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Outage Probability Comparisons for Diversity Systems With Cochannel Interference in Rayleigh Fading

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***Abstract*—Space diversity is an effective method to combat fading and cochannel interference (CCI) in wireless systems. In this paper, outage performances of several diversity schemes, including a practical variation of maximal-ratio combining that does not require signal-to-noise ratios at different antennas, equal-gain combining, and selection combining, are compared analytically for an interference-limited environment in a Rayleigh fading channel. Our analysis provides insight into the performance of diversity schemes in the presence of CCI as well as assesses the impact of cochannel interferer power distributions.**

***Index Terms*—Cochannel interference (CCI), diversity methods, Rayleigh channels.**

I. Introduction

IN SPACE diversity, the received signals at antenna branches are combined to combat fading and cochannel interference (CCI). The optimal scheme is optimum combining (OC), which achieves maximum signal-to-interference-plus-noise ratio (SINR) at the combiner output [1]. To implement OC, the second-order statistics of interference and noise needs to be known. For simplicity, suboptimal combining schemes such as maximal-ratio combining (MRC), equal-gain combining (EGC), and selection combining (SC) [2] are used. A number of papers have studied the outage performance of these diversity systems in fading and CCI [3]–[9]. However, to our knowledge, a comparative analysis of the relative outage performance for suboptimal combining schemes in fading and CCI has not been attempted. Such knowledge can be useful to better understand the design tradeoffs in practical cellular systems. The outage comparison for MRC, EGC, and SC with fading and additive white Gaussian noise was treated in the classical paper of Brennan [2]. In this paper, we provide a comparison study, both analytically and numerically, on the outage probability for suboptimal diversity systems with CCI and flat Rayleigh fading. The analysis considers an arbitrary number of interferers as well as arbitrary interferer power distributions.

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We assume that CCI is the dominant source of system degradation. Therefore, for simplicity, thermal noise was ignored in the analysis and an interference-limited environment was considered [4], [8]–[10]. The outage is defined as the event when the signal-to-interference ratio (SIR) at the combiner output drops below a threshold β, i.e., $P_{\mathrm{OUT}}(\beta) = \Pr\{\mathrm{SIR} < \beta\}$. In an interference-limited environment, MRC, which maximizes the output signal-to-noise ratio (SNR) and whose weights depend on noise powers on antenna branches [2], becomes invalid. Therefore, a variation of MRC was considered and denoted as channel-matched combining (CMC), whose weights are given as the desired user's channel response vector.[1] In practical systems where diversity branches are usually assumed to have the same noise powers, MRC is reduced to CMC [11].

II. Received Signal Vector

We consider a system where the desired signal is corrupted by L interfering signals, all transmitting data at a rate of $1/T$. Assuming perfect synchronization for the desired user and sampling the output of the receiver matched filter at time $t = nT$, we obtain the baseband signal vector at an M-element receiver antenna as [8]

$$\boldsymbol{r}[n] = \sqrt{P_s}\boldsymbol{c}_s a_s[n] + \sum_{i=1}^{L} \sqrt{P_i}\boldsymbol{c}_i \underbrace{\left(\sum_{m=-\infty}^{\infty} a_i[m] h(nT - mT - \tau_i)\right)}_{z_i[n]} \quad (1)$$

where P_s and P_i are, respectively, the transmitting powers of the desired and the ith interfering signals. Data symbols $a_s[n]$ and $a_i[m]$ are mutually independent with zero mean and unit variance. The delay of the ith interfering signal relative to the desired signal τ_i is assumed to be uniformly distributed over the interval $[0, T)$. The combined transmitter and receiver impulse response $h(t)$ is a Nyquist pulse with a raised cosine spectrum and roll-off factor ρ, where $0 \le \rho \le 1$ [12]. The channel vectors of the desired and the interfering signals $\boldsymbol{c}_s$ and $\boldsymbol{c}_i$'s are mutually independent. All channel vectors are assumed to be quasi-static (constant over a time frame [8]) and to have uncorrelated realizations in different frames. We further assume independent

[1] In [8], the combining scheme the authors called MRC is really CMC since an interference-limited environment was assumed.

Rayleigh fading among diversity branches, i.e., the elements of $\boldsymbol{c}_s$ and $\boldsymbol{c}_i$ are independent identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables (RV's) with zero mean and unit variance. In (1), $z_i[n]$ denotes the signal intersymbol interference from the ith interferer. It can be shown [13] that $\mathbb{E}[z_i[n]] = 0$, $\mathbb{E}[|z_i[n]|^2] = 1 - \rho/4$, and $\mathbb{E}[z_i[n] z_j^*[n]] = 0$ for $i \neq j$, where $\mathbb{E}[\cdot]$ denotes the expectation and the superscript * denotes the conjugate operation.

