Soft-Decision Multistage Multiuser Interference Cancellation

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***Abstract*—Successive interference cancellation (SIC) refers to a family of low-complexity multiuser detection methods for direct-sequence code-division multiple-access systems. The performance of multistage SIC depends on the decision function used in the interference cancellation iterations, e.g., hard, soft, or linear decision functions. Due to error propagation, multistage SIC with hard data bit decisions may perform more poorly than multistage SIC with linear or soft decision functions. We propose and analyze a family of generalized unit-clipper bit decision functions that better combine linear and hard decisions. Performance within 0.4 dB of the single-user bound can be obtained. We then make robust the above soft-decision SIC to time-delay errors as large as half a PN chip and evaluate its performance.**

***Index Terms*—Code-division multiple access, iterative methods, multiuser channels, successive interference suppression.**

I. Introduction

THE capacity of a code-division multiple-access (CDMA) system is limited by multiple-access interference (MAI) from other users. CDMA multiuser detection at the base station, which utilizes known user spreading codes, is an effective method to suppress MAI and improve receiver performance. Optimal multiuser detection has exponential computational complexity and is therefore impractical [1]. Several low-complexity multiuser detectors including decorrelation [4], minimum mean squared error, successive interference cancellation (SIC) [3], and parallel interference cancellation (PIC) have been proposed [2].

The SIC regenerates and cancels other users' signal before data decision of the desired user. The decision function used in the SIC may be hard, soft, or linear. If the regeneration and cancellation of other users' signals use a hard decision function, the interference could actually double from error propagation of incorrect hard decisions [9]. Methods including soft or linear interference cancellation and partial interference cancellation were proposed to mitigate this error propagation [5]. However, the linear SIC reduces to the decorrelating detector, which is inferior to the upper bound performance that SIC can achieve with an ideal decision function [7]. The performance of partial interference cancellation methods depends on the cancellation weights at each stage and the decision functions used. The selection of the optimum weights for the multiple stages can therefore be complex [8].

The SIC with hard or soft decision functions requires signal amplitude to perform interference cancellation. When the channel changes slowly, it is shown in [3] that an SIC receiver incorporating amplitude estimation by averaging over several bits can potentially result in a significant bit error rate (BER) performance improvement. In fact, the single-user BER lower bound may be reached if perfect amplitude information is available. Although amplitude averaging is a known technique, its performance depends on the decision function used in multistage SIC. For example, if hard decisions are used, error propagation may dominate over amplitude estimation errors.

While linear (soft) decision interference cancellation has no error propagation and will converge to the decorrelating detector, hard decision interference cancellation can completely cancel interference when the hard decisions are correct. We seek to combine the advantages of hard and soft decision functions. In our proposed decision function, when the instantaneous signal amplitude estimation is small compared to the averaged amplitude, linear decision cancellation is used. Otherwise, hard decision cancellation is employed. We therefore take advantage of amplitude averaging and achieve performance close to that of the single-user bound.

Our proposed detector is similar in principle to the two-stage decorrelating detector of [11], where hard decisions made from the first stage decorrelator are used only when highly reliable. While [11] uses either multidimensional search or decorrelation in the second stage, we propose to incorporate the two stages into the SIC iterations to gain a computational advantage, i.e., the two-stage decorrelator [11] has computational complexity proportional to the third power of the number of users [4] while the proposed multistage SIC has computational complexity linear in the number of users [7]. Moreover, the two-stage decorrelator performance is affected by time-delay estimation errors [18], while the soft-decision multistage SIC can be made robust to time-delay errors as described in Section V.

We consider the proposed decision function in the context of multistage SIC with amplitude averaging. We note that this technique may also be applied to PIC, but will not discuss this further. In the following sections, we describe the system model, propose a new decision function to be used in the multistage SIC receiver, and analyze its steady-state performance. To operate in practical nonperfect synchronization situations, the soft-decision multistage SIC is made robust for time-delay estimation errors. Finally, we provide comparisons through bit simulations.

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II. System Model

We consider the base-station receiver for the asynchronous uplink CDMA channel with binary phase-shift keying (BPSK) modulation.

It is assumed that the user data are transmitted in blocks, with a block length M. The equivalent baseband received signal for one block is

$$r(t)=\sum_{i=1}^{M}\sum_{k=1}^{K}a_k(i)e^{j\theta_k(i)}b_k(i)\tilde{s}_k(t-iT-\tau_k)+n(t) \quad (1)$$

where $a_k(i)\in\mathcal{R}$, $\theta_k(i)\in[0,2\pi)$, and $b_k(i)\in\{+1,-1\}$ are the kth user's received signal amplitude, phase shift, and data bit for the ith time interval, $\tau_k\in[0,T)$ is the kth user's propagation delay, T is the bit duration, K is the total number of users, and $n(t)$ is the white Gaussian noise. The time delays, phase shifts, and spreading codes of all users are assumed to be known at the receiver.

In (1), the normalized signature waveform of user k, $\tilde{s}_k(t)$, is

$$\tilde{s}_k(t)=\sum_{j=0}^{N-1}c_k(j)h(t-jT_c) \quad (2)$$

where $N=T/T_c$ is the spreading factor, T_c is the chip duration, $\{c_k(j)\}_{j=0}^{N-1}$ is user k's spreading code, and $h(t)$ is a rectangular chip pulse with duration [0, T_c).

Assuming that the channel changes relatively slowly compared to observation length $(M+1)T$, the received signal amplitude and phase shift parameters can be modeled as constants, i.e., $a_k(i)=a_k$ and $\theta_k(i)=\theta_k$ for $i=1,\ldots,M$. Due to asynchronism $\tau_k\in[0,T)$, we note that the observation interval must be [0, $(M+1)T$).

After chip-matched filtering and chip-rate sampling, the received signal is discretized and the $(M+1)T$ observations can be organized into the vector

$$\mathbf{r}=\sum_{i=1}^{M}\sum_{k=1}^{K}a_k e^{j\theta_k}b_k(i)\mathbf{d}_k(i)+\mathbf{n} \quad (3)$$

where $\mathbf{d}_k(i)$ is the discretized signature waveform of user k for the ith bit. The received vector $\mathbf{r}$ is the concatenation of $M+1$ vectors each of length N, i.e.,

$$\mathbf{r}=[\mathbf{r}^T(1)\ \mathbf{r}^T(2)\ \ldots\ \mathbf{r}^T(M+1)]^T\in\mathcal{C}^{(M+1)N} \quad (4)$$

where the mth vector $\mathbf{r}(m)$ in (4) corresponds to the mth observation interval $[mT,(m+1)T)$

$$\mathbf{r}(m)=[r(mN+1)\ \ldots\ r(mN+N)]^T\in\mathcal{C}^N. \quad (5)$$

Similarly, we may organize the zero-mean white Gaussian noise vector as

$$\mathbf{n}=[\mathbf{n}^T(1)\ \mathbf{n}^T(2)\ \ldots\ \mathbf{n}^T(M+1)]^T\in\mathcal{C}^{(M+1)N}. \quad (6)$$

The time delay of the kth user is decomposed into an integer p_k and fractional part δ_k, as $\tau_k=(p_k+\delta_k)T_c$, where $p_k\in\{0,1,\ldots,N-1\}$ and $\delta_k\in[0,1)$. The received discretized signature waveform of the ith bit of the kth user $\mathbf{d}_k(i)\in\mathcal{R}^{(M+1)N}$ can be expressed as a combination of two adjacent shifted versions of user spreading codes [15]

$$\mathbf{d}_k(i)=\delta_k\mathbf{c}_k(p_k+1,i)+(1-\delta_k)\mathbf{c}_k(p_k,i). \quad (7)$$

In (7), $\mathbf{c}_k(p_k,i)$ is defined as $\mathbf{c}_k$ right-shifted by $(i-1)N+p_k$ chips, where $\mathbf{c}_k\in\mathcal{R}^{(M+1)N}$ is the kth user's spreading code vector for the $(M+1)T$-length interval defined as

$$\mathbf{c}_k=[c_k(0)\ c_k(1)\ \ldots\ c_k(N-1)\ \underbrace{0\ 0\ \ldots\ 0}_{MN}]^T. \quad (8)$$

The received signal vectors $\mathbf{r}(i)$ over the $(M+1)T$ observation intervals, $i=1,\ldots,M+1$, provide sufficient statistics for detecting the transmitted data bits from the K users.

