Minimum BER Transmit Power Allocation and Beamforming for Two-Input–Multiple-Output Spatial Multiplexing Systems

Neng Wang and Steven D. Blostein, *Senior Member, IEEE*

***Abstract*—A two-input–multiple-output (TIMO) system represents an important special case of multiple-input–multiple-output (MIMO) systems in practical scenarios. In this paper, transmit optimization for TIMO spatial multiplexing systems is investigated. Minimum bit-error rate (MBER) is employed as the optimization criterion. Approximate MBER transmit power allocation for a variety of receiver structures is proposed, including zero-forcing, successive interference cancellation (SIC), and ordered SIC. Transmit-beamforming schemes using 4-ary pulse-amplitude modulation and quaternary phase-shift keying premixing are also proposed, which eliminate error floors in ill-conditioned TIMO channels. It is shown both analytically and by numerical simulations that the proposed schemes offer superior performance compared to existing schemes.**

***Index Terms*—Minimum bit-error rate (MBER), power allocation, precoding, spatial multiplexing, transmit beamforming, two-input multiple-output (TIMO).**

I. Introduction

Wireless communications using multiple transmit and receive antennas, known as multiple-input–multiple-output (MIMO) systems, offers key advantages over single-input–single-output (SISO) systems, such as diversity and spatial multiplexing gains [1]. Our goal of this correspondence is to investigate transmit optimization for a MIMO spatial multiplexing system with two transmit antennas, known also as two-input multiple-output (TIMO). The study of such a system can be motivated in a number of ways: 1) TIMO systems are important in practical scenarios where there are limitations on cost and/or space to install more antennas; 2) a virtual TIMO channel is created when two single-antenna mobiles operate in cooperative communication mode [2]; and 3) when transmit antenna selection is employed in MIMO to achieve diversity with reduced cost of transmit radio-frequency chains [3], selecting two out of multiple transmit antennas turns MIMO into TIMO.

Manuscript received December 18, 2004; revised August 4, 2005, January 31, 2006, and April 10, 2006. This work was supported in part by Grants from Samsung Electronics AIT, Korea, as well as from Bell Mobility, Canada. This paper was presented in part at IEEE Globecom 2005, St. Louis, MO, Nov.–Dec. 2005. The review of this paper was coordinated by X.-G. Xia.

N. Wang was with the Department of Electrical and Computer Engineering, Queen's University, Kingston, ON K7L 3N6, Canada. He is now with Nortel Networks Inc., Richardson, TX 75082 USA.

S. D. Blostein is with the Department of Electrical and Computer Engineering, Queen's University, Kingston, ON K7L 3N6, Canada (e-mail: steven.blostein@queensu.ca).

Digital Object Identifier 10.1109/TVT.2007.891416

Transmit optimization is receiver-dependent. Signal reception for spatial multiplexing can employ criteria such as maximum-likelihood (ML), zero-forcing (ZF), minimum mean-squared-error (MMSE), successive interference cancellation (SIC), or ordered SIC (OSIC) as, for example, in the case of the Vertical Bell Laboratories Layered Space-Time [4]. Efforts to optimize MIMO transceiver structures have involved, e.g., linear MMSE precoding/decoding [5] and minimum bit-error rate (MBER) precoding for ZF equalization (ZF-MBER) [6]. These schemes, however, generally require high feedback overhead and/or high-complexity processing, e.g., diagonalization of the channel matrix and/or matrix transformations at both the transmitter and the receiver.

In this correspondence, we consider simplified precoding by introducing structural constraints to precoding. We categorize TIMO channels into well- and ill-conditioned cases. Channel condition is determined by a number of factors, such as Ricean factor and/or spatial correlation. For well-conditioned TIMO channels, precoding is constrained to transmit power allocation, i.e., we optimize only the transmitted power of signal streams, resulting in reduced processing complexity compared to general precoding. Approximate MBER (AMBER) power allocation is proposed for TIMO spatial multiplexing in well-conditioned channels. When the TIMO channel is ill-conditioned, both ZF-MBER and MMSE precoding and the proposed power allocation are shown to experience error floors. We introduce added degrees of freedom and develop transmit-beamforming schemes under an MBER criterion, which can eliminate these error floors. Two transmit-beamforming methods are proposed based on 4-ary pulse-amplitude modulation (4-PAM) and quaternary phase-shift keying (QPSK) premixing. Compared to precoding which exploits spatial correlation of transmit antennas [7], the proposed transmit-beamforming method utilizes instantaneous channel state information (CSI) and does not depend on the channel model used. It is shown both analytically and by simulation that the proposed transmit-optimization schemes offer superior performance compared to existing precoding methods in general correlated fading channels.

