation demonstrates that in the case of no memory, the effect of this variation on the performance of Raptor codes is very small.

Fig. 4 shows the performance of Raptor codes in the special case of GE channels where the probability of erasure is 0 in the good state and 1 in the bad state. These GE channels correspond to non-cross-layer protocols and can well model the bursty behavior of packet losses due to network congestion or packet drops due to packet corruptions. The average erasure rate P_{era} , which is equal to the steady state probability of the bad state π_b , of the four curves are chosen to be the same ($P_{era} = 0.1$). It is not surprising that memory generally has a negative effect on the performance of Raptor codes. The performance loss is a result of the distribution of bad states (erasures) over one code block. Although the average numbers of bad states are the same

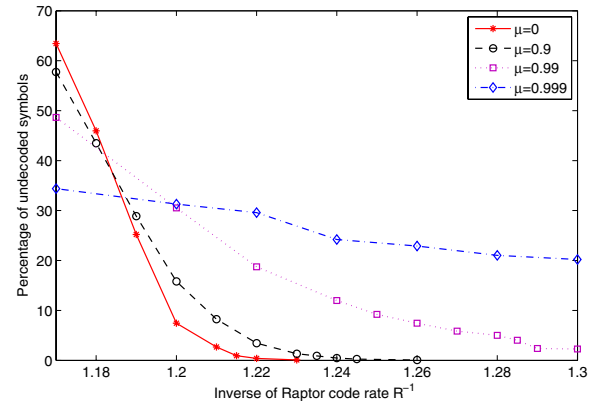


Fig. 4. Raptor code over erasure channels with memory

for the four different cases, a channel with higher correlation (memory) has higher variation of the number of bad states within one block length. The probability density function (PDF) of the number of bad states n_b that occur among n consecutive symbols is provided in [12]. It is interesting to observe that at the left part of each curve, i.e, when most of the information bits are not able to be decoded, the channel with higher memory actually shows better performance than channels with no memory. This can be explained by the fact that Raptor codes have a very steep performance curve in erasure channels. When not enough coded bits are received, most of the information bits can not be decoded. In this case, the higher variation of the number of bad states n_b is actually helpful.

Fig. 5 shows the performance of Raptor codes over more general GE channels. When CSI is available, the bad state

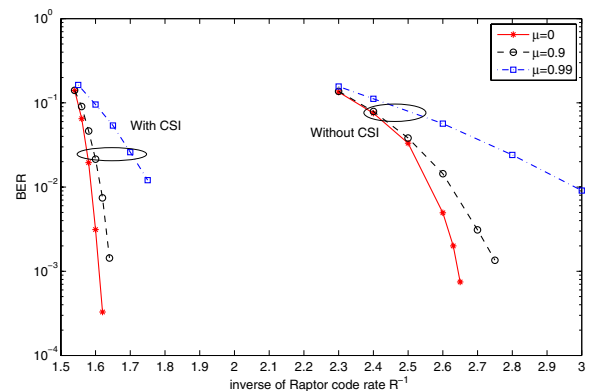


Fig. 5. Raptor code over GE channels ($P_G = 0.01$, $P_B = 0.5$, $\rho = g/b = 3$)

with $P_B = 0.5$ is actually the same as an erasure state. In this case, the GE channel with $\mu = 0$ is actually a

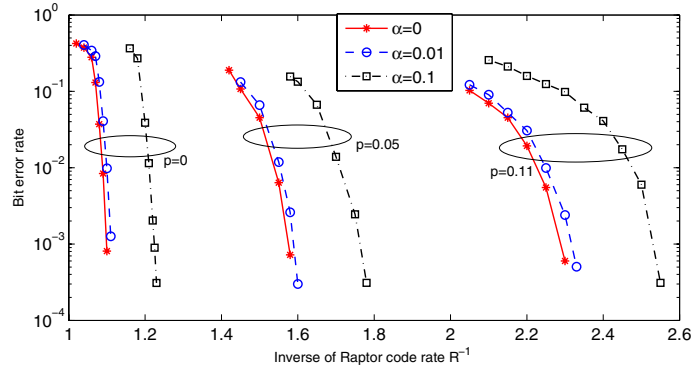


Fig. 3. Performance of Raptor code over BSCE (α, p) channels

$BSCE(\alpha = 0.25, p = 0.01)$ channel as discussed earlier. This GE channel can also be considered as a hybrid error-erasure channel with memory. When CSI is available, the decoding is performed using iterative BP decoding with initial LLR equal to 0 for the code symbol experiencing a bad state and $(-1)^y \ln((1 - P_b)/P_b)$ for the code symbol experiencing a good state. For GE channels when no CSI is available, the initial LLRs for all the code symbols are set according to the average crossover probability over all the states. It can be seen from Fig. 5 that there is a significant difference in code performance between the case when CSI is available and the case when it is not. It can also be observed that memory has a negative effect on performance regardless of the availability of CSI. The reason is the same as that for the case of erasure channels; memory increases the variation of the number of bad states in a given block length.

The results seem to be contrary to the capacity study [6] which shows that memory increases capacity when CSI is not available. To exploit the improved capacity, the decoder needs to utilize channel correlation for better estimation of CSI [6]. This improvement can possibly be obtained by employing similar estimation and decoding techniques used for LDPC codes, though applying such decoding methods to Raptor codes is beyond the scope of this paper. Since no attempt is made to estimate CSI, the performance of Raptor codes over GE channels with no CSI is still bounded by C_{NM} .

V. CONCLUSION AND FUTURE WORK

This paper considers the performance of Raptor codes for hybrid error-erasure channels with memory using iterative decoding. It has been shown that the overheads of Raptor codes are the same for BSCE and BSC channels with the same conditional cross-over probabilities. The effect of channel memory on the performance of Raptor codes is also investigated by considering the two-state GE channels. When channel correlation is not exploited in the decod-

ing process, memory degrades the performance of Raptor codes. More discussions on how channel memory affects the performance of application layer Raptor codes applied to practical wireless broadcasting services with different cross-layer protocols are future work and some simulation results are presented in [13].

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