III. SIC Multiuser Detector With Soft Decision

SIC is a low-complexity suboptimal multiuser detector for CDMA systems. The signal corresponding to a particular user is first estimated by subtracting other users' regenerated signals from the original received signal. After data bit decisions are successively made based on these estimated signals, the estimated signals are regenerated and then the process repeats. To obtain accurate interference cancellation performance, the regenerated signal subtractions occur in decreasing order of signal power. We note that 1) this ordering can be approximated by only sorting in the first SIC stage and 2) ordering with $O(K\log_2 K)$ complexity/stage does not substantially increase the $O(KN)$/stage computational complexity of the SIC.

The SIC needs users' amplitude information for data bit decisions and interference cancellation. Since the received signal amplitude is not known, it should be estimated. One approach is the *linear SIC receiver*, in which the ith signal's amplitude and data bit are estimated as the composite signal $\hat{b}_k(i)\hat{a}_k(i)$[3], [7]. This is equivalent to estimating amplitude in bit-by-bit fashion. The MAI and noise will affect the accuracy of the amplitude estimate, where the error may be modeled as zero-mean Gaussian noise. In [3], it was shown in theory that amplitude estimation by averaging over M bits can reduce the noise variance by a factor of M and results in a corresponding BER performance improvement. The single-user BER lower bound may also be approached for static channels if the number of bits used for averaging is large enough.

However, with averaged amplitudes, the multistage SIC receiver performance depends on the decision functions used in the interference cancellation iterations, as explained earlier. In the following, we will discuss some of the known decision functions and propose an improved decision function.

Suppose a multistage SIC receiver with amplitude averaging starts interference cancellation at stage $v=1$. During the $(v+1)$st stage, the SIC first performs Steps 1)–3) on user $k=1$, then repeats the same steps on users $k=2$ until user $k=K$.

Step 1) We estimate user k's received signal for bits $i=1,\ldots,M$ in one block. For the ith bit, the kth user's received signal is estimated by subtracting other users' regenerated signals from the received signal $\mathbf{r}(i)$ of (3)

$$\begin{aligned}\tilde{\mathbf{r}}_k^{v+1}=&\mathbf{r}-\sum_{l=1}^{k-1}\sum_{i=1}^{M}e^{j\theta_l}\bar{a}_l^{v+1}\hat{b}_l^{v+1}(i)\mathbf{d}_l(i)\\&-\sum_{l=k+1}^{K}\sum_{i=1}^{M}e^{j\theta_l}\bar{a}_l^{v}\hat{b}_l^{v}(i)\mathbf{d}_l(i).\end{aligned}$$

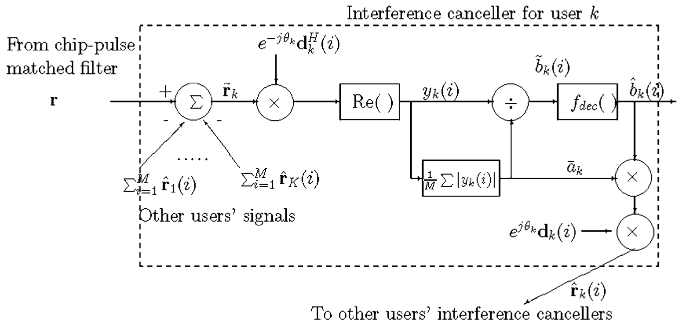

Fig. 1. The interference cancellation unit for user k.

Step 2) Obtain the averaged amplitude estimate by averaging the instantaneous estimate of user k's amplitudes over the M-bit block after despreading with PN sequence $\mathbf{d}_k(i)$

$$\bar{a}_k^{v+1} = \frac{1}{M}\sum_{i=1}^{M} \text{abs}\ \left(\text{Re}\ \left(e^{-j\theta_k}(\mathbf{d}_k(i))^H\ \tilde{\mathbf{r}}_k^{v+1}\right)\right)$$

where abs() and Re() denote the absolute value and the real part, respectively.

Step 3) For each bit in the block, $i = 1, \ldots, M$, obtain the normalized soft data bit estimate and make a data bit decision. For the ith bit, the soft data bit estimate is normalized with respect to the averaged amplitude $\bar{a}_k^{v+1}$

$$\tilde{b}_k^{v+1}(i) = \frac{\text{Re}\ \left(e^{-j\theta_k}(\mathbf{d}_k(i))^H\ \tilde{\mathbf{r}}_k^{v+1}\right)}{\bar{a}_k^{v+1}}.$$

The data bit decision is made by the decision function $f_{\text{dec}}(\cdot)$

$$\hat{b}_k^{v+1}(i) = f_{\text{dec}}(\tilde{b}_k^{v+1}(i)).$$

The interference canceller for user k is depicted in Fig. 1. The above multistage SIC either is performed for a desired number of cancellation stages or is terminated when there is no significant change from the previous stage. Note that if perfect amplitude information were available, Step 2) may be omitted.

Several possible decision functions $f_{\text{dec}}(\cdot)$ are depicted in Fig. 2. The hard-limiter decision function [6] of Fig. 2(a) utilizes only the sign of the soft data bit estimate $\hat{b}_k^{j+1}(i) = \text{sign}(\tilde{b}_k^{j+1}(i))$. Assume, for example, that the correct data bit is $+1$. If its soft estimate is a small negative number close to zero due to MAI and noise, i.e., -0.1, the hard decision will be -1. From this example, we can observe that interference may actually be amplified by the hard-limiter. This may cause error propagation, which could result in the SIC's converging to a local maximum. Partial interference cancellation [5] has been proposed to mitigate this error propagation, but its parameters can be difficult to optimize.

The hyperbolic tangent $(\tanh)$ [6] decision function of Fig. 2(c) has been shown to be optimum in the single-user case when the interference and noise are Gaussian, which may not accurately model the MAI of CDMA systems. In any case, hyperbolic tangent performance is only slightly better than that of the hard-limiter [6].

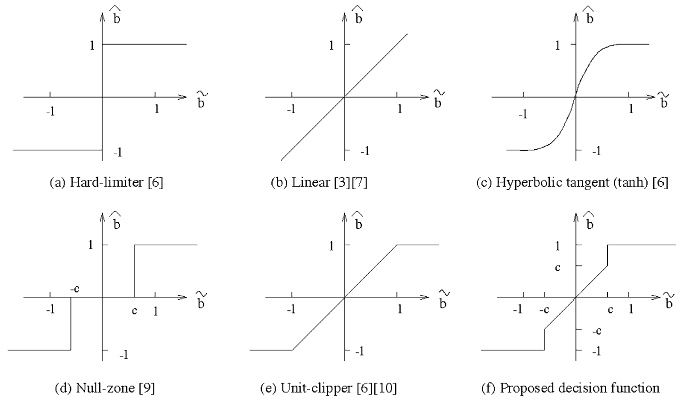

Fig. 2. The decision functions for SIC multiuser detectors.

