Recursive Maximum Likelihood Estimation of Maximum Doppler Frequency of a Sampled Fading Signal

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Abstract

Improved network resource management of personal communication systems require accurate estimation of mobile velocity. This can be done by estimating the Doppler frequency at the mobilestation (MS) based on the received fading signal. A conventional method is to compute the level crossing rate(LCR), which is a function of Doppler shift, and then to find the corresponding Doppler frequency. This is not easy in practice because of the difficulty in estimating a continuous signal's level crossing rate from a sampled signal. Zhou and Blostein have developed a LCR-based Doppler Frequency method using the discrete-time received signal. In this paper, we propose a recursive maximum likelihood approach to estimate the Doppler frequency from the received discrete-time fading signal envelope sequence by modelling the fading process as a first order discrete-time continuous-state Markov process.

I Introduction

Prior knowledge of mobile velocity is very important in many applications of wireless communications. The cellular mobile radio environment is characterized by the presence of fading that is related to the speed of the mobile. The maximum Doppler frequencey is defined as the ratio of mobile speed to the wavelength of the carrier. If this quantity can be extracted from the received fading signal, it can be used in measuring the mobile velocity. There are a number of different approaches developed for estimating the maximum Doppler frequency. We classify them into two main methods: signal envelope-based, and signal component-based. The signal envelope-based methods use the signal envelope $z(t) = \sqrt{r_I^2(t) + r_Q^2(t)}$ of a received sinusoid in noise to obtain mobile motion information. A well known approach is to compute the envelope level crossing rate (LCR) which is a function of the maximum Doppler frequency. The LCR is defined as the average number of positive-going crossings per second a signal makes for a given predetermined level R [1][2].

The signal component methods use the fading signal in-phase (or quadrature) component to estimate mobile velocity. Two types of signal component methods are referred as zero crossing rate (ZCR) [2] and autocorrelation function (ACF) [2][3][4]. The ZCR is defined as the average number of positive going zero crossings of a signal. The ACF is the autocorrelation function of fading signal in-phase component, a function of maximum Doppler frequency [1].

This paper models the discrete-time fading signal envelope in additive noise, presents the n^{th} order joint probability density function (pdf) that characterizes the property of fading signal envelope and obtains a recursive maximum likelihood estimation algorithm to estimate the maximun Doppler frequency. In Section 2 we present the received signal and envelope model. In Section 3 we briefly analyze previous approaches. In Sections 4 and 5 we outline our new approach. We provide simulation results and discussion in the final section.

II Discrete-time Model

The received signal at the MS consists of a number of horizontally travelling plane waves with random amplitudes and phases. Assuming that the transmitted signal is vertically polarized, the continuous-time received signal model is given in [1] [2]. The discrete-time signal model derived by Zhou and Blostein [4] [8] from the continuous model can be written as

$$r(k) = \sum_{n=1}^{N} \alpha_n \cos(\omega_c k + \omega_n k + \phi_n) \tag{1}$$

where α_n is the amplitude of the n^{th} incident wave, ω_c is the carrier frequency, $\omega_n = 2\pi f_m \cos\theta_n$ is the Doppler frequency due to the mobility of MS, f_m is the maximum Doppler frequency, θ_n is the incident angle of the n^{th} wave, and ϕ_n is the phase of the n^{th} incident wave.

The in-phase and quadrature components of the received signal can be written, respectively, as

$$r_I(k) = \sum_{n=1}^{N} \alpha_n \cos(\omega_n k + \phi_n) \tag{2}$$

$$r_Q(k) = \sum_{n=1}^{N} \alpha_n \sin(\omega_n k + \phi_n) \tag{3}$$

Assuming local scatter and a non-LOS path between the BS and MS, then $r_I(k)$ and $r_Q(k)$ both have zero mean and variance σ^2. The low-pass equivalent signal envelope