The channel vectors of the desired user and the ith interferer can be expressed component-wise as $\boldsymbol{c}_s = [\alpha_{s,1} e^{j\theta_{s,1}} \cdots \alpha_{s,M} e^{j\theta_{s,M}}]^{\mathrm{T}}$ and $\boldsymbol{c}_i = [\alpha_{i,1} e^{j\theta_{i,1}} \cdots \alpha_{i,M} e^{j\theta_{i,M}}]^{\mathrm{T}}$, respectively, where the superscript T denotes the transpose operation. The phase for the desired user channel $\theta_{s,j}$ and the phase for the interfering user channel $\theta_{i,j}$ are uniformly distributed over $[0, 2\pi)$. The fading amplitudes $\alpha_{s,j}$ and $\alpha_{i,j}$ are modeled as Rayleigh with probability density function (pdf) $f_\alpha(\alpha) = 2\alpha e^{-\alpha^2}$, $\alpha \geq 0$.

III. Outage Probabilities of CMC, EGC, and SC With CCI

A. Outage Probability of CMC

Using the weight vector $\boldsymbol{w}_{\mathrm{CMC}} = \boldsymbol{c}_s$, the output of CMC becomes

$$\boldsymbol{w}_{\mathrm{CMC}}^{\mathrm{H}} \boldsymbol{r}[n] = \sqrt{P_s}\left(\boldsymbol{c}_s^{\mathrm{H}} \boldsymbol{c}_s\right) a_s[n] + \sum_{i=1}^{L} \sqrt{P_i}\left(\boldsymbol{c}_s^{\mathrm{H}} \boldsymbol{c}_i\right) z_i[n]$$

where the superscript H denotes the conjugate transpose operation. The SIR can be written as

$$\begin{aligned}\mathrm{SIR}_{\mathrm{CMC}} &= \frac{P_s \left|\boldsymbol{c}_s^{\mathrm{H}} \boldsymbol{c}_s\right|^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L} P_i \left|\boldsymbol{c}_s^{\mathrm{H}} \boldsymbol{c}_i\right|^2} \\ &= \frac{|\boldsymbol{c}_s|^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L} \frac{1}{\Lambda_i}\frac{|\boldsymbol{c}_s^{\mathrm{H}} \boldsymbol{c}_i|^2}{|\boldsymbol{c}_s|^2}} \\ &= \frac{\sum_{j=1}^{M} \alpha_{s,j}^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L} \frac{\eta_i}{\Lambda_i}}\end{aligned} \tag{2}$$

where $\Lambda_i \triangleq P_s/P_i$, $i = 1, \ldots, L$, is the power ratio of the desired signal to the ith interfering signal and $\eta_i \triangleq |\boldsymbol{c}_s^H \boldsymbol{c}_i|^2/|\boldsymbol{c}_s|^2$. It has been shown in [8] that $\boldsymbol{c}_s^{\mathrm{H}} \boldsymbol{c}_i/|\boldsymbol{c}_s|$ is a circularly symmetric complex Gaussian RV with zero mean and unit variance and is independent of $\boldsymbol{c}_s$. Hence, η_i is exponentially distributed with unit mean.

In Appendix A, we derive new outage probability expressions of CMC for both equal $(\Lambda_1 = \cdots = \Lambda_L = \Lambda)$ and distinct $(\Lambda_i \neq \Lambda_j$ for $i \neq j)$ interferer powers as shown in the following:

$$P_{\mathrm{OUT,CMC}}(\beta) = \begin{cases} \left(\dfrac{\beta_0}{\beta_0+\Lambda}\right)^M \displaystyle\sum_{k=0}^{L-1} \frac{(k+M-1)!}{k!(M-1)!}\left(\frac{\Lambda}{\beta_0+\Lambda}\right)^k & \text{equal interferer powers} \\ \displaystyle\sum_{i=1}^{L} \pi_i \left(\frac{\beta_0}{\beta_0+\Lambda_i}\right)^M & \text{distinct interferer powers} \end{cases} \tag{3}$$

where $\beta_0 = (1 - \rho/4)\beta$ and $\pi_k = \prod_{i=1, i\neq k}^{L} (\Lambda_i)/(\Lambda_i - \Lambda_k)$. It can be verified that (3) is numerically equivalent to the outage expressions derived by Aalo and Chayawan [7, eqs. (13) and (14)], as well as the outage expression recently derived by Shah and Haimovich [8, eq. (43)]. However, as shown in Section IV, the present expressions are more suitable for analytical outage probability comparison.

B. EGC Outage Probability

Using combining weight vector $\boldsymbol{w}_{\mathrm{EGC}} = [e^{j\theta_{s,1}} \cdots e^{j\theta_{s,M}}]^{\mathrm{T}}$, the EGC output is

$$\boldsymbol{w}_{\mathrm{EGC}}^{\mathrm{H}} \boldsymbol{r}[n] = \sqrt{P_s}\left(\sum_{j=1}^{M} \alpha_{s,j}\right) a_s[n] + \sum_{i=1}^{L} \sqrt{P_i}\left(\sum_{j=1}^{M} \underbrace{\alpha_{i,j} e^{j(\theta_{i,j}-\theta_{s,j})}}_{g_{i,j}}\right) z_i[n]$$

where $g_{i,j}$ is a circularly symmetric complex Gaussian RV with zero mean and unit variance [9].

