# Antenna Array Mutual Coupling Effects on Cellular CDMA Communication Systems


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## *Abstract*

The electromagnetic behaviour of antenna array structures is usually modeled by combining the beampatterns of the individual antenna elements through superposition. Unfortunately, superposition models do not take into account the effects of mutual coupling and thus potentially lead to less accurate system performance predictions. Furthermore, methods for computing beampatterns which consider mutual coupling effects are computationally intensive and closed-form expressions can only be found for certain special cases that are rarely encountered in practice. In this paper, we outline and compare three methods which consider these effects during beampattern synthesis in our study of cellular CDMA communication systems using smart antennas.


## I Introduction

Code-Division Multiple Access (CDMA) is a technology by which greater cell capacity is achieved by increasing the number of users within a given bandwidth as compared with other multiple access techniques [7]. Although CDMA on its own is enough to satisfy the current mobile communications market, new techniques must be developped which can further increase cell capacity to meet future demands. One such technique is to increase capacity via digital beamforming and base station antenna arrays [5] [6] [11] [15].

One method of studying the capacity of CDMA communications systems is investigating the electromagnetic behaviour of antenna array structures, usually modelled by combining the beampatterns of the individual antenna elements through superposition [4] [5] [6] [8] [11]. One commonly found phenomenon associated with the electromagnetic behaviour of antenna arrays is mutual coupling. By definition, when a pair of antennas are in close proximity, whether one or both antennas are transmitting and/or receiving, some of the energy that is primarily intended for one antenna ends up at the other. Thus, mutual coupling effects do have an effect on the beampatterns of the array. Unfortunately, most beampattern synthesis models do not consider the effects of mutual coupling [5] [6] [11] thus potentially leading to less accurate system performance predictions.


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In this paper, we outline and compare three methods which consider mutual coupling effects in smart antenna arrays. These three methods are the Induced EMF Method, the Method of Moments, and a finite-element numerical computation method (also based on a Method of Moments approach). With each method, we perform beampattern synthesis and compare the results to the case where mutual coupling effects are neglected. Furthermore, cell capacity estimates of models which incorporate mutual coupling effects, generated using the Induced EMF Method and the Method of Moments, shall be examined and compared against models which do not consider these effects.

Througout this paper, all antenna array models are of circular geometry comprising of half-wavelength dipoles in a side-by-side arrangement [5] [6] [11].

## II Induced EMF Method

The Induced EMF (electromagnetic fields) Method [2] [3] [9] [10] [12] is a classical method of computing the self and mutual impedances of a collection of two-port networks, such as antenna arrays. Using the mutual impedance matrix generated by the Induced EMF Method, $[Z_{mn}]$, where $1 \leq n, m \leq N$ for $N$ array elements, we perform beampattern synthesis. Although this method is restricted to straight, parallel, and in-echelon elements and does not account for the radius of the wires and the gaps at the feeds, its advantage is that it leads to closed-form solutions which provide adequate design data. After extensive derivation, we obtain the following expressions for the $N \times N$ impedance matrix $[Z_{mn}]$ with components

$$Z_{mn} = \begin{cases} 30\left(0.5772 + \ln(2\beta l) - Ci(2\beta l)\right) \\ \quad + j30\left(Si(2\beta l)\right) & , m = n \\ \\ R_{mn} + jX_{mn} & , m \neq n \end{cases} \quad (1)$$

where

$$\begin{aligned} R_{mn} = {} & 30\cos(2\beta l)(Ci(u_0) + Ci(v_0) - 2Ci(u_1) - 2Ci(v_1) \\ & + 2Ci(\beta d)) + 30\sin(2\beta l)(-Si(u_0) + Si(v_0) + 2Si(u_1) \\ & - 2Si(v_1)) + 30(-2Ci(u_1) - 2Ci(v_1) + 4Ci(\beta d)) \end{aligned}$$

$$\begin{aligned} X_{mn} = {} & 30\cos(2\beta l)(-Si(u_0) - Si(v_0) + 2Si(u_1) + 2Si(v_1) \\ & - 2Si(\beta d)) + 30\sin(2\beta l)(-Ci(u_0) + Ci(v_0) + 2Ci(u_1) \\ & - 2Ci(v_1)) + 30(2Si(u_1) + 2Si(v_1) - 4Si(\beta d)) \end{aligned}$$

$$u_0 = \beta(\sqrt{d^2+4l^2}-2l) \qquad v_0 = \beta(\sqrt{d^2+4l^2}+2l)$$
$$u_1 = \beta(\sqrt{d^2+l^2}-l) \qquad v_1 = \beta(\sqrt{d^2+l^2}+l)$$
$$Ci(u) = \int_\infty^u \frac{\cos(x)}{x}dx \qquad Si(u) = \int_0^u \frac{\sin(x)}{x}dx$$

and $d$ is the horizontal distance between two dipole antennas, $l$ is half the length of the dipole antenna, and $\beta$ is the wave number, namely, $\beta = 2\pi/\lambda$.