$z(k) = \sqrt{r_I^2(k) + r_Q^2(k)}$ has Rayleigh *pdf*

$$p_z(x) = \frac{x}{\sigma^2} exp\left\{-\frac{x^2}{2\sigma^2}\right\} \quad (x \geq 0) \tag{4}$$

The joint *pdf* of two adjacent envelope samples z_k and z_{k+1} is given by [8]

$$p(z_k, z_{k+1}) = \frac{z_k z_{k+1} exp\left\{-\frac{z_k^2+z_{k+1}^2}{2\sigma^2(1-J_0^2)}\right\}}{\sigma^4(1-J_0^2)} I_0\left(\frac{z_k z_{k+1}}{\sigma^2(1-J_0^2)}\right) \tag{5}$$

In (5), $J_0(2\pi f_m)$ is simplified as J_0, for the sake of notational convenience, where $J_0(\cdot)$ is the zero-order Bessel function of the first kind. $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind.

In the case that the received fading signal is corrupted by band-limited white Gaussian noise, $y(k) = r(k) + n(k)$, the in-phase and quadrature components are given by

$$y_I(k) = r_I(k) + n_I(k) \tag{6}$$
$$y_Q(k) = r_Q(k) + n_Q(k) \tag{7}$$

Both $y_I(k)$ and $y_Q(k)$ still have zero mean and variance $\sigma^2 + \sigma_n^2$. The low-pass equivalent signal envelope $z(k) = \sqrt{y_I^2(k) + y_Q^2(k)}$ is Rayleigh distributed. We derive the joint *pdf* $p(z_k, z_{k+1})$ for the additive white noise case as

$$p(z_k, z_{k+1}) = \frac{z_k z_{k+1} \exp\left\{-\frac{\phi_0(z_k^2+z_{k+1}^2)}{2(\phi_0^2-\phi_1^2)}\right\}}{(\phi_0^2-\phi_1^2)} I_0\left(\frac{\phi_1 z_k z_{k+1}}{(\phi_0^2-\phi_1^2)}\right) \tag{8}$$

where

$$\phi_0 = E\{y_I^2\} = E\{y_Q^2\} = \sigma^2 + \sigma_n^2 \tag{9}$$

is the variance of both in-phase and quadrature components, and

$$\begin{aligned} \phi_1 &= E\{y_I(k)y_I(k+1)\} = E\{y_Q(k)y_Q(k+1)\} \\ &= \sigma^2 J_0 + \sigma_n^2 sin(\pi B_n/F_s)/(\pi B_n/F_s) \end{aligned} \tag{10}$$

where σ_n^2 is the variance of band-limited noise in both in-phase and quadrature components. B_n is the bandwidth of the front-end low-pass filter. F_s is the sampling rate.

III LCR and ACF Estimation

A LCR-based Estimation

For a continuous-time fading signal, the envelope's level crossing rate at a specified power level R, is defined as the expected rate at which the envelope crosses level R in the positive (or negative) going direction. The theoretical LCR is given in [1] [2]. In practice, measuring the LCR is difficult in the presence of noise.

Zhou and Blostein have developed a LCR-based Doppler frequency estimator using discrete-time signal samples. They define the discrete-LCR as the the ratio of the expected number of level crossings, X, to the number of given samples N. That is

$$\text{LCR} = \frac{E\{X\}}{N} \tag{11}$$

The number of level crossings

$$X = \sum_{k=n}^{n+N-1} X_k \tag{12}$$

where X_k is the level crossing state between two samples. If there is no level crossing, $X_k = 0$, if there is a level crossing, $X_k = 1$.