II. TIMO Channel and Transceiver

Consider a TIMO system with $N_r \geq 2$ receive antennas. The received signal can be modeled as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \boldsymbol{\eta} = s_1\mathbf{h}_1 + s_2\mathbf{h}_2 + \boldsymbol{\eta} \tag{1}$$

where $\mathbf{s} = [s_1, s_2]^{\mathrm{T}}$ denotes a transmitted signal vector, $\mathbf{H} = [\mathbf{h}_1 \vdots \mathbf{h}_2]$ is an $N_r \times 2$ channel matrix, which is assumed to be general correlated Ricean fading [8], and $\boldsymbol{\eta}$ is an $N_r \times 1$ additive Gaussian noise vector. For simplification of analysis purposes, we assume white noise and input, i.e., $\mathbb{E}[\mathbf{s}\mathbf{s}^{\mathrm{H}}] = E_s\mathbf{I}_2$ and $\mathbb{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}] = N_0\mathbf{I}_{N_r}$, and define the signal-to-noise ratio (SNR) $\gamma_s \stackrel{\mathrm{def}}{=} E_s/N_0$. Binary phase-shift keying (BPSK) modulation is assumed. Extension to other constellations will be addressed later.

A. TIMO Signal Reception

1) ZF Receiver: With ZF equalization, the transmitted signal is estimated as $\hat{\mathbf{s}} = \mathbf{H}^{\dagger}\mathbf{r} = \mathbf{s} + \mathbf{H}^{\dagger}\boldsymbol{\eta}$, where $(\cdot)^{\dagger}$ denotes Moore–Penrose pseudoinverse. The decision-point SNR of the kth signal stream is obtained as

$$\gamma_{Z,k} = \frac{2E_s}{N_0\left[(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\right]_{k,k}} \stackrel{\mathrm{def}}{=} 2\gamma_s g_{Z,k}^2$$

where $g_{Z,k}^2 \stackrel{\mathrm{def}}{=} [(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}]_{k,k}^{-1}$ denotes the power gain of kth stream and can be calculated as

$$g_{Z,1}^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_2\|^2}, \qquad g_{Z,2}^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_1\|^2} \tag{2}$$

where $\Delta_{\mathbf{H}} \stackrel{\mathrm{def}}{=} \|\mathbf{h}_1\|^2\|\mathbf{h}_2\|^2 - |\mathbf{h}_2^{\mathrm{H}}\mathbf{h}_1|^2$.

2) SIC Receiver: Without loss of generality (w.o.l.g.), we assume that stream $k = 1$ is detected first. Assuming ZF equalization is employed, the power gain in detecting s_1 is given by

$$g_{S,1}^2 = g_{Z,1}^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_2\|^2}. \tag{3}$$

Assuming that $\hat{s}_1 = s_1$, the interference due to the first stream is then regenerated and subtracted, i.e., $\mathbf{r}' = \mathbf{r} - \hat{s}_1\mathbf{h}_1 = s_2\mathbf{h}_2 + \boldsymbol{\eta}$. The detection of s_2 in $\mathbf{r}'$ with ZF equalization is given by $\hat{s}_2 = \mathbf{h}_2^{\dagger}\mathbf{r}' = s_2 + (\mathbf{h}_2^{\mathrm{H}}\boldsymbol{\eta}/\|\mathbf{h}_2\|^2)$, which is equivalent to maximal ratio combining, with power gain

$$g_{S,2}^2 = \|\mathbf{h}_2\|^2. \tag{4}$$

3) OSIC Receiver: To improve SIC performance, the streams can be reordered based on SNR at each stage. The SNR-based ordering scheme [4] detects the stream with largest decision-point SNR first. The stream to be detected first is

$$k_1 = \arg\max_{l\in\{1,2\}} \gamma_{Z,l} = \arg\max_{l\in\{1,2\}} g_{Z,l}^2 = \arg\max_{l\in\{1,2\}} \|\mathbf{h}_l\|^2 \tag{5}$$

i.e., SNR-based ordering is equivalent to norm-based ordering in TIMO systems.[1] Therefore, we obtain the power gains as

$$g_{O,1}^2 = \frac{\Delta_{\mathbf{H}}}{\min\left\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\right\}} \tag{6}$$

$$g_{O,2}^2 = \min\left\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\right\}. \tag{7}$$

[1]We note that this does not apply to general MIMO with $N_t \geq 3$ transmit antennas.

The average BER of the above receivers can be calculated as [9]

$$\bar{P}\left(\gamma_s; g_1^2, g_2^2\right) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2}\right) \tag{8}$$

where the power gains g_1^2 and g_2^2 depend on the receiver structure and are given in (2)–(4), (6), and (7); $\mathcal{Q}(x) \stackrel{\mathrm{def}}{=} (1/\sqrt{2\pi})\int_x^{\infty} e^{-y^2/2}dy$. We note that for SIC and OSIC receivers, (8) is only a lower bound due to the neglecting of error propagation. However, at moderate-to-high SNR regimes, this lower bound closely approximates the average BER, since error propagation is minimal.

B. Ill-Conditioned TIMO Channels

We consider TIMO without transmit optimization first and address precoding later. Since the $\mathcal{Q}(\cdot)$ function decreases rapidly in its argument, the average BER in (8) is dominated by the term with smaller power gain. In the extreme case with vanishing power gain, the system experiences an error floor. We refer to this as an ill-conditioned TIMO channel. From gains in (2)–(4), (6), and (7), the channel is ill-conditioned when either $\Delta_{\mathbf{H}} \cong 0$ or $\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\} \cong 0$.