After creating the mutual impedance matrix $[Z_{mn}]$, we must solve the matrix equation $[Z_{mn}]^{-1}[V_n] = [I_m]$ to use in our beampattern synthesis, where $[V_n]$ is a vector of the voltages along each antenna, $[I_m]$ is a vector of the currents along each antenna, and $[Z_{mn}]^{-1}$ is the inverse matrix of $[Z_{mn}]$. Provided we define the elements of $[V_n]$, we can determine the current for each antenna using the above equation. Using $[I_m]$, we can perform beampattern synthesis for a circular array using [12]

$$A(\theta) = \left| \sum_{m=1}^{N_A} I_m e^{j\left(\frac{\pi \cos(\theta - m(2\pi/N_A))}{2\sin(\pi/N_A)}\right)} \right|^2 \tag{2}$$

where $I_m$ is an element of $[I_m]$, $\theta$ is the AOA (angle-of-arrival), and $N_A$ is the number of antenna elements.

## III Method of Moments

The Method of Moments is another technique of obtaining the mutual impedance matrix $[Z_{mn}]$, which is used in beampattern synthesis [1] [8] [13] [14]. Using basic equations from electromagnetics and assuming that the current flows in one direction, the current and charge densities are approximated by filaments of current and charge on the wire axis, and viewing the antenna array as a system of $N$ small increments which constitute an $N$-port network, we get

$$\begin{aligned} Z_{mn} = j\omega\mu\Delta\vec{l}_n \cdot \Delta\vec{l}_m \psi(n,m) + \tfrac{1}{j\omega\varepsilon}[\psi(n^+,m^+) \\ -\psi(n^-,m^+) - \psi(n^+,m^-) + \psi(n^-,m^-)] \end{aligned} \tag{3}$$

$$\psi(n,m) = \frac{1}{\Delta l_n}\int_{\Delta l_n} \frac{e^{-jkR_m}}{4\pi R_m}dl \tag{4}$$

$$R_m = \begin{cases} \sqrt{\rho^2 + (z-z_m)^2} & , \quad m \neq n \\ \sqrt{a^2+z^2} & , \quad m = n \end{cases} \tag{5}$$

where $\rho$ is the horizontal distance between antennas containing points $n$ and $m$, $a$ is the dipole antenna radius, $k$ is the wave number, $z_m$ is the vertical distance between points $n$ and $m$, $\mu$ is the permeability, $\epsilon$ is the permittivity, $\Delta l_n$ is the length of the $n^{th}$ increment, $\omega$ is the frequency of operation (in radians per second), $n^-$ and $n^+$ denote the starting and terminating points of the $n^{th}$ increment, and $m^-$ and $m^+$ denote the starting and terminating points of the $m^{th}$ increment.

Numerically integrating $\psi(n,m)$, we obtain $[Z_{mn}]$ and solve the matrix equation $[Z_{mn}]^{-1}[V_n] = [I_m]$, where $[V_n]$ specifies a vector of the increment voltages and $[I_m]$ specifies a vector of the increment currents. If the antennas are center-fed, only the increments corresponding to the centers of the antennas will have a non-zero voltage. Upon obtaining $[I_m]$, we perform beampattern synthesis by determining the far-zone field at a point $i$ using [1]

$$A_i = \frac{\mu e^{-jkr_i}}{4\pi r_i}\sum_{j=1}^{n} I_j e^{jkr_j\cos\xi_{ij}}\Delta l_j \tag{6}$$

where $r_i$ and $r_j$ are the radius vectors to the distant field and source points, respectively, and $\xi_{ij}$ is the angle between them.

