Random Antenna Selection & Antenna Swapping Combined with OSTBCs

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***Abstract*— This paper proposes a novel and efficient iterative antenna selection algorithm based on an SNR selection criterion for a multi-input-multi-output (MIMO) system employing orthogonal space time block codes (OSTBCs), specifically Alamouti codes. The proposed algorithm addresses the open problem of finding a suboptimal set of transmit and receive antennas that performs close to the globally optimum configuration selection solution at significantly reduced complexity. Also, the paper is proposing a method to incrementally update the selected set of antennas, so as to enable even greater complexity reduction, and in particular, for the case of channels with temporal or time correlation. To date, antenna selection has not been assessed under temporal correlation. Simulation results show promising average bit error rate (ABER) performance gain after only a small number of iterations.**

***Index Terms*— multi-input-multi-output (MIMO), orthogonal space time block codes (OSTBCs), Average Bit Error Rate (ABER), antenna swapping (AS), Antenna selection, exhaustive search (ES).**

I. Introduction

In wireless communications employing multiple transmit and receive antennas can bring promising improvement to the quality of the communication link or to the capacity of the system [1]. The potential gain in performance for a multiple-input multiple-output (MIMO) system is mitigated by the increased cost of the number of expensive radio-frequency (RF) hardware and complexity. To reduce cost and enable the feasibility of deploying MIMO technology, a complexity reduction technique known as antenna selection can be applied. In antenna selection, only a subset of the full array of transmit and receive antennas are chosen based on some criterion. The antennas are connected to a limited number of RF chains by a low-cost RF switch. The resulting system enjoys many benefits offered by the full complexity MIMO system but with fewer RF resources.

There exist many antenna selection algorithms in the literature, and they can be broadly classified into transmit side, receive side, or joint transmit and receive antenna selection. Selection criteria typically maximize system capacity or diversity. Some early work on antenna selection with a focus on diversity includes a receive antenna selection algorithm known as hybrid selection/maximal ratio combining [2]. Transmit antenna selection based on minimizing error rate is also studied in [3].

More recently, receive antenna selection algorithms based on a capacity maximization criterion are proposed and studied in [4] & [5]. The algorithms proposed in [4] iteratively remove antennas from a full set of receive antennas that result in minimum capacity loss. Iterative algorithms based on adding antennas that result in maximum capacity gain are proposed in [5]. Low complexity norm-based selection algorithms are also proposed in [6]. These receive antenna selection algorithms can also be applied for transmit antenna selection.

The study of joint transmit and receive antenna selection has been limited. In [7], a suboptimal algorithm based on the capacity maximization criterion is proposed, and antenna selection is performed for one side of the communication link at a time.

As noted in [8], an efficient or systematic method for finding the optimal set of transmit and receive antennas remains an open problem. However we show that the proposed suboptimal scheme results in performance gains comparable to the optimal scheme by first searching through a subset of all the available antenna configurations and secondly using an efficient scheme to incrementally update the antenna configuration over time. Simulation results show promising ABER gain and the gains are more significant in the case of temporally correlated channels. To date little has been done in terms of analyzing the effects of temporal correlation on the performance of antenna selection schemes. This paper addresses this issue by using the models presented in [9] and [10] to examine the performance of the proposed antenna selection scheme in temporally correlated channels.

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II. System Model

For a communication system with multiple antennas at both ends, let N_{tx}, L_{tx}, N_{rx}, and L_{rx} represent the number of transmit antennas, available transmit RF chains, receive antennas, and available receive RF chains, respectively, with $N_{tx} \leq N_{rx}$, $L_{tx} \leq L_{rx}$, $L_{tx} \leq N_{tx}$, and $L_{rx} \leq N_{rx}$. The received signal vector $\mathbf{r}$ can be represented as

$$\mathbf{r} = H\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{s}$ is the white input signal vector of dimension $L_{tx} \times 1$ with $E[\mathbf{s}\mathbf{s}^H] = E_s I_{L_{tx}}$; H is an $L_{rx} \times L_{tx}$ antenna selected MIMO channel with i.i.d. complex Gaussian channel gains and flat Rayleigh quasi-static fading; and $\mathbf{n}$ is the spatially white additive Gaussian noise vector of dimension $L_{rx} \times 1$ with $E[\mathbf{n}\mathbf{n}^H] = N_o I_{L_{rx}}$. The input signal-to-noise ratio (SNR) is defined as $\gamma_o = E_s/N_o$.