The null-zone decision function [9] of Fig. 2(d) improves the hard-limiter by using sign information only when the soft bit estimate has a large enough amplitude.

The linear decision function [3], [7] of Fig. 2(b) does not make hard bit decisions. This *linear SIC* converges to the decorrelating detector as the number of interference cancellation stages goes to infinity [7]. Linear SIC performance is therefore limited by decorrelating detector noise enhancement [4].

The limiter in the unit-clipper decision function [6], [10] of Fig. 2(e) improves performance over the linear SIC. However, the unit-clipper cancels only the part of the noise above the amplitude limit. It has been shown in [12] that a multistage interference cancellation receiver with a unit-clipper function is equivalent to the (0,1)-constrained maximum-likelihood (ML) solution of the optimum multiuser detection, subject to a box constraint.

To improve the tradeoff between linear SIC noise enhancement and error propagation from hard limiting, we propose to generalize the unit-clipper to the following decision function depicted in Fig. 2(f):

$$\hat{b} = f_{dec}(\tilde{b}) = \begin{cases} 1, & \tilde{b} > c \\ \tilde{b}, & \tilde{b} \in [-c, c] \\ -1, & \tilde{b} < -c \end{cases} \tag{9}$$

where the threshold $0 \leq c \leq 1$. The effect of the choice of c on the performance of the SIC using the above proposed decision function will be analyzed in Section V-C and simulated in Section VI.

The decision function (9) makes a linear (soft) bit decision when the value of the normalized soft bit estimate is small, and so will exhibit desirable convergence similar to that of the linear SIC. Otherwise, it makes a hard bit decision, which will be correct with high probability.

The performance of the proposed SIC in (9) can also be compared to an SIC using a Gibbs sampler [13]. The Gibbs sampler introduces randomness into the SIC cancellation, where the hard data bit decision is made by choosing a sample from a conditional probability density function (pdf) of the soft data bit estimate. For example, if the soft bit estimate is $\tilde{b} = 0.5$, the Gibbs sampler draws a sample that will be $+1$ with probability 88%. With perfect power control and perfect amplitude information, the SIC using a Gibbs sampler achieves BER performance within 0.5 dB of the single-user bound [13]. While our SIC uses deterministic soft decisions, it may reach a fixed point faster than [13], although [13] may convergence to a lower steady-state error. Under a 10-dB near–far ratio and with imperfect amplitude information, the soft-decision SIC achieves a BER performance within 0.4 dB of the single-user bound, as will be described in Section VI. While the number of iterations may not be identical, the Gibbs sampler has the same order of computation as that of the proposed SIC.

IV. A Steady-State Performance Analysis

In this section, we analyze the steady-state performance of the proposed SIC detector after convergence. It has been shown by simulation [9], [14] that convergence is approximately achieved after about five iterations for multistage SIC with null-zone and hard-limiter decision functions. The multistage SIC with proposed soft-decision function also converges in about five iterations, as will be shown by the simulation results in Fig. 7 of Section VI.

After convergence, the residual interference can be assumed to be Gaussian-distributed, and the interference introduced by individual users can be assumed to be mutually independent [14]. Let the interference variance from one bit of user k be σ_k^2. The total interference and noise variance σ^2 is the sum of the K users' interference variances and the channel noise variance σ_N^2, i.e., $\sigma^2 = \sum_{k=1}^K \sigma_k^2 + \sigma_N^2$.

For the multistage linear SIC detector, denote the interference and noise variance of the estimated received signal of user k at the input of the correlator as σ^2 at convergence. After correlation, the variance of the reconstructed signal $e^{j\theta_k}\bar{a}_k\hat{b}_k(i)\mathbf{d}_k(i)$ will be $\sigma_k^2 = \sigma^2/N$ due to spreading gain N. Therefore, it can be shown [14] that σ^2 is the solution to

$$\sigma^2 = K\frac{\sigma^2}{N} + \sigma_N^2. \tag{10}$$

That is, $\sigma^2 = 1/(1-(K/N))\sigma_N^2$. For a spreading factor $N = 31$ and $K = 20$ users, the performance loss of the linear decision SIC detector relative to the single-user lower bound is 4.5 dB.

For the proposed decision function Fig. 2(f), let user k's amplitude be a_k. Without loss of generality, let user k's ith transmitted data bit be $b_k(i) = +1$. Its unnormalized correlator output $y_k(i) = \text{Re}\ (e^{-j\theta_k}(\mathbf{d}_k(i))^H\ \hat{\mathbf{r}}_k(i)) = \bar{a}_k\tilde{b}_k(i)$ can be modeled as a Gaussian random variable with mean a_k and variance σ^2. User k's decision region for the unnormalized correlator output $y_k(i)$ can be partitioned into 1) a hard-decision region (ca_k, ∞), 2) a linear decorrelator region $[-ca_k, ca_k]$, and 3) a bit-error region $(-\infty, -ca_k)$. The reconstructed signal of user k for interference cancellation is $e^{j\theta_k}\bar{a}_k\hat{b}_k(i)\mathbf{d}_k(i)$. This leads to three cases.

Case 1) The unnormalized correlator output $y_k(i)$ falls in hard-decision region (ca_k, ∞) with probability $[1 - Q((1-c)a_k/\sigma)]$, where $\mathcal{Q}(x) = \int_x^\infty (1/\sqrt{2\pi})e^{-(y^2/2)}dy$. The data bit decision is correct, i.e, $\hat{b}_k(i) = b_k(i)$. Its regenerated signal for interference cancellation is $e^{j\theta_k}\bar{a}_k b_k(i)\mathbf{d}_k(i)$, which uses the averaged amplitude for all $i = 1, 2, \ldots, M$. The introduced interference variance can be calculated as the second moment of the difference between the reconstructed signal and the true signal $e^{j\theta_k}a_k b_k(i)\mathbf{d}_k(i)$, i.e.,

$$\text{Var}_1 = \frac{1}{N}E\left[(\bar{a}_k b_k(i) - a_k b_k(i))^2\right] = \frac{\sigma^2}{MN} \tag{11}$$

where N is due to spreading gain and M is due to averaging gain.

Case 2) The unnormalized correlator output $y_k(i)$ falls in the linear decorrelator region $[-ca_k, ca_k]$ with probability $[Q((1-c)a_k/\sigma) - Q((1+c)a_k/\sigma)]$. Its regenerated signal $e^{j\theta_k}y_k(i)\mathbf{d}_k(i)$ uses the instantaneous amplitude estimate $\text{abs}(y_k(i))$, which has a variance $\text{Var}_2 = \sigma^2/N$ due to spreading gain only.