1) The condition $\Delta_{\mathbf{H}} \cong 0$ is equivalent to $\|\mathbf{h}_1\|^2 \cdot \|\mathbf{h}_2\|^2 \cong |\mathbf{h}_1^{\mathrm{H}}\mathbf{h}_2|^2$. We have $(|\mathbf{h}_1^{\mathrm{H}}\mathbf{h}_2|^2/\|\mathbf{h}_2\|^2) = \mathbf{h}_1^{\mathrm{H}}\Upsilon_{\mathbf{h}_2}\mathbf{h}_1 = \|\Upsilon_{\mathbf{h}_2}\mathbf{h}_1\|^2 \cong \|\mathbf{h}_1\|^2$, where $\Upsilon_{\mathbf{X}} \stackrel{\mathrm{def}}{=} \mathbf{X}(\mathbf{X}^{\mathrm{H}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{H}}$, is the projection matrix. Therefore, w.o.l.g., we can assume $\mathbf{h}_2 \cong a \cdot \mathbf{h}_1$ with $a \in \mathbb{C}$, and the channel matrix
$$\mathbf{H} \cong \mathbf{h}_1[1 \;\; a] \tag{9}$$
which is also an example of a "pinhole" channel [10]. The least squares (LS) estimate of a can be found to be
$$\hat{a}_{\mathrm{LS}} = \mathbf{h}_1^{\dagger}\mathbf{h}_2 = \frac{\mathbf{h}_1^{\mathrm{H}}\mathbf{h}_2}{\|\mathbf{h}_1\|^2}. \tag{10}$$
2) Denote $m = \arg\min_k\{\|\mathbf{h}_k\|^2\}$. The condition $\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\} \cong 0$ implies that $\mathbf{h}_m \cong 0$, i.e., the link from the mth transmit antenna to all receive antennas is blocked. Note that, in this case, the model (9) is still valid with $a \cong 0$.

Furthermore, it can be shown that the model (9) is valid for ill-conditioned TIMO with linear ZF-MBER and MMSE precoding/decoding as well.

III. Approximate MBER (AMBER) Transmit Power Allocation for TIMO

Denote the power allocated to the kth stream as $p_k^2 (k = 1, 2)$. The received signal is given by $\mathbf{r} = p_1 s_1 \mathbf{h}_1 + p_2 s_2 \mathbf{h}_2 + \boldsymbol{\eta}$. We assume that the total transmit power is constrained via $p_1^2 + p_2^2 = 2$. The average BER of TIMO with power allocation can be obtained by generalizing (8) to

$$\bar{P}\left(\gamma_s; \{g_k^2\}; \{p_k^2\}\right) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2 p_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2 p_2^2}\right). \tag{11}$$

To minimize the average BER in (11) under transmit-power constraint, no closed-form solution exists. However, taking the approach in [11], we approximate the objective function to obtain a closed-form AMBER solution with performance very close to the MBER solution. For general constellations, the BER can be approximated as $P_b(\gamma) \approx (1/5)e^{-c\gamma}$, where c is a constellation-specific constant [12]. For BPSK modulation, $c = 1$, and the AMBER power allocation is obtained as [11]

$$p_k^2 = \gamma_s^{-1} g_k^{-2} \left(\ln g_k^2 + \nu\right)_+, \qquad k = 1, 2 \tag{12}$$

where $(x)_+ \stackrel{\text{def}}{=} \max\{0, x\}$ and ν is chosen to satisfy power constraint. Note the fact that the total transmit power $p_1^2 + p_2^2$ is a piecewise-linear function in ν, with breakpoints at $-\ln g_1^2$ and $-\ln g_2^2$. W.o.l.g., we assume $g_1^2 \geq g_2^2$. We can simplify the solution (12) as

$$\begin{cases} p_1^2 = 2, \;\; p_2^2 = 0, & \text{if } \ln\left(g_1^2 - g_2^2\right) \geq 2\gamma_s g_1^2 \\ p_1^2 = \frac{\ln g_1^2 + \nu_a}{\gamma_s g_1^2}, \;\; p_2^2 = \frac{\ln g_2^2 + \nu_a}{\gamma_s g_2^2}, & \text{otherwise} \end{cases} \tag{13}$$

where $\nu_a = (2\gamma_s g_1^2 g_2^2 - g_1^2 \ln g_2^2 - g_2^2 \ln g_1^2)/(g_1^2 + g_2^2)$. A similar power-allocation scheme for OSIC was proposed [13]. The solution in [13] is actually a special case of our proposed solution (12), corresponding to the second case of (13). Although our discussion of power allocation is focused on TIMO channels and BPSK modulation, extension to general MIMO systems and modulations is straightforward [14], [15].

1) Feedback Overhead and Complexity: For a TIMO system using a general precoding method, either the channel or precoding matrix is required at the transmitter. The proposed power-allocation scheme requires only transmitted power information. Precoding schemes require diagonalization of a channel matrix [5], [6]. Using power allocation, operations performed at the transmitter are trivial. Therefore, power allocation has less overhead and complexity compared to general precoding.