## IV Finite-Element Numerical Computation using IE3D

One commercially-available finite-element numerical computation software package is known as IE3D [16]. It is capable of modeling both the electric current on a metallic structure as well as a magnetic current representing the field distribution on a metallic aperture. The computation process of IE3D is based upon solving an integral equation derived using Green's Functions and the Method of Moments [16]. The $N \times N$ matrix equation is given as

$$[Z_{mn}][I_m] = [V_n] \tag{7}$$

such that

$$Z_{mn} = \int_S \{Z_s\vec{B_m}\cdot\vec{B_n}\}ds + \int_S ds\int_S\{\vec{B_m}\cdot\vec{G}(\vec{r}|\vec{r}')\cdot\vec{B_n}\}ds' \tag{8}$$

and

$$V_n = \int_S\{\vec{E_i}(\vec{r})\cdot\vec{B_n}\}ds \tag{9}$$

where we compute the voltage vector $[V_n]$ and mutual impedance matrix $[Z_{mn}]$ and use (7) to solve for the current vector $[I_m]$. The differences among finite elements formulations are based on the choice of basis functions $\vec{B_n}(\vec{r})$ for the current distribution representation and the Green's function $\vec{G}(\vec{r}|\vec{r}')$.

The rationale for a finite-element approach is that it is capable of predicting the patterns of larger classes of antenna arrays which cannot be analytically studied, such as those that have directional radiation which could be employed in fixed wireless applications.

## V Cell Capacity Estimates

We extend the derivation of cell capacity of CDMA communication systems in [7] for interference limited systems to take into account mutual coupling effects. The mean mobile power for an $N_A$ antenna array is [5]

$$E\{P_R\} = \left(\frac{E_b}{N_0}\right)N_A\sigma_n^2\left\{\frac{1}{R_B} - \frac{E\{\phi_k\}N_I}{B}\left(\frac{E_b}{N_0}\right)\right\} \tag{10}$$

where $B$ is the bandwidth of the spread signal, $E_b/N_0$ is the signal-to-noise ratio, $R_B$ is the single data bit rate for all users, $\sigma_n^2$ is the variance of the background thermal noise which is proportional to $N_o$, and $E\{\phi_k\}$ is the expected fraction of the interferer's $k$'s signal power passed by the beamforming weights. In this study, we use $B = 1.2288\text{MHz}$, $E_b/N_0 = 7\text{dB}$, and $R_B = 9600\text{bits/second}$.

To find the predicted capacity of the system, we must find the minimum number of mobiles which makes (10) negative. Thus, for the multiple antenna element case, the expression becomes

$$N_M = N_I + 1 = \left\lfloor \frac{1}{E\{\phi_k\}} \left( \frac{B}{R_B\left(E_b/N_0\right)} \right) \right\rfloor \quad (11)$$

where $N_I$ is the number of interfering mobiles.

The determination of the expected interference factor $E\{\phi_k\}$ is similar to that found in [5] with the exception that instead of an integration over the beampattern, we discretize the problem in one degree increments over $360^o$ and consider the effects of mutual coupling. We obtain

$$E\{\phi_k\} = \frac{1}{(360)^2} \sum_{i=0}^{359} \sum_{l=0}^{359} \left| \frac{\vec{w}_{MC}^H(\theta_{di})}{\|\vec{w}_{MC}^H(\theta_{di})\|} \cdot \frac{\vec{a}(\theta_{kl})}{\|\vec{a}(\theta_{kl})\|} \right|^2 \quad (12)$$

where $\vec{w}_{MC}(\theta_{di})$ is the beamforming weight vector for a desired user $d$ at an angle $\theta_{di}$ that includes the effects of mutual coupling, $\vec{a}(\theta_{kl})$ is the array response vector for a signal arriving from the direction $\theta_{kl}$, and $\|.\|$ is the vector norm. Both the beamforming weight and array response vectors result from our work described earlier concerning beampattern synthesis.

It should be noted that a circular array geometry is used and may not be optimum; other geometries, such as linear arrays, might perform relatively better for capacity predictions with and without mutual coupling. Furthermore, perfect power control and uniform distribution of mobiles are assumed throughout these models. Although more accurate capacity estimates may be obtained with imperfect power control and scattering due to multipath, the relative effects of mutual coupling will remain approximately the same, although the absolute capacity values will drop. We are currently investigating scattering effects, as explained in the future work section.

## VI Results

Using the Induced EMF Method and the Method of Moments beampattern synthesis models, as well as finite element numerical computation, beampattern synthesis can be performed for a number of desired angle-of-arrivals based on array geometries consisting of equal length dipoles positioned in a side-by-side configuration. For example, Figures 1, 2, 3, and 4 are the normalized beampatterns of four half wavelength dipoles arranged in a circular array, shown in Figure 5, focusing on an angle-of-arrival of $45^o$. Figure 1 does not consider the effects of mutual coupling while Figures 2, 3, and 4 do include these effects. Moreover, the effects of mutual coupling are generated in Figure 2 using the Induced EMF Method, with its associated normalized admittance matrix shown in Table 2, while in Figure 3 the Method of Moments was used to model these effects. In Figure 4, the beampattern is generated using IE3D's finite-element numerical computation, which is also based on a Method of Moments approach.