Case 3) The unnormalized correlator output $y_k(i)$ falls in bit-error region $(-\infty, -ca_k)$ with probability $Q((1+c)a_k/\sigma)$. Since a wrong hard bit decision is made, i.e., $\hat{b}_k(i) = -b_k(i)$, its regenerated signal for interference cancellation is $e^{j\theta_k}\bar{a}_k(-b_k(i))\mathbf{d}_k(i)$. Assuming that the data bit error and the amplitude estimation error are independent, the introduced interference variance can be calculated as

$$\begin{aligned}\text{Var}_3 =& \frac{1}{N}E\left[(\bar{a}_k(-b_k(i)) - a_k b_k(i))^2\right]\\ =& \frac{1}{N}\left\{E\left[(2a_k b_k(i))^2\right]\right.\\ &\left.+E\left[(\bar{a}_k b_k(i) - a_k b_k(i))^2\right]\right\}\\ =& \frac{(2a_k)^2}{N} + \frac{\sigma^2}{MN} \approx \frac{(2a_k)^2}{N}.\end{aligned} \tag{12}$$

Combining the above cases, the average interference variance contribution from one bit of user k conditioned on its amplitude a_k is

$$\begin{aligned}\sigma_k^2(a_k) =& \left[1 - Q\left(\frac{(1-c)a_k}{\sigma}\right)\right]\frac{\sigma^2}{MN}\\ &+\left[Q\left(\frac{(1-c)a_k}{\sigma}\right) - Q\left(\frac{(1+c)a_k}{\sigma}\right)\right]\frac{\sigma^2}{N}\\ &+Q\left(\frac{(1+c)a_k}{\sigma}\right)\frac{(2a_k)^2}{N}.\end{aligned} \tag{13}$$

If the received user signals have unequal powers, we may assume that the received amplitudes a_k are uniformly distributed between $a_{\min}$ and $a_{\min}X$, where $a_{\min} = \min\{a_1, \ldots, a_K\}$ is the amplitude of the weakest user and $X > 1$ is the ratio of $\max\{a_1, \ldots, a_K\}/a_{\min}$. The average interference variance contribution from user k can be calculated by averaging (13) over the distribution of a_k, which is assumed uniform in $[a_{\min}, a_{\min}X]$.

Denote the expectation

$$\begin{aligned}f(b) \equiv& E_{a_k}\left[Q\left(\frac{ba_k}{\sigma}\right)\right] = \frac{1}{a_{\min}(X-1)}\\ &\left[a_{\min}XQ\left(\frac{ba_{\min}X}{\sigma}\right)\right.\\ &\left.-a_{\min}Q\left(\frac{ba_{\min}}{\sigma}\right)\right]\\ &+\frac{\sigma}{\sqrt{2\pi}b}\left[e^{-(b^2a_{\min}^2/2\sigma^2)} - e^{-(b^2(a_{\min}X)^2/2\sigma^2)}\right]\end{aligned} \tag{14}$$

and by using the approximation $Q(t) \approx (1/\sqrt{2\pi}t)e^{-t^2}$

$$\begin{aligned} g(b) \equiv& E_{a_k}\left[a_k^2 Q\left(\frac{ba_k}{\sigma}\right)\right] \\ =& \frac{1}{a_{\min}(X-1)}\int_{a_{\min}}^{a_{\min}X} a_k^2 Q\left(\frac{ba_k}{\sigma}\right) da_k \\ \approx& \frac{\sigma^3}{\sqrt{2\pi}b^3}\frac{1}{a_{\min}(X-1)} \\ &\times\left(e^{-(b^2a_{\min}^2/2\sigma^2)} - e^{-(b^2(a_{\min}X)^2/2\sigma^2)}\right). \end{aligned} \quad (15)$$

Substituting (14) and (15) into (13), the total interference σ^2 for all K users including the channel noise variance σ_N^2 is the solution to

$$\begin{aligned} \sigma^2 =& \sum_{k=1}^{K} E_{a_k}[\sigma_k^2(a_k)] + \sigma_N^2 \\ \approx& \left\{(1-f(1-c))\frac{\sigma^2}{MN} + (f(1-c)-f(1+c))\frac{\sigma^2}{N}\right. \\ &\left.+\frac{4}{N}g(1+c)\right\}K + \sigma_N^2. \end{aligned} \quad (16)$$

For example, for an amplitude averaging length of $M = 9$ bits, signal-to-noise ratio (SNR) of 10 dB, near–far ratio of 10 dB, spreading factor of $N = 31$, and number of users $K = 20$, the loss to the single-user bound is about 0.35 dB for threshold $c = 0.5$, 0.68 dB for $c = 0.8$, and 1.93 dB for $c = 1.0$. The value $c = 1.0$ is a special case where our proposed decision function reduces to the unit-clipper decision function.

Alternatively, if the received user powers are all equal under ideal power control, i.e., $a_k = a$ for $k = 1, \ldots, K$, then (13) need not be averaged. Instead of (16), the total interference and noise variance is given as

$$\begin{aligned} \sigma^2 =& \sum_{k=1}^{K} \sigma_k^2(a_k) + \sigma_N^2 \\ =& \left\{\left[1 - Q\left(\frac{(1-c)a}{\sigma}\right)\right]\frac{\sigma^2}{MN}\right. \\ &+\left[Q\left(\frac{(1-c)a}{\sigma}\right) - Q\left(\frac{(1+c)a}{\sigma}\right)\right]\frac{\sigma^2}{N} \\ &\left.+Q\left(\frac{(1+c)a}{\sigma}\right)\frac{(2a)^2}{N}\right\}K + \sigma_N^2. \end{aligned} \quad (17)$$

Modifying the above example to a near–far ratio of 0 dB corresponding to equal user powers, the loss to the single-user bound is about 0.51 dB for $c = 0.5$, 1.18 dB for $c = 0.8$, and 1.93 dB for $c = 1.0$. Compared to the previous example, the proposed SIC detector performs more poorly under equal received power conditions.

It is also interesting to calculate the performance loss to the single-user bound when the decision function used is ideal, i.e., decision is error free, with the amplitudes averaged. Similar to the decorrelator, after correlation, the variance of the reconstructed signal $e^{j\theta_k}\bar{a}_k\hat{b}_k(i)\mathbf{d}_k(i)$ will be $\sigma_k^2 = \sigma^2/(MN)$ due to spreading gain N and averaging gain M. Therefore, σ^2 is the solution to

$$\sigma^2 = K\frac{\sigma^2}{MN} + \sigma_N^2. \quad (18)$$

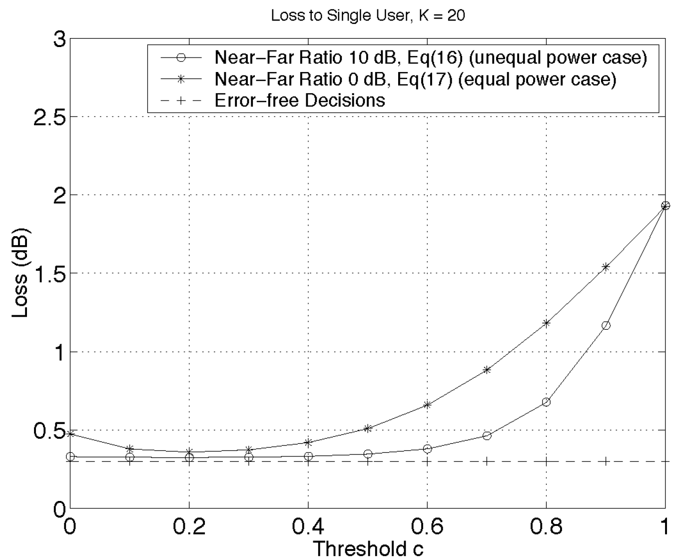

Fig. 3. The SNR loss for the proposed SIC detector compared to the single-user detector as a function of the thresholds $0 \leq c \leq 1$. $K = 20$ users. SNR= 10 dB. $c = 1$ represents the unit clipper.

That is, $\sigma^2 = 1/(1-(K/MN))\sigma_N^2$. For a spreading factor $N = 31$ and $K = 20$ users, the performance loss of the error-free decision SIC detector relative to the single-user bound is 0.3 dB. This loss is due to the noise term in the averaged amplitude compared to the noise-free amplitude information.