Levels crossings in one direction are counted, and the LCR is given by [4]

$$\begin{aligned} \text{LCR} &= P\{z_{k-1} \leq R, z_k > R\} \\ &= \int_0^R \int_R^\infty p(z_{k-1}, z_k) dz_{k-1} dz_k \end{aligned} \tag{13}$$

where $p(z_{k-1}, z_k)$ is the joint pdf of two adjacent envelope samples. In practice, the LCR value is computed as the number of crossings counted on the MS divided by the total number of samples. The results in [4] [8] show that the LCR estimator underestimates Doppler frequency when fading is fast for a fixed sampling rate. This is due to the assumption that only one crossing occurs between two samples. This fails in case of fast fading or low sampling rates when multiple crossing occur during one sample period.

Another problem is the choice of the specified envelope power level R. In the case that the received signal is corrupted by band-limited white noise, when R chosen is low or sampling rate is high, the noise power will affect the LCR value. When R is chosen too high, there are not enough samples to obtain a reliable LCR estimate. We remark that the value of the LCR is an intermediate quantity which itself is not critical.

B ACF Estimation

For Rayleigh distributed fading channels, the autocorrelation function $\phi(k)$ of the quadrature components is a product of the average envelope power and a Bessel function, of which the maximum Doppler frequency f_m is the variable. At $k = 0, 1$, ϕ_0 and ϕ_1 are approximated as

$$\begin{aligned} \hat{\phi}(0) &= \frac{1}{N}\sum_{k=1}^{N} x_k^2 \\ \hat{\phi}(1) &= \frac{1}{N-1}\sum_{k=1}^{N-1} x_k x_{k+1} \end{aligned} \tag{14}$$

where x_k is the quadrature component of the received fading signal. By computing the ratio $R = \phi_1/\phi_0$ with the known signal-to-noise ratio $\gamma_s = \sigma^2/\sigma_n^2$, the maximum Doppler frequency can then be estimated. The main problems in this method are that it requires prior knowledge of signal variance and noise variance, as well as coherent demodulation.

IV Recursive Frequency Estimator

In the previous section, we have pointed out that the fading signal envelope at any time t_k is Rayleigh distributed. While the envelope *pdf* is not a function of the Doppler frequency, the joint *pdf* of z_k and z_{k+1} is a function of the Doppler frequency. Our goal is to estimate the Doppler frequency f_m as a *non-random parameter* directly from the observed samples of the fading signal envelope. In the following approach, we determine the n^{th}-order joint *pdf* of the n observed samples conditioned on f_m and then maximize this joint *pdf*, to obtain a maximum likelihood estimate of f_m. This is difficult because the observed signal is correlated. Wang and Chang have verified a first-order Markov model for a Rayleigh fading channel [5]. With the assumption that the random fading signal envelope is first-order Markov, the fading signal envelope can be described completely by the n^{th}-order joint *pdf* of n consecutive samples of the fading signal envelope. The n^{th}-order joint pdf can be written as

$$f_n(z_1, z_2, ..., z_n) = f(z_1) \prod_{k=1}^{n-1} \frac{f(z_k, z_{k+1})}{f(z_k)} \tag{15}$$

We maximize the likelihood function

$$L(z_1, z_2, ..., z_n; f_m) = f_n(z_1, z_2, ..., z_n) \tag{16}$$

Setting $\frac{\partial \ln L(z_1, z_2, ..., z_n; f_m)}{\partial f_m}$ to zero, in the noiseless case, we obtain the likelihood equation

$$\sum_{k=1}^{n-1} -\frac{2\pi J_1}{\sigma^2(1-J_0^2)^2}[-(z_k^2 + z_{k+1}^2)J_0 + 2\sigma^2 J_0(1-J_0^2) + z_k z_{k+1}(1+J_0^2)\frac{I_1}{I_0}] = 0 \tag{17}$$

where J_0 and J_1, function of $2\pi f_m$, are zero and first-order Bessel functions of the first kind respectively, and I_0 and I_1, functions of $z_k z_{k+1}/\sigma^2(1-J_0^2)$, are zero and first-order modified Bessel function of the first kind respectively. Since $f_m > 0$, $J_0 < 1$, and $J_1 \neq 0$. In addition, J_0 and J_1 are not functions of samples z_k, then (17) can be simplified as

$$\sum_{k=1}^{n-1}[-(z_k^2+z_{k+1}^2)J_0+2\sigma^2 J_0(1-J_0^2)+z_k z_{k+1}(1+J_0^2)\frac{I_1}{I_0}] = 0 \tag{18}$$