It is observed that Figures 2, 3, and 4 compare closely to one another. When compared to Figure 1, we see that Figures 2, 3, and 4 all exhibit higher sidelobe levels and wider beamwidths relative to Figure 1.

The cell capacity estimates of models which include mutual coupling effects are listed in Table 1 alongside results which do not incorporate mutual coupling. Columns (a) and (b), which contain capacity predictions of models that include mutual coupling effects generated by the Method of Moments and Induced EMF Method, respectively, compare closely with each other. On the other hand, they are noticeably lower when compared to columns (c) and (d), which do not contain mutual coupling effects and are derived in [5] [6], respectively. This makes sense when one examines the beampattern synthesis results shown earlier, especially the effect of mutual coupling on the behaviour of a pattern's sidelobe levels and beamwidths. Thus, an increase beamwidth and/or higher sidelobe levels would certainly cause a decrease of cell capacity.

Finally, finite element numerical computations agree strongly with the analytically derived antenna array models which consider mutual coupling effects. Therefore, finite-element methods might be used to predict beampatterns of larger classes of antenna arrays which cannot be analytically studied, including directional antennas.

## VII Summary

In this paper, several beampattern synthesis models which incorporate the effects of mutual coupling in the antenna array have been presented. Moreover, the beampattern generation and cell capacity prediction results of these models have been exhibited and compared against one another and to models which do not consider mutual coupling effects. From these results, we see that the Induced EMF Method, the Method of Moments, and the finite-element numerical computation all provide beampatterns and capacity predictions which compare closely. Furthermore, all the beampatterns which include mutual coupling effects have higher sidelobe levels and wider beamwidths relative to beampatterns which do not consider mutual coupling effects. Also, the system capacity estimates decrease with respect to ideal estimates when mutual coupling effects are considered. Finally, the finite-element numerical computations have a strong correlation to the analytically derived antenna array models which consider mutual coupling effects. Thus, finite-element methods might be used to predict beampatterns of larger classes of antenna arrays which cannot be analytically studied, including directional antennas.

## VIII Future Work

In the future, an investigation of beamforming performance of CDMA networks involving a ring of scatterers and mutual coupling will be carried out. Furthermore, the assumption of perfect power control will be dropped. Time variation in SINR due to imperfect power control will be included as well as scatter and mutual coupling. Finally, we shall apply these results to improve our cell capacity prediction studies of CDMA networks which use signal array processing algorithms that do not require array manifold knowledge.

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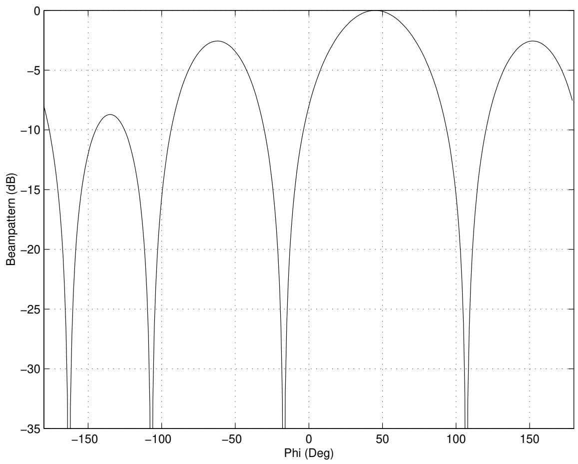


Figure 1: A sample beampattern focusing on an angle-of-arrival (AOA) of $45^o$ in a circular array with 4 elements which does not consider mutual coupling effects

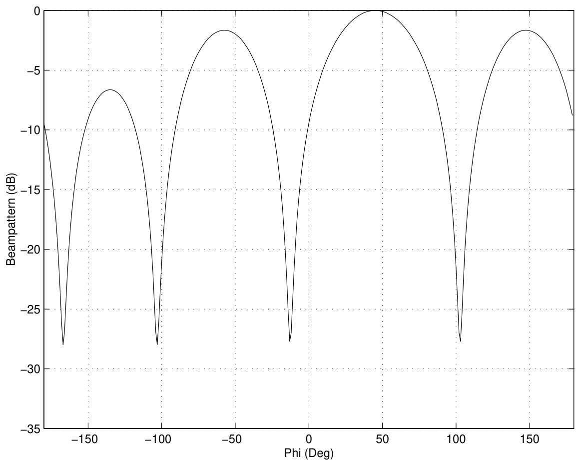


Figure 2: A sample beampattern focusing on an angle-of-arrival (AOA) of $45^o$ in a circular array with 4 elements which considers mutual coupling effects generated using the Induced EMF Method