In Fig. 3, the SNR loss to the single-user detector as a function of the thresholds at SNR= 10 dB is shown. The curve for the near–far ratio 10-dB case is calculated using (16), while the curve for the near–far ratio 0-dB case is calculated using (17). Since our analysis may underestimate the SNR loss when c is close to zero, we should choose c as large as possible when the performance loss is roughly the same. From Fig. 3, a suitable choice of the threshold c is near 0.5 for near–far ratio 10-dB case. Under a near–far ratio of 10 dB, the analyzed SNR loss compared to the single-user case is 0.35 and 1.93 dB for thresholds $c = 0.5$ and 1.0, respectively. Thus, the generalized unit-clipper results in a 1.6-dB improvement.

V. Robustification to Time Delay Errors

When there are time-delay estimation errors, the robust multiuser detection method presented in [16] based on linear SIC can be improved by the proposed soft-decision framework. Robustness here is defined as the accurate estimation and cancellation of interference introduced by the time-delay estimation error. The impact of robustness on system capacity for linear SIC can be found in [16] and is not discussed here. We first briefly review robustness to time-delay error results in [16]. Following this, we incorporate the proposed soft-decision function.

A. Delay-Robust SIC

Denote the estimated time delay of the kth user as $\hat{\tau}_k = (\hat{p}_k + \hat{\delta}_k)T_c$. It is assumed that all users are acquired so that the estimated time delays are within $\pm 0.5T_c$ of the true time delays, i.e., $|\hat{\tau}_k - \tau_k| \leq 0.5T_c$ [15].

Since the chip-rate sampling time instants are arbitrarily chosen at the receiver, the relative position of the estimated and true time delays can be divided two cases: in the same sampling interval and in two adjacent sampling intervals.

If the true delay and the estimated delay are in the same chip sampling interval, then they have the same integer part, i.e, $p_k = \hat{p}_k$ for $1 \leq k \leq K$. The kth user's discretized signature waveform for the ith interval $\mathbf{d}_k(i)$ in (7) can be expressed in a *prediction error form* [16] as the weighted sum of two signals $\hat{\mathbf{d}}_k(i)$ and $\Delta\mathbf{d}_k(i)$

$$\begin{aligned}\mathbf{d}_k(i) =& \delta_k \mathbf{c}_k(p_k+1,i) + (1-\delta_k)\mathbf{c}_k(p_k,i)\\ =& \left[\hat{\delta}_k \mathbf{c}_k(\hat{p}_k+1,i) + (1-\hat{\delta}_k)\mathbf{c}_k(\hat{p}_k,i)\right]\\ &+ (\delta_k - \hat{\delta}_k)\left[\mathbf{c}_k(\hat{p}_k+1,i) - \mathbf{c}_k(\hat{p}_k,i)\right]\\ \stackrel{\text{def}}{=}& \hat{\mathbf{d}}_k(i) + (\delta_k - \hat{\delta}_k)\Delta\mathbf{d}_k(i). \end{aligned} \tag{19}$$

We denote the $(M+1)N$-dimensional vector $\Delta\mathbf{d}_k(i)$ as the *error vector*. Note that MN entries of (19) have zero value. Since a rectangular chip-pulse is used, the expression in (19) is exact [17].

Alternatively, if the true delay and the estimated delay happen to fall in adjacent sampling intervals, without loss of generality, we have the situation where $\hat{p}_k = p_k - 1$. The kth user's discretized signature waveform for the ith interval $\mathbf{d}_k(i)$ in (7) can instead be expressed as the weighted sum of three signals $\hat{\mathbf{d}}_k(i)$, $\Delta\mathbf{d}_k(i)$, and $\mathbf{c}_k(p_k+1,i)$

$$\begin{aligned}\mathbf{d}_k(i) =& (1-\delta_k)\mathbf{c}_k(p_k,i) + \delta_k \mathbf{c}_k(p_k+1,i)\\ =& (1-\delta_k)\left[(\hat{\delta}_k \mathbf{c}_k(\hat{p}_k+1,i) + (1-\hat{\delta}_k)\mathbf{c}_k(\hat{p}_k,i))\right.\\ &\left. + (1-\hat{\delta}_k)(\mathbf{c}_k(\hat{p}_k+1,i) - \mathbf{c}_k(\hat{p}_k,i))\right]\\ &+ \delta_k \mathbf{c}_k(\hat{p}_k+2,i)\\ \stackrel{\text{def}}{=}& (1-\delta_k)\left[\hat{\mathbf{d}}_k(i) + (1-\hat{\delta}_k)\Delta\mathbf{d}_k(i)\right]\\ &+ \delta_k \mathbf{c}_k(\hat{p}_k+2,i). \end{aligned} \tag{20}$$

We denote the vector $\mathbf{c}_k(p_k+1,i)$ as the *guard vector*.

Since the receiver cannot know whether the estimated and true time delays are in the same sampling interval, the robust SIC detector uses (20) to cancel two residual MAI terms for each user, corresponding to the error vector and the guard vector. If the estimated and true time delays are in the same sampling interval, then the estimated signal corresponding to the guard vector will contribute noise terms only, i.e., the negative effect of using (20) instead of (19) is the noise enhancement.

At each SIC stage, the nonzero terms of error vectors of each user in (19) are concatenated into an $\mathcal{R}^{(M+1)N}$ *long* error vector based on the tentative data bit decisions $\hat{b}_k(i)$ as

$$\mathbf{e}_k = \sum_{i=1}^{M} \Delta\mathbf{d}_k(i)\hat{b}_k(i). \tag{21}$$

Similarly, the M guard vectors in (20) are combined as

$$\mathbf{g}_k = \sum_{i=1}^{M} \mathbf{c}_k(p_k+1,i)\hat{b}_k(i). \tag{22}$$

B. Soft-Decision Delay-Robust SIC

Denote the *long* error vector of the kth user at the vth SIC stage as $\mathbf{e}_k^v$ and its amplitude estimate as $\hat{f}_k^v$. Denote the corresponding *long* guard vector as $\mathbf{g}_k^v$ and its amplitude estimate as $\hat{h}_k^v$. The SIC in Section III can be made robust by subtracting the estimated signals due to timing errors in Step 1). Step 1) can be replaced by the following, denoted Step 1R):

$$\begin{aligned}\hat{f}_k^{v+1} =& \frac{1}{M}(\mathbf{e}_k^{v+1})^H(\breve{\mathbf{r}}_k^{v+1})\\ \hat{h}_k^{v+1} =& \frac{1}{M}(\mathbf{g}_k^{v+1})^H(\breve{\mathbf{r}}_k^{v+1})\\ \breve{\mathbf{r}}_k^{v+1} =& \mathbf{r} - \sum_{l=1}^{k-1}\Big(\hat{f}_l^{v+1}\mathbf{e}_l^{v+1} + \hat{h}_l^{v+1}\mathbf{g}_l^{v+1}\\ &+ \sum_{i=1}^{M} e^{j\theta_l}\bar{a}_l^{v+1}\hat{b}_l^{v+1}(i)\mathbf{d}_l(i)\Big)\\ &- \sum_{l=k+1}^{K}\left(\hat{f}_l^v\mathbf{e}_l^v + \hat{h}_l^v\mathbf{g}_l^v + \sum_{i=1}^{M} e^{j\theta_l}\bar{a}_l^v\hat{b}_l^v(i)\mathbf{d}_l(i)\right). \end{aligned} \tag{23}$$

C. Performance Analysis—Comparison to CRLB

To assess the proposed detector's robustness to time-delay errors, we compare the observed time-delay error variance to the Cramér–Rao lower bound (CRLB), which is derived as follows.