2) Performance in Ill-Conditioned Channels: W.o.l.g., we assume $|a| \leq 1$ in the ill-conditioned channel (9). The power gains of OSIC can be obtained as $g_{O,1}^2 \cong 0$ and $g_{O,2}^2 = |a|^2\|\mathbf{h}_1\|^2$. Applying power allocation in (12), we obtain $p_1^2 = 0$, and $p_2^2 = 2$. The average BER of power allocation for ill-conditioned channels can be approximated as $\bar{P}(\gamma_s; \mathbf{h}_1, a) \cong (1/10) + (1/10)\exp\{-4\gamma_s|a|^2\|\mathbf{h}_1\|^2\}$, which experiences an obvious error floor. This motivates our study of transmit beamforming for ill-conditioned TIMO channels.

IV. Transmit Beamforming for Ill-Conditioned TIMO Channels

We consider BPSK modulation for derivation of transmit-beamforming schemes. The idea can be extended to other constellations.

A. Transmit-Beamforming Method

Consider general precoding for ill-conditioned TIMO channels. Denote the precoding matrix $\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$. The normalized transmit-power constraint is given by $\text{tr}(\mathbf{P}\mathbf{P}^{\text{H}}) = |p_{11}|^2 + |p_{12}|^2 + |p_{21}|^2 + |p_{22}|^2 = 2$. The received signal is $\mathbf{r} = \mathbf{H}\mathbf{P}\mathbf{s} + \boldsymbol{\eta}$. With ZF equalization, the estimate of the transmitted signal is given by

$$\begin{aligned} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} &= \mathbf{H}^\dagger \mathbf{r} \\ &= \frac{1}{1+|a|^2}\left(\begin{bmatrix} 1 & a \\ a^* & |a|^2 \end{bmatrix}\begin{bmatrix} p_{11}s_1 + p_{12}s_2 \\ p_{21}s_1 + p_{22}s_2 \end{bmatrix}\right. \\ &\quad \left. + \frac{\mathbf{h}_1^{\text{H}}\boldsymbol{\eta}}{\|\mathbf{h}_1\|^2}\begin{bmatrix} 1 \\ a^* \end{bmatrix}\right). \end{aligned} \tag{14}$$

From (14), we observe that the transmitted signals s_1 and s_2 are coupled in z_1 and z_2, and $z_2 = a^* z_1$. Therefore, it suffices to process z_1 only. Consider

$$\begin{aligned} z_1' &\stackrel{\text{def}}{=} \left(1 + |a|^2\right) z_1 \\ &= (p_{11} + ap_{21})s_1 + (p_{12} + ap_{22})s_2 + \frac{\mathbf{h}_1^{\text{H}}\boldsymbol{\eta}}{\|\mathbf{h}_1\|^2}. \end{aligned} \tag{15}$$

Signal detection can be performed in either 1-D or 2-D signal space.

1) 1-D Signal Detection: W.o.l.g., we assume that s_1 is detected first. The average BER can be approximated as

$$\begin{aligned} \tilde{P}_{1D}(\gamma_s; \mathbf{h}_1, a; \mathbf{P}) &\cong \frac{1}{20}e^{-\gamma_s\|\mathbf{h}_1\|^2[\Re(p_{11}+ap_{21}-p_{12}-ap_{22})]^2} \\ &\quad + \frac{1}{20}e^{-\gamma_s\|\mathbf{h}_1\|^2[\Re(p_{11}+ap_{21}+p_{12}+ap_{22})]^2} \\ &\quad + \frac{1}{10}e^{-\gamma_s\|\mathbf{h}_1\|^2[\Re(p_{12}+ap_{22})]^2} \\ &\stackrel{\text{def}}{=} \tilde{P}_{1D}. \end{aligned} \tag{16}$$

It can be shown that, w.o.l.g., the precoder minimizing (16) under the transmit-power constraint satisfies $p_{11} \geq 0$, $p_{12} \geq 0$, $ap_{21} \geq 0$, and $ap_{22} \geq 0$. Denote $p_{21} = a^* q_{21}$ and $p_{22} = a^* q_{22}$, where $q_{21} \geq 0$ and $q_{22} \geq 0$. To find a closed-form solution, we further approximate average BER (16) as

$$\begin{aligned} \tilde{P}_{1D}(\gamma_s; \mathbf{h}_1, a; \mathbf{P}) &\cong \frac{1}{10}e^{-\gamma_s\|\mathbf{h}_1\|^2\left(p_{11}+|a|^2 q_{21}-p_{12}-|a|^2 q_{22}\right)^2} \\ &\quad + \frac{1}{10}e^{-\gamma_s\|\mathbf{h}_1\|^2\left(p_{12}+|a|^2 q_{22}\right)^2}. \end{aligned} \tag{17}$$