In the case that the received fading signal is corrupted by band-limited white Gaussian noise, we obtain the likelihood equation

$$\sum_{k=1}^{n-1}[-(z_k^2+z_{k+1}^2)\phi_0\phi_1+2\phi_1(\phi_0^2-\phi_1^2)+z_k z_{k+1}(\phi_0^2+\phi_1^2)\frac{I_1}{I_0}] = 0 \tag{19}$$

where ϕ_0 is given in (9), ϕ_1 is given in (10), and I_0 and I_1, are functions of $\phi_1 z_k z_{k+1}/(\phi_0^2 - \phi_1^2)$.

We cannot solve f_m in (18) and (19) in closed-form. Instead, we use the following recursive stochastic approximation technique to estimate parameter f_m [6]:

Assume there is one unknown consistant parameter sequence $\left\{\hat{f_m}\right\}_{n=1}^{\infty}$, and the likelihood equation

$$\sum_{k=1}^{n} \psi(Z_k; \hat{f_m}) = 0 \tag{20}$$

Under mild conditions [6], the difference, $\hat{f_{mn}} - \hat{f_{mn-1}}$, converges to zero as $n \to \infty$. Then equation (20) can be approximated by expanding $\hat{f_{mn-1}}$ as

$$\sum_{k=1}^{n} \psi(Z_k; \hat{f_{mn-1}}) + (\hat{f_{mn}} - \hat{f_{mn-1}}) \sum_{k=1}^{n} \psi'(Z_k; \hat{f_{mn-1}}) \approx 0 \tag{21}$$

where ψ' is $\frac{\partial \psi}{\partial f_m}$. Then, we obtain

$$\hat{f_{mn}} \approx \hat{f_{mn-1}} - \frac{\sum_{k=1}^{n} \psi(Z_k; \hat{f_{mn-1}})}{\sum_{k=1}^{n} \psi'(Z_k; \hat{f_{mn-1}})} \tag{22}$$

Since $\hat{f_{mn-1}}$ solves $\sum_{k=1}^{n-1} \psi(Z_k; \hat{f_m}) = 0$, the numerater sum on the right hand of (22) has only one term, $\psi(Z_n; \hat{f_{mn-1}})$, thus (22) becomes

$$\hat{f_{mn}} \approx \hat{f_{mn-1}} + \frac{\psi(Z_n; \hat{f_{mn-1}})}{-\sum_{k=1}^{n} \psi'(Z_k; \hat{f_{mn-1}})} \tag{23}$$

Writing the denominater sum of (23) as

$$n\left[-\frac{1}{n}\sum_{k=1}^{n} \psi'(Z_k; \hat{f_{mn-1}})\right] \tag{24}$$

the Weak Law of Large Numbers implies that

$$-\frac{1}{n}\sum_{k=1}^{n} \psi'(Z_k; \hat{f_{mn-1}}) \to i_{f_m} \tag{25}$$

in probability, where

$$i_{f_m} = -E_{f_m}\left\{\psi'(Z_k; \hat{f_{mn-1}})\right\} = E\left\{\psi^2(Z_k; f_m)\right\} \tag{26}$$

is Fisher's information per sample. Since $\hat{f_m} \to f_m$, as n gets large, we can approximate

$$-\frac{1}{n}\sum_{k=1}^{n} \psi'(Z_k; \hat{f_{mn-1}}) \approx i_{\hat{f_{mn-1}}} \tag{27}$$