The proposed algorithm is also applicable to transmit antenna correlated MIMO channels, and can be modeled as [3]

$$H = H_w R_t^{1/2} \tag{2}$$

where R_t is the $L_{tx} \times L_{tx}$ covariance matrix between the rows of H_w, and $(.)^{\frac{1}{2}}$ represents the square root of a matrix.

A. Orthogonal Space Time Block Codes

The proposed system does not require the availability of any form of channel state information at the transmitter, therefore to achieve transmit OSTBCs have been employed. In this work Alamouti's [11] popular OSTBC has been employed however for $L_{tx} > 2$ and/or $L_{rx} > 2$ there exists a variety of OSTBCs that can be used with the proposed scheme. References [12] and [11] provide more information on OSTBCs and Alamouti coding respectively.

B. BER Expressions

The space-time channel decouples into multiple independent scalar channels, in the case of Alamouti coding there are two resulting sub-channels each with the following SNR:

$$SNR = \frac{||\mathbf{h}||^2}{\sigma^2}.E[|s_n|^2], \tag{3}$$

where $||\mathbf{h}||^2$ is the 1×2 vector h_{11}, h_{12} or h_{21}, h_{22}, σ^2 is the variance of the white Gaussian noise, and $E[|s_n|^2]$ is the average transmitted power.

$$P_0 = c.Q\left(\sqrt{g.\gamma_b 2\frac{|\mathbf{h}|^2}{\sigma^2}}\right), \tag{4}$$

where c and g are constants ($c = 1$ and $g = 2$ for BPSK) and $\gamma_b 2\frac{|h|^2}{\sigma^2} \simeq SNR$. Using (3) and Using (4) one can calculate the BER associated with each subchannel and consequently the overall ABER. It is assumed that the receiver estimates the channel, while the transmitter has no channel knowledge.

C. Selection Criterion

In this paper, the average ABER of the system in is used as the antenna selection criterion. However since the Q function is a monotonically decreasing function, the computational complexity has been significantly reduced and SNR is used a benchmark for antenna selection.

D. Temporally Correlated Channels

The models provided in [9] and [10] were used to generate a vector of correlated Rayleigh variates with the desired maximum doppler frequency. The channel matrix was then generated as follows:

$$H_n = vec^{-1}(\mathbf{h}_n), \tag{5}$$

where $\mathbf{h}_n^T$ is the $1 \times N_{tx} \times N_{rx}$ vector of timely correlated Rayleigh random variables with maximum doppler frequency f_d at time instance n and H_n is the desired channel matrix at time n.

III. Antenna Swapping

The operation of swapping a pair of receive (transmit) antennas is equivalent to replacing a row (column) in H_{n-1} with another row of channel gains to form H_n. The modification matrices for these operations will be presented below.

A. Matrix Modification for Receive Antenna Swapping

From [5], it can be seen that adding a receive antenna is equivalent to performing a rank-1 modification to the $H^H H$ matrix as follows

$$H_n^H H_n = H_{n-1}^H H_{n-1} + \mathbf{h}\mathbf{h}^H \tag{6}$$

where $H_{n-1}^H H_{n-1}$ is the original matrix in the $(n-1)^{th}$ step of the algorithm, $H_n^H H_n$ is the modified matrix in the n^{th} step, and $\mathbf{h}\mathbf{h}^H$ is the rank-1 modification. Therefore, if the j^{th} row is swapped out and the i^{th} row is swapped in, then the contribution from the j^{th} row can be subtracted and the contribution from the i^{th} row can be added to $H_n^H H_n$ to obtain $H_{n+1}^H H_{n+1}$ as follows