By using the method of Lagrange multipliers, the solution to the problem of minimizing (17) can be obtained as $q_{21} = p_{11}$, $q_{22} = p_{12}$, and $p_{11} = 2p_{12}$. To satisfy the power constraint, we have

$$\begin{cases} p_{11} = \sqrt{8/5}\left(1 + |a|^2\right)^{-1/2} \\ p_{12} = p_{11}/2 \\ p_{21} = a^* p_{11} \\ p_{22} = a^* p_{12} \end{cases} \tag{18}$$

from which the precoder is obtained as

$$\begin{aligned} \mathbf{P}_{1D} &= \underbrace{\left(1 + |a|^2\right)^{-1/2}\begin{bmatrix} 1 \\ a^* \end{bmatrix}}_{\mathbf{v}_{\text{BF}}} \times \underbrace{\sqrt{\frac{2}{5}}\begin{bmatrix} 2 & 1 \end{bmatrix}}_{\mathbf{v}_{\text{PM1}}^{\text{T}}} \\ &\stackrel{\text{def}}{=} \mathbf{v}_{\text{BF}}\mathbf{v}_{\text{PM1}}^{\text{T}}. \end{aligned} \tag{19}$$

The precoder (19) has rank one and can be viewed as premixing $\mathbf{v}_{\mathrm{PM1}}$ followed by transmit beamforming $\mathbf{v}_{\mathrm{BF}}$ pointing in the AMBER direction. Note that $\mathbf{v}_{\mathrm{PM1}}$ premixes two BPSK streams with different allocated powers into a 4-PAM stream. We refer to this scheme as 4-PAM beamforming.

2) Two-Dimensional Signal Detection: W.o.l.g., we assume s_1 and s_2 are, respectively, in-phase and quadrature components of (15). We can assume, w.o.l.g., that $p_{11} \geq 0$, $jp_{12} \geq 0$, $ap_{21} \geq 0$, and $jap_{22} \geq 0$. Accordingly, the average BER can be calculated as

$$\bar{P}_{2D}(\gamma_s; \mathbf{h}_1, a; \mathbf{P}) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s\|\mathbf{h}_1\|^2(p_{11}+ap_{21})^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{-2\gamma_s\|\mathbf{h}_1\|^2(p_{12}+ap_{22})^2}\right). \tag{20}$$

By using Lagrange multipliers, the exact MBER solution that minimizes (20) under the transmit-power constraint is obtained as

$$\begin{cases} p_{11} = \left(1+|a|^2\right)^{-1/2} \\ p_{12} = jp_{11} \\ p_{21} = a^* p_{11} \\ p_{22} = ja^* p_{11}. \end{cases} \tag{21}$$

Therefore, the precoder is given by

$$\mathbf{P}_{2D} = \underbrace{\left(1+|a|^2\right)^{-1/2}\begin{bmatrix} 1 \\ a^* \end{bmatrix}}_{\mathbf{v}_{\mathrm{BF}}} \times \underbrace{[1 \quad j]}_{\mathbf{v}_{\mathrm{PM2}}^{\mathrm{T}}} \stackrel{\mathrm{def}}{=} \mathbf{v}_{\mathrm{BF}}\mathbf{v}_{\mathrm{PM2}}^{\mathrm{T}}. \tag{22}$$

Note that $\mathbf{v}_{\mathrm{PM2}}$ premixes two BPSK streams into a QPSK stream. We refer to this scheme as QPSK beamforming.

Feedback Overhead and Complexity: From (19) and (22), only an estimate of a is required at the transmitter, which can be obtained using (10). Operations performed at the transmitter are also trivial.

Connection to ML–Minimum Distance (MD) Precoding: We note that both ML–MD precoding [16] and QPSK beamforming (22) premix two BPSK streams into a QPSK stream. Minimum distance precoding applies to ML detection using Euclidean distance criterion. Channel diagonalization is required for ML–MD precoding, and the premixed QPSK stream is transmitted over the dominant channel eigenmode. On the other hand, the proposed QPSK beamforming applies to ZF receivers and is derived based on ill-conditioned channel model (9) under an MBER criterion.

B. Performance Analysis

1) Performance of Beamforming in Ill-Conditioned Channels: Substituting (19) into (16) and (22) into (20), we obtain the approximate average BER as, respectively

$$\bar{P}_{4-\mathrm{PAM}}^{\mathrm{BF}}(\gamma_s; \mathbf{h}_1, a) \cong \frac{3}{20}\exp\left\{-\frac{2}{5}\gamma_s\|\mathbf{h}_1\|^2\left(1+|a|^2\right)\right\} \stackrel{\mathrm{def}}{=} \tilde{P}_{4-\mathrm{PAM}}^{\mathrm{BF}}(\gamma_s; \mathbf{h}_1, a) \tag{23}$$

$$\bar{P}_{\mathrm{QPSK}}^{\mathrm{BF}}(\gamma_s; \mathbf{h}_1, a) \cong \frac{1}{5}\exp\left\{-\gamma_s\|\mathbf{h}_1\|^2\left(1+|a|^2\right)\right\} \stackrel{\mathrm{def}}{=} \tilde{P}_{\mathrm{QPSK}}^{\mathrm{BF}}(\gamma_s; \mathbf{h}_1, a). \tag{24}$$

Therefore, both 4-PAM and QPSK beamforming do not experience error floors in ill-conditioned channels. Comparing (24) with (23), we observe an approximate SNR gain of 2.5 (≅4 dB) at the expense of increased detection complexity. This will be verified by simulations in Section V.