where, for the noiseless case, $\psi(Z_k; \hat{f_m})$ is given as

$$\begin{aligned}\psi(Z_k; \hat{f_m}) &= -(z_k^2 + z_{k+1}^2)J_0 + 2\sigma^2 J_0(1-J_0^2) \\ &\quad + z_k z_{k+1}(1+J_0^2)\frac{I_1}{I_0}\end{aligned} \tag{28}$$

and $\psi'(Z_k; \hat{f_m})$ is given as

$$\begin{aligned}\psi'(Z_k; \hat{f_m}) &= -2\pi J_1\Big\{-(z_k^2 + z_{k+1}^2) + 2\sigma^2 - 6\sigma^2 J_0^2 + \\ &\quad 2z_k z_{k+1} J_0 \frac{I_1}{I_0} + \frac{z_k^2 z_{k+1}^2 (1+J_0^2)^2}{\sigma^2(1-J_0^2)^2} \\ &\quad \times\big[1 - \frac{\sigma^2(1-J_0^2)}{z_k z_{k+1} J_0}\frac{I_1}{I_0} - \frac{I_1^2}{I_0^2}\big]\Big\}\end{aligned} \tag{29}$$

for the case that the received signal is corrupted by bandlimited white noise, (28) and (29) can be generalized to

$$\begin{aligned}\psi(Z_k;\hat{f_m}) &= -(z_k^2+z_{k+1}^2)\phi_0\phi_1+2\phi_1(\phi_0^2-\phi_1^2) \\ &\quad +z_kz_{k+1}(\phi_0^2+\phi_1^2)\frac{I_1}{I_0} \qquad (30)\end{aligned}$$

and

$$\begin{aligned}\psi'(Z_k;\hat{f_m}) &= -2\pi\sigma^2J_1\Big\{-(z_k^2+z_{k+1}^2)\phi_0+2\phi_0^2-6\phi_1^2+ \\ &\quad 2z_kz_{k+1}\phi_1\frac{I_1}{I_0}+\frac{z_k^2z_{k+1}^2(\phi_0^2+\phi_1^2)^2}{(\phi_0^2-\phi_1^2)^2} \\ &\quad \times\big[1-\frac{(\phi_0^2-\phi_1^2)}{\phi_1z_kz_{k+1}}\frac{I_1}{I_0}-\frac{I_1^2}{I_0^2}\big]\Big\} \qquad (31)\end{aligned}$$

Choice of Sampling Rate: The discrete-time Doppler frequency to be estimated is $f_m = F_m/F_s$, where F_s is the sampling rate. The smallest positive solution of $J_0(2\pi f_m) = 0$ is $f_m = 0.3827$. To guarantee a unique estimate of f_m, we must choose a sampling rate F_s that places f_m in the range (0, 0.3827). We choose F_s to be larger than F_m, so that a one-to-one mapping from $J_0(2\pi f_m)$ to f_m exists.

V ACF-Based Estimation

The autocorrelation function (ACF) ϕ_k of the in-phase and quadrature components is given by

$$\phi(k) = \sigma^2J_0(2\pi f_mk)+\sigma_n^2\frac{\sin\pi B_nkT_s}{\pi kT_s} \qquad (32)$$

where $T_s = 1/F_s$ is the sampling period. When $k = 0, 1$, ϕ_0 and ϕ_1 are given in (9),(10) respectively. In practice, the ϕ_0 can be computed by the ensemble average $\hat{\phi}_0 = \langle|z|^2\rangle/2$ [1] [2]. ϕ_1 can be estimated using our recursive maximum likelihood estimation algorithm (23). The $\psi(Z_k;\hat{\phi}_1)$ is given as

$$\begin{aligned}\psi(Z_k;\hat{\phi}_1) &= -(z_k^2+z_{k+1}^2)\phi_0\phi_1+2\phi_1(\phi_0^2-\phi_1^2) \\ &\quad +z_kz_{k+1}(\phi_0^2+\phi_1^2)\frac{I_1}{I_0} \qquad (33)\end{aligned}$$