$$\begin{aligned} H_{n+1}^H H_{n+1} &= H_n^H H_n + \mathbf{h}_i \mathbf{h}_i^H - \mathbf{h}_j \mathbf{h}_j^H \\ &= H_n^H H_n + UV^H \end{aligned} \tag{7}$$

$$\text{where } U = \begin{bmatrix} \mathbf{h}_i & \mathbf{h}_j \end{bmatrix} \;\; ; \;\; V = \begin{bmatrix} \mathbf{h}_i & -\mathbf{h}_j \end{bmatrix} \tag{8}$$

and the modification matrix $M = UV^H$ is rank-2. To swap k pairs of receive antennas at at time, a rank-$2k$ modification matrix would be required.

B. Matrix Modification for Transmit Antenna Swapping

When the i^{th} column in H_n is changed, it will affect the i^{th} row and i^{th} column in $H_n^H H_n$. Therefore, changing the i^{th} column in H_n, requires simultaneous changes to the symmetric i^{th} row and i^{th} column of $H_n^H H_n$. When replacing the i^{th} transmit antenna in H_n with transmit antenna j from the full complexity MIMO matrix, the modification matrix C is found to be of the form $C = PQ^H$ with

$$P = \begin{bmatrix} \mathbf{p} & \mathbf{e}_i \end{bmatrix} \;\; ; \;\; Q = \begin{bmatrix} \mathbf{e}_i & \mathbf{p} \end{bmatrix} \tag{9}$$

where P and Q are L_{tx} x 2 matrices, with $\mathbf{p}$ containing the adjustment values to $H_n^H H_n$, and $\mathbf{e}_i$ is a vector with a 1 in the i^{th} position and zeros everywhere else. Expanding C and comparing its elements to the terms in $H_n^H H_n$, the values for $\mathbf{p}$ can be found to be

$$p_{k,1} = \mathbf{h}_k^H \mathbf{h}_j - \mathbf{h}_k^H \mathbf{h}_i \tag{10}$$

for $k = 1, ..., (i-1), (i+1), ..., L_{tx}$, and the i^{th} element in $\mathbf{p}$ can be determined as

$$p_{i,1} = \frac{1}{2}\left(\mathbf{h}_j^H \mathbf{h}_j - \mathbf{h}_i^H \mathbf{h}_i\right) \tag{11}$$

With the correction matrix C defined, the new $H_{n+1}^H H_{n+1}$ can be computed as

$$H_{n+1}^H H_{n+1} = H_n^H H_n + C = H_n^H H_n + PQ^H \tag{12}$$

C. Overall Algorithm

Table I provides an overview of the outlined algorithm for the simplified antenna selection scheme. Temporally correlated channel matrices are generated separately to test the outlined antenna selection scheme.

TABLE I
ANTENNA SELECTION WITH AS ALGORITHM PSEUDOCODE

Initialization:
Randomly select L_{tx} transmit, L_{rx} receive antennas to form H_0
Calculate $\left(H_0^H H_0\right)$ and SNR_0 using (3)
Initialize $\left(H^H H\right)_{best} = \left(H_0^H H_0\right)$ and $SNR_{best} = SNR_0$
Main Loop:
Iterate through the antenna configurations {
Swap a pair of transmit or receive antennas
Update $\left(H_n^H H_n\right)$ with (7) or (12)
Calculate SNR_n using (3)
if (SNR$_n <$ SNR$_{n-1}$) then
Current antenna configuration is the best
$\left(H^H H\right)_{best} = \left(H_n^H H_n\right)$
end if }

IV. ANTENNA SELECTION & AS PERFORMANCE ANALYSIS AND SIMULATION RESULTS

For a (4:4, 2:4) MIMO system, the ABER performance of the the proposed antenna selection with AS algorithm under uncorrelated Rayleigh flat fading for MIMO OSTBC (Alamouti) using BPSK modulation is shown in Figure 1. Simulations are performed for $1,000,000$ channel realizations with a maximum doppler frequency $f_d\tau = 333Hz$..