Let the kth user's signal amplitude be a_k. Then by (19), the kth user's signal can be decomposed into two terms as

$$a_k\mathbf{d}_k(i) = a_k\hat{\mathbf{d}}_k(i) + a_k(\delta_k - \hat{\delta}_k)\Delta\mathbf{d}_k(i). \tag{24}$$

Define the amplitudes of the error signal as $\Delta a_k = a_k(\delta_k - \hat{\delta}_k)$. Clearly, the time-delay error is proportional to Δa_k.

For the problem we are considering, the parameters to be estimated are noise variance σ^2, user amplitudes $\mathbf{a} = [a_1\ a_2 \ldots a_K]^T$, and the amplitudes of the error signals $\Delta\mathbf{a} = [\Delta a_1\ \Delta a_2 \ldots \Delta a_K]^T$. These parameters to be estimated are organized in a vector ψ

$$\psi = [\sigma^2\ \ \mathbf{a}^T\ \ \Delta\mathbf{a}^T]^T. \tag{25}$$

The observed data are the received vector $\mathbf{r} = [\mathbf{r}^T(1)\ \ \mathbf{r}^T(2)\ \ \ldots\ \ \mathbf{r}^T(M+1)]^T \in \mathcal{C}^{(M+1)N}$ in (4). The log-likelihood function is

$$\begin{aligned} ln\Omega(\mathbf{r}) = -(M+1)Nln\sigma^2 - \frac{1}{\sigma^2}&(\mathbf{r} - \mathbf{d}\ \mathbf{a} - \Delta\mathbf{d}\ \Delta\mathbf{a})^H\\ &\cdot(\mathbf{r} - \mathbf{d}\ \mathbf{a} - \Delta\mathbf{d}\ \Delta\mathbf{a}) \end{aligned} \tag{26}$$

where

$$\begin{aligned}\mathbf{d} = \Bigg[&\sum_{i=1}^{M}\mathbf{d}_1(i)b_1(i)e^{j\theta_1}\ldots\sum_{i=1}^{M}\mathbf{d}_k(i)b_k(i)e^{j\theta_k}\ldots\\ &\sum_{i=1}^{M}\mathbf{d}_K(i)b_K(i)e^{j\theta_K}\Bigg] \end{aligned} \tag{27}$$

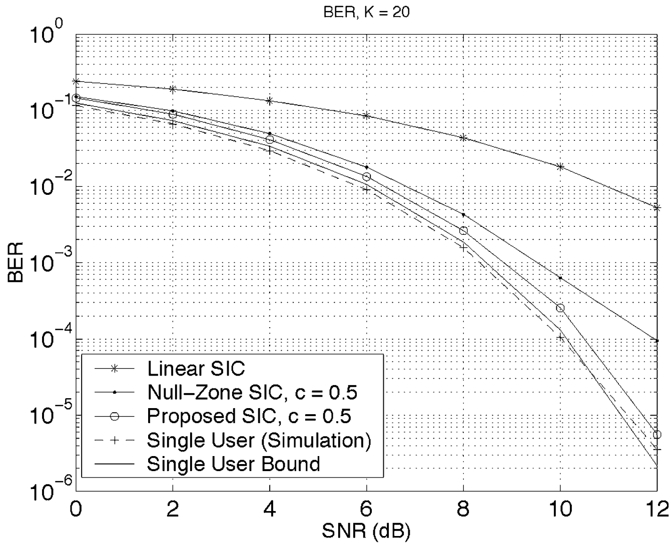

Fig. 4. BER of user 1 for proposed SIC detector and other SIC detectors. $K = 20$ users. Near–far ratio $= 10$ dB. The threshold is $c = 0.5$.

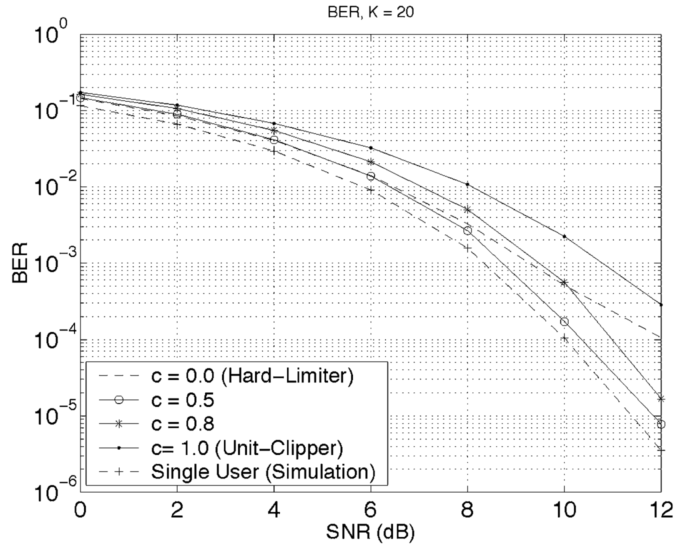

Fig. 5. BER of user 1 for proposed SIC detector with $K = 20$ users. Near–far ratio $= 10$ dB. The thresholds are $c = 0.0, 0.5, 0.8$ and 1.0, respectively. $c = 0.0$ represents the hard limiter. $c = 1.0$ represents the unit clipper.

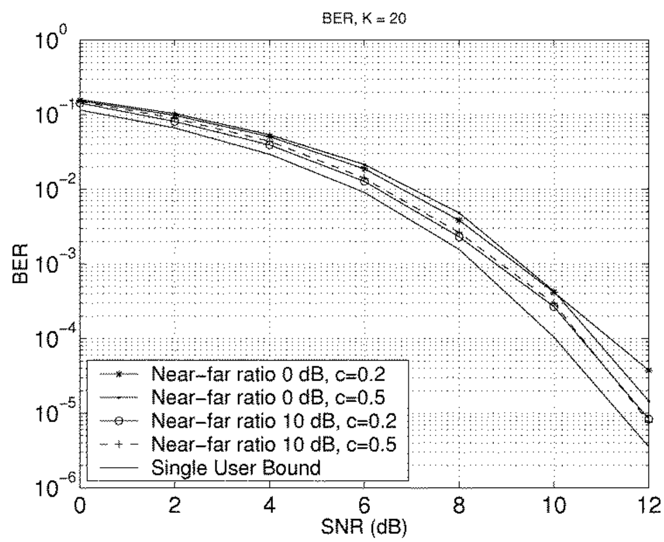

Fig. 6. BER of user 1 for proposed SIC detector with $K = 20$ users. Near–far ratio $= 10$ and 0 dB. The thresholds are $c = 0.2$ and 0.5, respectively.

and

$$\Delta\mathbf{d} = \left[\sum_{i=1}^{M} \Delta\mathbf{d}_1(i) b_1(i) e^{j\theta_1} \ldots \sum_{i=1}^{M} \Delta\mathbf{d}_k(i) b_k(i) e^{j\theta_k} \ldots \sum_{i=1}^{M} \Delta\mathbf{d}_K(i) b_K(i) e^{j\theta_K}\right]. \quad (28)$$

The derivation details of the CRLB are found in the Appendix . It is shown that the CRLB is the inverse of the Fisher information matrix $\mathbf{J} = E\left[(\partial ln\Omega(\mathbf{r})/\partial\psi)(\partial ln\Omega(\mathbf{r})/\partial\psi)^H\right] \in \mathcal{C}^{(1+2K)\times(1+2K)}$, which can be written as

$$\mathbf{J} = \begin{bmatrix} (M+1)\frac{N}{\sigma^4} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J_{aa}} & \mathbf{J_{a\Delta a}} \\ \mathbf{0} & \mathbf{J_{a\Delta a}^H} & \mathbf{J_{\Delta a\Delta a}} \end{bmatrix} \quad (29)$$

where the matrices $\mathbf{J_{aa}}, \mathbf{J_{a\Delta a}}, \mathbf{J_{\Delta a\Delta a}} \in \mathcal{C}^{K\times K}$ are defined as

$$\mathbf{J_{aa}} = \frac{2}{\sigma^2}\mathbf{a}^H\mathbf{d}^H\mathbf{d}\ \mathbf{a} \quad (30)$$

$$\mathbf{J_{a\Delta a}} = \frac{2}{\sigma^2}\mathbf{a}^H\mathbf{d}^H\Delta\mathbf{d}\ \Delta\mathbf{a} \quad (31)$$

$$\mathbf{J_{\Delta a\Delta a}} = \frac{2}{\sigma^2}\Delta\mathbf{a}^H\Delta\mathbf{d}^H\Delta\mathbf{d}\ \Delta\mathbf{a}. \quad (32)$$

We note that the CRLB is conditioned on known data symbols $b_k(i)$.