and $\psi'(Z_k;\hat{\phi}_1)$ is given as

$$\begin{aligned}\psi'(Z_k;\hat{\phi}_1) &= -(z_k^2+z_{k+1}^2)\phi_0+2\phi_0^2-6\phi_1^2+ \\ &\quad 2z_kz_{k+1}\phi_1\frac{I_1}{I_0}+\frac{z_k^2z_{k+1}^2(\phi_0^2+\phi_1^2)^2}{(\phi_0^2-\phi_1^2)^2} \\ &\quad \times\big[1-\frac{(\phi_0^2-\phi_1^2)}{\phi_1z_kz_{k+1}}\frac{I_1}{I_0}-\frac{I_1^2}{I_0^2}\big] \qquad (34)\end{aligned}$$

Then the ratio $R = \phi_1/\phi_0$ can be computed by

$$\hat{R} = \frac{\hat{\phi}_1}{\hat{\phi}_0} \qquad (35)$$

Since the received signal is a mixture of a pure fading signal and additive white Gaussian noise, the noise variance σ_n^2 usually is unknown and needs to be estimated. From (33), the $\psi(Z_k;\hat{\sigma_n^2})$ is given as

$$\begin{aligned}\psi(Z_k;\hat{\sigma_n^2}) &= \phi_n(1)\Big[-(z_k^2+z_{k+1}^2)\phi_0\phi_1+2\phi_1(\phi_0^2-\phi_1^2) \\ &\quad +z_kz_{k+1}(\phi_0^2+\phi_1^2)\frac{I_1}{I_0}\Big] \qquad (36)\end{aligned}$$

and $\psi'(Z_k;\hat{\sigma_n^2})$ is given as

$$\begin{aligned}\psi'(Z_k;\hat{\phi}_1) &= \phi_n(1)^2\Big\{-(z_k^2+z_{k+1}^2)\phi_0+2\phi_0^2-6\phi_1^2+ \\ &\quad 2z_kz_{k+1}\phi_1\frac{I_1}{I_0}+\frac{z_k^2z_{k+1}^2(\phi_0^2+\phi_1^2)^2}{(\phi_0^2-\phi_1^2)^2} \\ &\quad \times\big[1-\frac{(\phi_0^2-\phi_1^2)}{\phi_1z_kz_{k+1}}\frac{I_1}{I_0}-\frac{I_1^2}{I_0^2}\big]\Big\} \qquad (37)\end{aligned}$$

where, $\phi_n(1) = sin(\pi B_n/F_s)/(\pi B_n/F_s)$ is a constant determined by known B_n and F_s. The fading signal variance σ^2 is easily computed as $\hat{\sigma^2} = \hat{\phi}_0 - \hat{\sigma_n^2}$. Define the signal to noise ratio $\gamma_s = \sigma^2/\sigma_n^2$, we can estimate the f_m by

$$f_m = \frac{1}{2\pi}J_0^{-1}\left(R(1+\frac{1}{\gamma_s})-\frac{1}{\gamma_s}\phi_n(1)\right) \qquad (38)$$

As we can see, in the whole procedure of estimating f_m, we only need collect the samples of the fading signal envelope. We do not need to demodulate received signal.

VI Simulation Results

We first show results on a noiseless fading signal simulated using Young's method [7]. Figure 1 and 2 show preliminary estimation results for $20km/h$ and $40km/h$, respectively, for the cases of no additive noise and no front-end filtering. The horizontal line is the actual Doppler frequency, and the other curve is the recursive estimate. From these figures, we can see that the estimation of f_m converges to the true f_m.

Figure 3 depicts results for 60 km/h in 0 dB additive bandlimited white noise with known SNR. Figure 4 shows a Doppler frequency estimate of 200 km/h in 30 dB known SNR. In Figure 5, we observe Fisher's information per sample in Eq. (26) corresponding to Figure 4 initially increases and quickly converges to a steady-state. This demonstrates the effectiveness of an empirical estimate of Fisher's information when analytical results are intractible.