The number of iterations are varied as a percentage of an exhaustive search (ES). The proposed algorithm's ABER performance converges to the ES performance as the number of iterations increase. However it is important to note that at the same time, the ABER performance gain diminishes. In Figure 1 the top most curve is when no antenna selection is performed, and a fixed subset of antennas are used for all the channel realizations, which is equivalent to a 2×2 system. The second curve represents 50% of the ES iterations performed at every 5^{th} frame, thus requiring the channel to be estimated less frequently (less overhead). However clearly there is no performance gain when the channel is uncorrelated overtime. The third curve achieves the performance gains of the ES algorithm within 1 dB by only performing half the required iterations. These are the performance gain for a (4:4, 2:2) system and the gain would be more significant at higher SNRs as can be seen in Figure 1. Figure 2 represents the ABER performance of the proposed antenna selection under temporally correlated Rayleigh flat fading for MIMO OSTBC (Alamouti) using BPSK modulation. The top curve represents the performance results for a system with no antenna selection. The two subsequent curves represent the performance of the system when antenna swapping is performed at every 5th and 20th frame respectively. There is a performance gain 1dB in the system (in the SNR range 6-12 dB and increasing) by performing only 25% of the ES iterations at every 20th frame. This is a significant gain considering the minimally added overhead. Similar to the uncorrelated scheme the system achieves the ES performance (optimal configuration of antennas) within 1dB by performing 50% of ES iterations.

V. Conclusion

An efficient implementation of the antenna selection algorithm based on antenna swapping is introduced. The combined scheme is a novel joint transmit and receive antenna selection algorithm based on a minimum average ABER criterion. Analysis shows that the algorithm results in promising performance gains by introducing minimal overhead. The proposed AS algorithm allows for the channel estimation to be performed incrementally over time thus significantly reducing the start-up complexity associated with perviously outlined antenna selection algorithms. This allows for the selection of antennas to be optimized overtime. It is also for the first time demonstrated that the performance gains associated with antenna selection can be achieved as significantly reduced complexity (in terms of channel estimation and antenna selection) if the channel is timely correlated from one instance to another. Further analysis of the proposed algorithm's outage probability and throughput are subjects of future study.

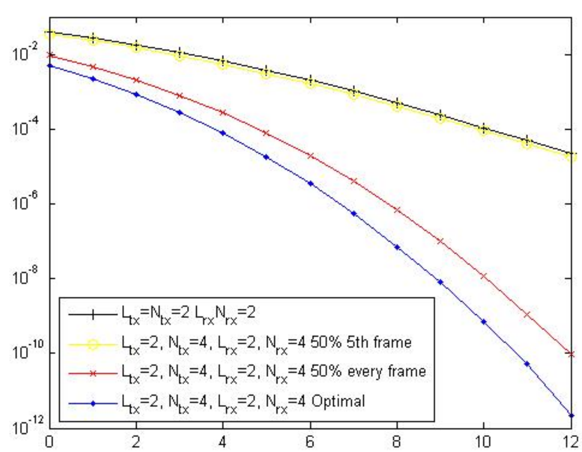

Fig. 1. Average BER Performance of (4:4, 2:2) i.i.d.

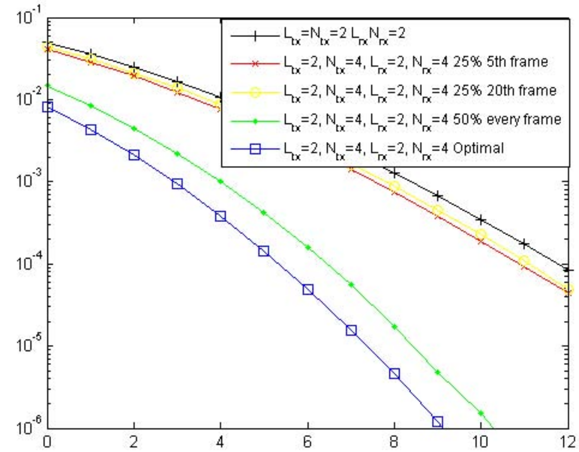

Fig. 2. Average BER Performance of (4:4, 2:2) temporally correlated.

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