VI. Numerical and Simulation Results

Throughout the simulations, Gold code sequences of length $N = 31$ and a block size of $M = 9$ bits are used. An additive white Gaussian noise channel is simulated. The number of users is $K = 20$ to account for a highly loaded system. The SNR is defined with respect to the user of interest, denoted as user 1. The near–far ratio is defined as the power ratio between the strongest user and user 1, which is fixed at 10 dB. All other users have an amplitude uniformly distributed between that of the strongest user and the weakest user.

Fig. 4 compares the BER performance of the linear SIC, null-zone SIC, and proposed SIC detector with threshold $c = 0.5$. The proposed SIC with $c = 0.5$ has the smallest distance to the single-user BER curve. The BER curve of the SIC using the null-zone decision function with fixed threshold $c = 0.5$ exhibits an error floor due to the error propagation effects. Adaptive adjustment of c for each user at each stage is required to improve null-zone performance [9].

In Fig. 5, the proposed SIC detector with various threshold values $c = 0.0$ (hard-limiter), $0.5, 0.8, 1.0$(unit-clipper) are shown. The BER curve of the hard-limiter also exhibits an error floor due to error propagation. At a BER of 10^{-3}, the losses relative to the single-user bound are 0.40 dB for $c = 0.5$ and 2.1 dB for $c = 1.0$, which are very close to the analytically derived results of 0.35 and 1.93 dB, as shown in Fig. 3.

In Fig. 6, we compared the BER for $c = 0.2$ and 0.5 at near–far ratios of 0 and 10 dB, respectively. For the 10-dB near–far ratio, the BERs for $c = 0.2$ and 0.5 are almost

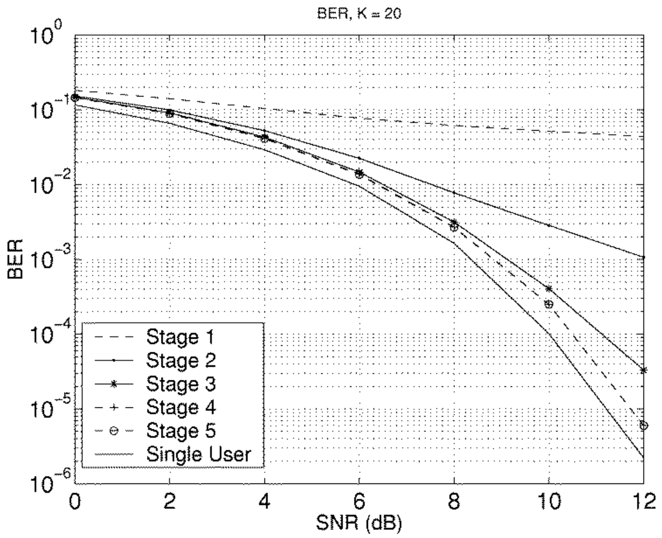

Fig. 7. BER of user 1 for proposed SIC detector as a function of the number of SIC stages. $K = 20$ users. Near–far ratio $= 10$ dB. The threshold is $c = 0.5$.

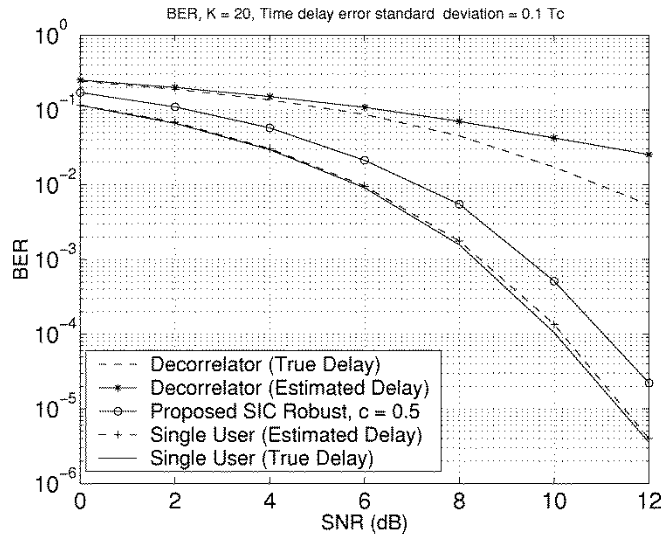

Fig. 9. BER of user 1 for robustified SIC detector. $K = 20$ users. Near–far ratio $= 10$ dB. The threshold is $c = 0.5$. The time delay has an error of $\sigma_\tau = 0.1T_c$.

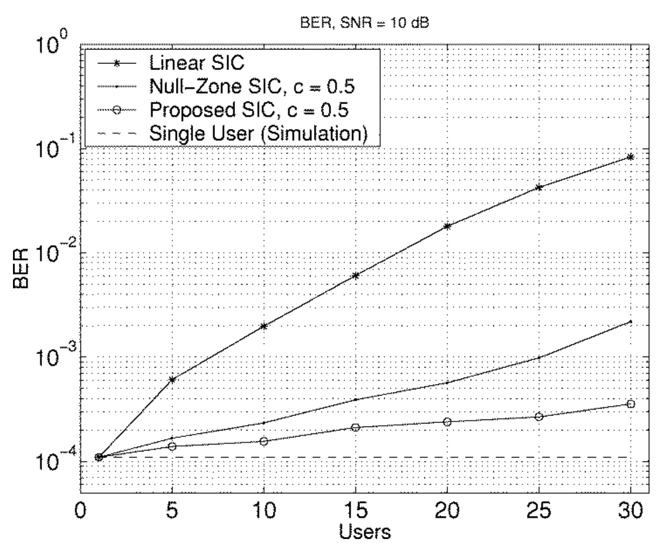

Fig. 8. BER of user 1 for proposed SIC detector and other SIC detectors as a function of the number of users. SNR$=$ 10 dB. Near–far ratio $= 10$ dB. The threshold is $c = 0.5$.

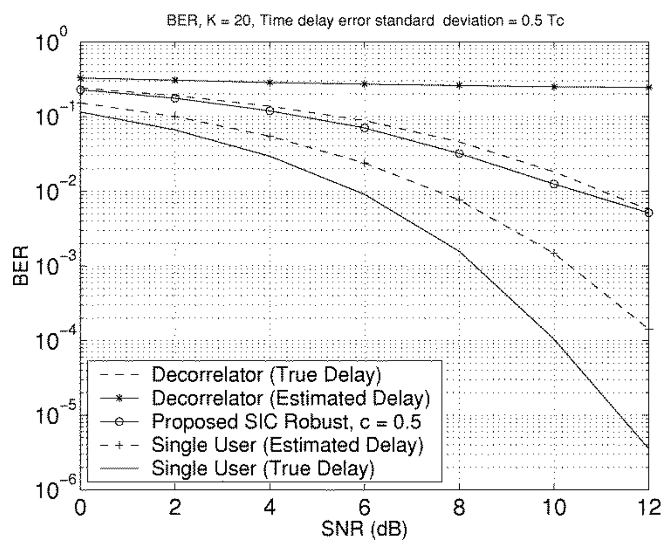

Fig. 10. BER of user 1 for robustified SIC detector. $K = 20$ users. Near–far ratio $= 10$ dB. The threshold is $c = 0.5$. The time delay has an error of $\sigma_\tau = 0.5T_c$.

identical, which agrees with Fig. 3. However, for 0-dB near–far ratio, the analysis results of Fig. 3 underestimate SNR loss for small c, at large SNR. So, in the following simulations, we select $c = 0.5$.