In the future, we will investigate joint estimation of Doppler frequency and SNR using recursions presented in Section IV. The results will be compared to the ACF method presented in Section V as well as the LCR method in [4].

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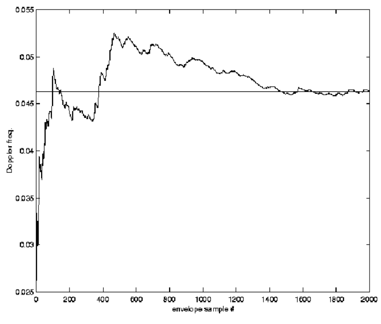

Figure 1: Estimation of Doppler frequency $\hat{f}_m$. True $f_m = 0.0463$ corresponding to a MS moving at $20km/h$, sampling rate 800 Hz, no additive noise, single trial.

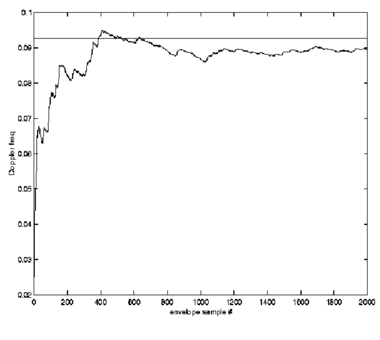

Figure 2: Estimation of Doppler frequency $\hat{f}_m$. True $f_m = 0.0926$ corresponding to a MS moving at $40km/h$ sampling rate 800 Hz, no additive noise, single trial.

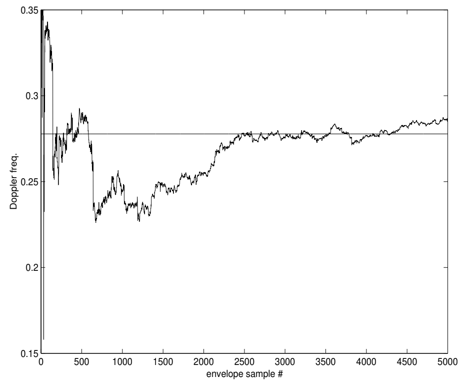

Figure 3: Estimation of Doppler frequency $\hat{f}_m$. True $f_m = 0.2778$ corresponding to a MS moving at $60km/h$, sampling rate 400 Hz, SNR 0 dB. Average of 2 Monte Carlo trials

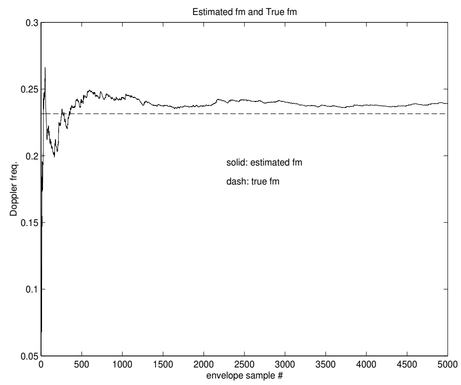

Figure 4: Estimation of Doppler frequency $\hat{f}_m$. True $f_m = 0.2315$ corresponding to a MS moving at $200km/h$, sampling rate 1600 Hz, SNR 30 dB, single trial.

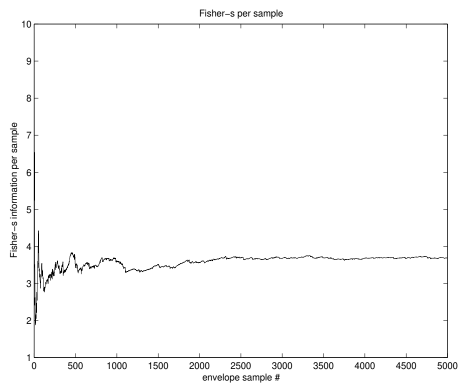

Figure 5: Fisher's information per sample corresponding to Figure 4.