2) Performance of Beamforming in Well-Conditioned Channels: It is of interest to study performances of beamforming in well-conditioned channels. The performances of 4-PAM and QPSK beamforming can be obtained by substituting $\hat{a}_{\mathrm{LS}}$ in (10) into (23) and (24), respectively

$$\tilde{P}_{4-\mathrm{PAM}}^{\mathrm{BF}}(\gamma_s; \mathbf{h}_1, \mathbf{h}_2) = \frac{3}{20}\exp\left\{-\gamma_s \cdot \frac{2}{5} \cdot \frac{\|\mathbf{h}_1\|^4 + |\mathbf{h}_1^{\mathrm{H}}\mathbf{h}_2|^2}{\|\mathbf{h}_1\|^2}\right\} \tag{25}$$

$$\tilde{P}_{\mathrm{QPSK}}^{\mathrm{BF}}(\gamma_s; \mathbf{h}_1, \mathbf{h}_2) = \frac{1}{5}\exp\left\{-\gamma_s \cdot \frac{\|\mathbf{h}_1\|^4 + |\mathbf{h}_1^{\mathrm{H}}\mathbf{h}_2|^2}{\|\mathbf{h}_1\|^2}\right\}. \tag{26}$$

From (25) and (26), we observe that 1-D detection has a 4-dB power penalty relative to 2-D detection in well-conditioned channels as well.

V. Numerical Results and Discussions

We compare BER performance of the proposed MBER transmit power-allocation and beamforming methods with existing precoding methods in well- and ill-conditioned channels, respectively. In our simulations, we adopt the spatial fading correlation model for general nonisotropic scattering given in [8]. The following parameters are chosen: $N_t = 2$ transmit and $N_r = 4$ receive antennas; transmit and receive antenna spacings expressed in wavelength are 0.5 and 10, respectively; angles of arrival and departure of the deterministic components are 30° and 0°, respectively; angle spread 10°; $K = 8$ dB for Ricean fading channels; and BPSK modulation is used for the purposes of comparison with [6].

Fig. 1 is a plot of the average BER for a variety of transceivers in an uncorrelated Rayleigh fading channel. To clarify the plot, performances of ZF with power allocation and SIC without power allocation are not shown, since they are nearly identical to that of MMSE precoding/decoding; OSIC without power allocation (also not shown) has performance close to that of ZF-MBER precoding. We observe that at a BER of 10^{-3}, the proposed power-allocation scheme offers 0.5, 1.2, and 0.6 dB gains over ZF, SIC, and OSIC receivers, respectively. Both SIC and OSIC with power allocation outperform precoding schemes, e.g., at a BER of 10^{-3}, OSIC with power allocation offers 0.9- and 1.9-dB SNR gains over ZF-MBER and MMSE precoding/decoding, respectively. In Fig. 1, it is also observed that QPSK beamforming offers superior performance to all other simulated schemes except ML–MD precoding, e.g., at a BER of 10^{-3}, its SNR gain over OSIC with power

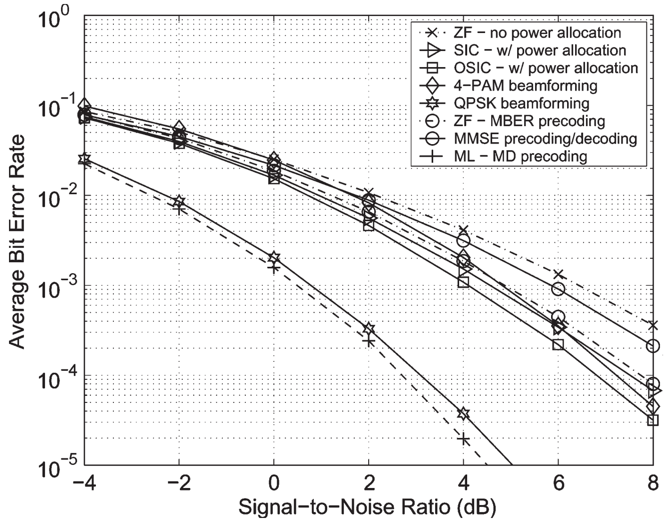

Fig. 1. Average BER performance in uncorrelated Rayleigh fading TIMO channel ($N_t = 2$, $N_r = 4$).

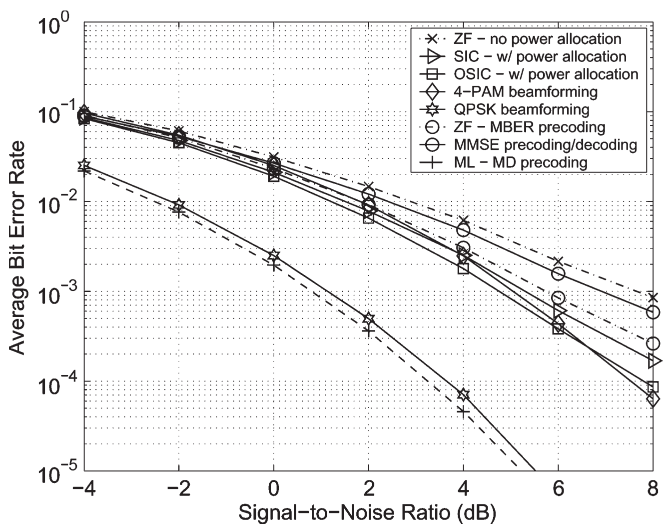

Fig. 2. Average BER performance in correlated Rayleigh fading TIMO channel ($N_t = 2$, $N_r = 4$).