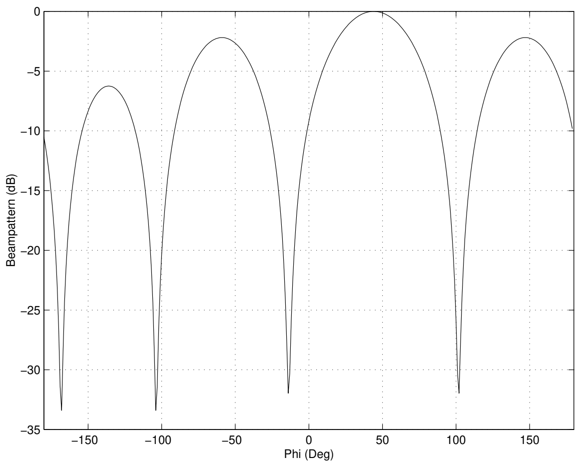


Figure 3: A sample beampattern focusing on an angle-of-arrival (AOA) of $45^o$ in a circular array with 4 elements which considers mutual coupling effects generated using the Method of Moments

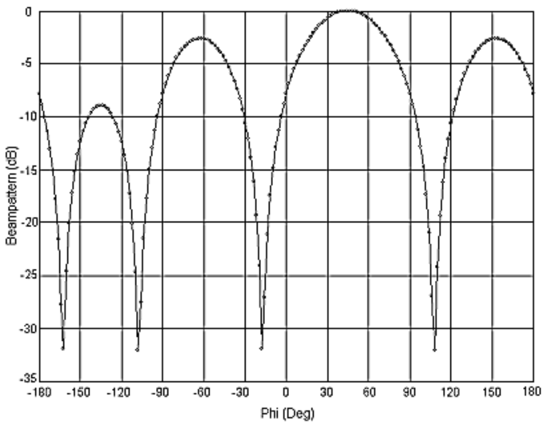


Figure 4: A sample beampattern focusing on an angle-of-arrival (AOA) of $45^o$ in a circular array with 4 elements which considers mutual coupling effects generated using finite-element numerical computation (IE3D)

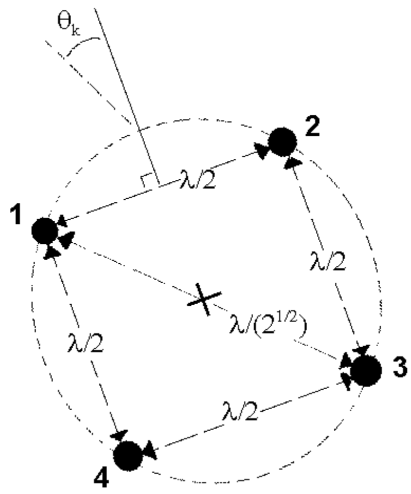


Figure 5: A circular array consisting of four half wavelength dipole antennas arranged in a side-by-side configuration, each of thickness $0.01\lambda$, receiving a signal from an angle-of-arrival of $\theta_k$

| $N_A$ | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 1 | 26 | 26 | 26 | 26 |
| 2 | 46 | 43 | 47 | 37 |
| 3 | 62 | 60 | 65 | 59 |
| 4 | 78 | 78 | 79 | 82 |
| 5 | 93 | 97 | 104 | 104 |
| 6 | 112 | 111 | 125 | 128 |

Table 1: Capacity predictions for circular arrays comprising of $N_A$ antenna elements: (a) with mutual coupling effects generated with the Method of Moments, (b) with mutual coupling effects generated with the Induced EMF Method, (c) without mutual coupling effects [5], (d) without mutual coupling effects [6]

$$[Y_{mn}] = [Z_{mn}]^{-1} = \begin{bmatrix} -0.6303-0.7763i & 0.3735+0.3009i & 0.1583+0.2039i & 0.3735+0.3009i \\ 0.3735+0.3009i & -0.6303-0.7763i & 0.3735+0.3009i & 0.1583+0.2039i \\ 0.1583+0.2039i & 0.3735+0.3009i & -0.6303-0.7763i & 0.3735+0.3009i \\ 0.3735+0.3009i & 0.1583+0.2039i & 0.3735+0.3009i & -0.6303-0.7763i \end{bmatrix}$$

Table 2: The normalized admittance matrix generated by the Induced EMF method for four half wavelength dipole antennas arranged in a circular array