Fig. 7 shows the BER curves of the proposed SIC detector with threshold value $c = 0.5$ from stages 1 to 5. The largest improvements are in early stages, while the BER curves of stages 4 and 5 are almost identical, showing that convergence is approximated after five stages.

In Fig. 8, the BERs of SIC receivers with different decision functions are compared as a function of the number of users at 10-dB SNR. A threshold of $c = 0.5$ is used for both the null zone and the proposed decision function.

In the following simulations, the conditions are the same as described before, except that estimated time delays are used at the receiver. The time-delay errors are modeled as zero-mean Gaussian random variables truncated to be within the interval $\pm 0.5T_c$.

In Fig. 9, the standard deviation of the timing error is $\sigma_\tau = 0.1T_c$, which is typical of current timing estimation methods for CDMA. Our robust SIC (that employs (23)) performs within 1.2 dB of the single-user bound.

In Fig. 10, the extreme case of $\sigma_\tau = 0.5T_c$ is shown. Usually the estimated time delay will have an error much smaller than in this case. However, our robustified SIC performs almost the same as a decorrelating detector containing true time-delay information, although it exhibits an error floor as the SNR gets larger.

In Fig. 11, we compare the root mean square error (RMSE) of the delay-robust SIC to the CRLB for $\sigma_\tau = 0.1T_c$ and $0.5T_c$. As the CRLB is conditioned on the user amplitudes, data symbols, and delays, it is averaged over 500 different runs. For comparison, we also show the RMSE of the unbiased estimator as-

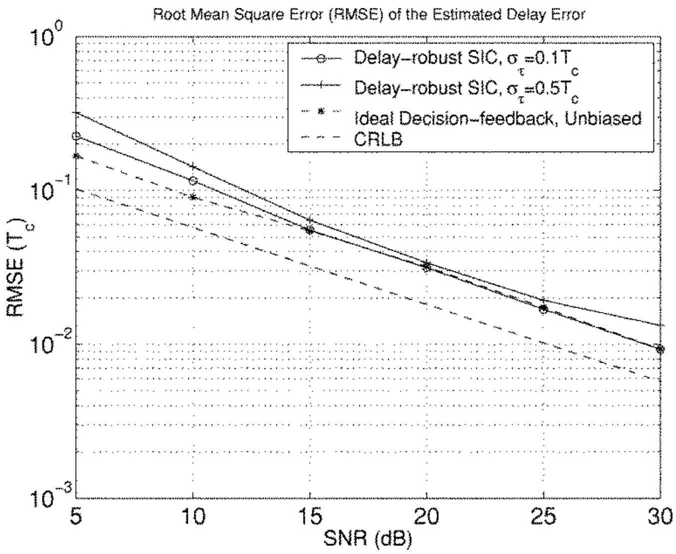

Fig. 11. RMSE of user 1 for delay-robust SIC detector compared to the CRLB. $K = 20$ users. Near–far ratio $= 10$ dB. Soft decision function is used with threshold $c = 0.5$.

suming ideal decision feedback. The CRLB and the RMSE of the unbiased estimator are not affected by the value of σ_τ. When SNR is larger than 15 dB, the RMSEs of the delay-robust SIC and the unbiased estimator are almost identical for $\sigma_\tau = 0.1T_c$, so the delay-robust SIC based estimator is approximately unbiased, and it is meaningful to compare its RMSE to the CRLB. The almost constant gap between the RMSE and the CRLB is due to the decorrelator noise enhancement. The robustness of the delay-robust SIC is justified by its decreased RMSE as the SNR increases, since the time-delay error introduced interference is increased as we increase the SNR while keeping the near–far ratio fixed. Even with $\sigma_\tau = 0.5T_c$, the RMSE also decreases as the SNR increases, so robustness is achieved.

VII. Conclusion

We have proposed and analyzed a family of improved bit decision procedures for the SIC. This new decision function combines the advantages of the unit-clipper and the hard-limiter decision functions. BER performance within 0.4 dB of the single-user bound has been shown by both simulation and analysis. The previously proposed unit clipper ($c = 1$) [6], [10] can incur a performance loss of more than 2 dB. Our analysis enables the design of an appropriate threshold parameter for the decision function. This soft-decision multistage SIC was then made robust to time-delay estimation errors up to half a PN chip.

Appendix
Derivation of the CRLB

The derivation of the CRLB follows the procedure of [15]. The log-likelihood function is

$$ln\Omega(\mathbf{r}) = -(M+1)Nln\sigma^2 - \frac{1}{\sigma^2}(\mathbf{r} - \mathbf{d}\,\mathbf{a} - \Delta\mathbf{d}\,\Delta\mathbf{a})^H (\mathbf{r} - \mathbf{d}\,\mathbf{a} - \Delta\mathbf{d}\,\Delta\mathbf{a}). \quad (33)$$

The gradients

$$\frac{\partial ln\Omega(\mathbf{r})}{\partial\psi} = \begin{bmatrix} \frac{\partial ln\Omega(\mathbf{r})}{\partial\sigma^2} \\ \frac{\partial ln\Omega(\mathbf{r})}{\partial\mathbf{a}} \\ \frac{\partial ln\Omega(\mathbf{r})}{\partial\Delta\mathbf{a}} \end{bmatrix} \quad (34)$$

are given as

$$\frac{\partial ln\Omega(\mathbf{r})}{\partial\sigma^2} = -\frac{(M+1)N}{\sigma^2} + \frac{1}{\sigma^4}\mathbf{n}^H\mathbf{n} \quad (35)$$

$$\frac{\partial ln\Omega(\mathbf{r})}{\partial\mathbf{a}} = \frac{2}{\sigma^2}\mathbf{a}^H\mathbf{d}^H\mathbf{n} \quad (36)$$

$$\frac{\partial ln\Omega(\mathbf{r})}{\partial\Delta\mathbf{a}} = \frac{2}{\sigma^2}\Delta\mathbf{a}^H\Delta\mathbf{d}^H\mathbf{n}. \quad (37)$$

It can be shown that the (1,1) block of matrix $\mathbf{J}$ is

$$E\left[\left(\frac{\partial ln\Omega(\mathbf{r})}{\partial\sigma^2}\right)^2\right] = \frac{(M+1)N}{\sigma^4}. \quad (38)$$

Since $\partial ln\Omega(\mathbf{r})/\partial\sigma^2$ is uncorrelated with all other gradients, the (1,2) and (1,3) blocks of matrix $\mathbf{J}$ are all zeros.

To calculate the other blocks in matrix $\mathbf{J}$, the general expression of the calculation is

$$E\left[\left(\frac{2}{\sigma^2}\mathbf{f}_1^H\mathbf{n}\right)\left(\frac{2}{\sigma^2}\mathbf{f}_2^H\mathbf{n}\right)^H\right] = \frac{2}{\sigma^2}\mathbf{f}_1^H\mathbf{f}_2 \quad (39)$$

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