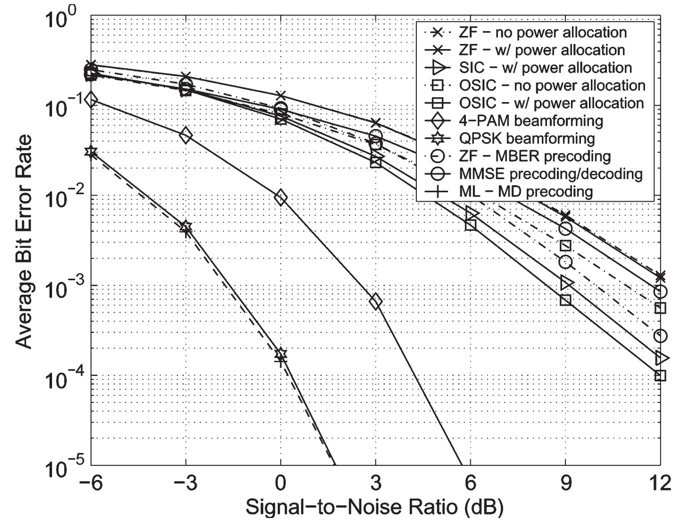

Fig. 3. Average BER performance in uncorrelated Ricean fading TIMO channel ($N_t = 2$, $N_r = 4$, $K = 8$ dB).

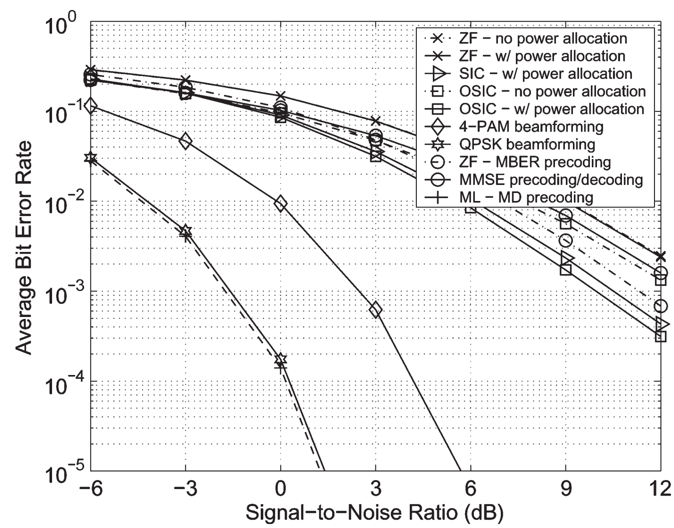

Fig. 4. Average BER performance in correlated Ricean fading TIMO channel ($N_t = 2$, $N_r = 4$, $K = 8$ dB).

allocation is 3.3 dB. Fig. 2 illustrates average BER performance in correlated Rayleigh fading. Similar relationships among the simulated schemes as in uncorrelated Rayleigh fading channels are observed.

Figs. 3 and 4 illustrate average BERs in uncorrelated and correlated Ricean fading channels, respectively. Performance of SIC without power allocation (not shown) is nearly identical to that of MMSE precoding/decoding. Again, SIC and OSIC with power allocation outperform ZF-MBER and MMSE precoding/decoding. We also observe that the proposed 4-PAM and QPSK beamforming offer significant gain over the power-allocation schemes shown: At a BER of 10^{-3}, 6.3- and 10.2-dB SNR gain over OSIC with power allocation in uncorrelated fading, and 7.5 dB and 11.5 dB correlated fading are observed. This is as excepted since in Ricean fading, due to the existence of a line-of-sight component, the channel matrix is likely to be ill-conditioned.

1) General Observations: From Figs. 1–4, we observe a 4-dB power penalty for 4-PAM beamforming relative to QPSK beamforming. In all simulated channels, SNR losses of QPSK beamforming relative to ML–MD precoding are less than 0.5 dB.

VI. Conclusion

MBER transmit power allocation and beamforming for TIMO spatial multiplexing are proposed in this paper. It is shown that SIC and OSIC with AMBER power allocation outperform linear ZF-MBER and MMSE precoding/decoding schemes in Rayleigh fading channels, e.g., at a BER of 10^{-3}, OSIC with power allocation offers 0.9- and 1.9-dB SNR gains over ZF-MBER and MMSE precoding/decoding, respectively. The proposed 4-PAM and QPSK transmit beamforming eliminates error floors and offers superior performance over both power allocation and ZF-MBER and MMSE precoding/decoding in Ricean fading channels, e.g., at a BER of 10^{-3}, 4-PAM beamforming offers 6.3- and 7.5-dB SNR gains over OSIC with power allocation in uncorrelated and correlated Ricean fading, respectively, and QPSK beamforming offers 10.2- and 11.5-dB gains, respectively. Compared to more general precoding methods, the proposed transmit optimization schemes provide a simple and efficient way to exploit partial channel-state information at the transmitter.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewer for pointing out reference [16] and their helpful comments that enabled us to significantly improve the correspondence.

REFERENCES

[1] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bölcskei, "An overview of MIMO communications—A key to gigabit wireless," *Proc. IEEE*, vol. 92, no. 2, pp. 198–218, Feb. 2004.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[3] D. A. Gore, R. W. Heath, Jr., and A. J. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Commun. Lett.*, vol. 6, no. 11, pp. 491–493, Nov. 2002.

[4] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 11, pp. 1841–1852, Nov. 1999.

[5] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and equalizers," *IEEE Trans. Signal Process.*, vol. 50, no. 5, pp. 1051–1064, May 2002.

[6] Y. Ding, T. N. Davidson, Z.-Q. Luo, and K. M. Wong, "Minimum BER block precoders for zero-forcing equalization," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2410–2423, Sep. 2003.

[7] J. Akhtar and D. Gesbert, "A closed-form precoder for spatial multiplexing over correlated MIMO channels," in *Proc. IEEE Globecom*, San Francisco, CA, Dec. 2003, vol. 4, pp. 1847–1851.

[8] A. Abdi and M. Kaveh, "A space-time correlation model for multielement antenna systems in mobile fading channels," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 3, pp. 550–560, Apr. 2002.

[9] J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.

[10] D. Gesbert, H. Bölcskei, D. Gore, and A. J. Paulraj, "Outdoor MIMO wireless channels: Models and performance prediction," *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 1926–1934, Dec. 2002.

[11] N. Wang and S. D. Blostein, "Power loading for CP-OFDM over frequency-selective fading channels," in *Proc. IEEE Globecom*, San Francisco, CA, Dec. 2003, vol. 4, pp. 2305–2309.

[12] S. Zhou and G. B. Giannakis, "Adaptive modulation for multi-antenna transmissions with channel mean feedback," in *Proc. IEEE ICC*, Anchorage, AK, May 2003, vol. 4, pp. 2281–2285.

[13] S. H. Nam, O.-S. Shin, and K. B. Lee, "Transmit power allocation for a modified V-BLAST system," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1074–1079, Jul. 2004.

[14] N. Wang and S. D. Blostein, "Approximate minimum BER power allocation for MIMO spatial multiplexing systems," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 180–187, Jan. 2007.

[15] ——, "Minimum BER power allocation for MIMO spatial multiplexing systems," in *Proc. IEEE ICC*, Seoul, Korea, May 2005, vol. 4, pp. 2282–2286.

[16] L. Collin, O. Berder, P. Rostaing, and G. Burel, "Optimal minimum distance-based precoder for MIMO spatial multiplexing," *IEEE Trans. Signal Process.*, vol. 52, no. 3, pp. 617–627, Mar. 2004.

Neng Wang received the B.Eng. degree from Shanghai Jiao Tong University, Shanghai, China, in 1994, the M.Eng. degree from Nanjing University of Posts and Telecommunications (NUPT), Nanjing, China, in 1999, and the Ph.D. degree from Queen's University, Kingston, ON, Canada, in 2005, all in electrical engineering.

He was an Electronics Engineer at Suzhou Lambeau High-Tech Enterprise Group, Suzhou, China, from 1994 to 1996. He was a Research and Teaching Assistant at the Department of Information Engineering, NUPT, from 1996 to 1999, and at the Department of Electrical and Computer Engineering, Queen's University, from 2000 to 2005. He was a Research Scientist at the Communications Research Centre, Ottawa, ON, in 2006. Since 2007, he has been with Nortel Networks, Richardson, TX, as a Wireless Access Standards Engineer. His research interests include general areas of wireless communications and signal processing.

Steven D. Blostein (S'83–M'88–SM'96) received the B.S. degree in electrical engineering from Cornell University, Ithaca, NY, in 1983 and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Illinois, Urbana-Champaign, in 1985 and 1988, respectively.

He has been with the Faculty at Queen's University, Kingston, ON, Canada, since 1988 and is currently a Professor and the Head of the Department of Electrical and Computer Engineering. From 1994 to 1995, he was with Lockheed Martin Electronic Systems, Montreal, QC, Canada. In 1995, he was a Visiting Associate Professor at the Department of Electrical Engineering, McGill University, Montreal. From 1999 to 2003, he was the Leader of the Multirate Wireless Data Access Major Project sponsored by the Canadian Institute for Telecommunications Research. He has also been a Consultant to the industry and government in the areas of image compression, target tracking, and wireless communications. His current research interests include the application of signal processing to wireless communications systems, including smart antennas, MIMO systems, and space-time frequency processing for MIMO-OFDM systems.

Dr. Blostein has been a member of the World 4G Wireless Forum as well as an Invited Distinguished Speaker at Ryerson University, Toronto, ON, and at the Samsung Advanced Institute of Technology. He served as Chair of the IEEE Kingston Section from 1993 to 1994, Chair of the Biennial Symposium on Communications in 2000 and 2006, Associate Editor for the IEEE TRANSACTIONS ON IMAGE PROCESSING from 1996 to 2000, and Publications Chair for the IEEE ICASSP in 2004. He is currently an Associate Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He is a registered Professional Engineer in the Province of Ontario.