The SIR can be found from the above expression as

$$\begin{aligned}\mathrm{SIR}_{\mathrm{EGC}} &= \frac{P_s\left(\sum_{j=1}^{M} \alpha_{s,j}\right)^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L} P_i \left|\sum_{j=1}^{M} g_{i,j}\right|^2} \\ &= \frac{\left(\sum_{j=1}^{M} \alpha_{s,j}\right)^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L} \frac{\mu_i}{\Lambda_i}}\end{aligned} \tag{4}$$

where $\mu_i = |\sum_{j=1}^{M} g_{i,j}|^2$. It can be shown that μ_i is exponentially distributed with mean M. An exact outage analysis for EGC in Rayleigh fading and CCI has recently been reported in [9, eqs. (4)–(7)].

C. SC Outage Probability

In SC, the outage event occurs when the branch with maximum SIR value drops below a predefined threshold. The outage probability of SC can therefore be calculated from $P_{\mathrm{OUT,SC}}(\beta) = \Pr\{\mathrm{SIR}_{\mathrm{SC},1} < \beta, \cdots, \mathrm{SIR}_{\mathrm{SC},M} < \beta\}$, where $\mathrm{SIR}_{\mathrm{SC},i}$ is the SIR at the output of the ith receiving antenna. The outage probability expressions of SC can be obtained from [3],[2] [4], and [5] as

$$P_{\mathrm{OUT,SC}}(\beta) = \begin{cases} \left[1-\left(\dfrac{\Lambda}{\beta_0+\Lambda}\right)^L\right]^M & \text{equal interferer powers} \\ \left[\displaystyle\sum_{k=1}^{L} \pi_k \frac{\beta_0}{\beta_0+\Lambda_k}\right]^M & \text{distinct interferer powers.} \end{cases} \tag{5}$$

[2]Note that the SC outage probability expression presented in [3, eq. (1)] is only valid for the equal interferer power case.

IV. Analytical Outage Probability Comparisons

A. Outage Probability Comparison for CMC and EGC

This section proves that CMC has a lower outage probability than that of EGC. To show this, we first rewrite the output SIR expression of CMC in (2) as

$$\mathrm{SIR_{CMC}} = \frac{M\sum_{j=1}^{M}\alpha_{s,j}^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L}\frac{M\eta_i}{\Lambda_i}} = \frac{M\sum_{j=1}^{M}\alpha_{s,j}^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L}\frac{\nu_i}{\Lambda_i}} \quad (6)$$

where $\nu_i = M\eta_i$. Since η_i is exponentially distributed with unit mean, ν_i is exponentially distributed with mean M. Comparing (6) with (4), we recognize that $\xi_1 \triangleq \sum_{i=1}^{L}\nu_i/\Lambda_i$ in (6) and $\xi_2 \triangleq \sum_{i=1}^{L}\mu_i/\Lambda_i$ in (4) have the same distribution. In (4) and (6), the denominator is independent of the numerator. Thus, we express the respective outage probabilities as

$$\begin{aligned} P_{\mathrm{OUT,CMC}}(\beta) &= \Pr\left\{M\sum_{j=1}^{M}\frac{\alpha_{s,j}^2}{\xi_1} < \beta_0\right\} \\ &= \int \Pr\left\{M\sum_{j=1}^{M}\alpha_{s,j}^2 < \beta_0\xi\right\} f_{\xi_1}(\xi)\mathrm{d}\xi \\ P_{\mathrm{OUT,EGC}}(\beta) &= \Pr\left\{\frac{\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2}{\xi_2} < \beta_0\right\} \\ &= \int \Pr\left\{\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2 < \beta_0\xi\right\} f_{\xi_2}(\xi)\mathrm{d}\xi. \end{aligned}$$

Since pdf $f_{\xi_1}(\xi) = f_{\xi_2}(\xi)$ and $(\sum_{j=1}^{M}\alpha_{s,j})^2 \le M\sum_{j=1}^{M}\alpha_{s,j}^2$ due to the Cauchy–Schwarz inequality, we have $P_{\mathrm{OUT,CMC}}(\beta) \le P_{\mathrm{OUT,EGC}}(\beta)$, where equality is achieved when $M = 1$ (single antenna). When $M > 1$, the outage probability for CMC is strictly lower than that for EGC. The above conclusion holds for arbitrary interferer power distributions.

B. Outage Probability Comparison for CMC and SC

We first consider the case of one interfering signal, which approximates the situation where the system has one strong dominant interfering user. Setting $L = 1$ in (3) and (5), we obtain

$$P_{\mathrm{OUT,CMC}}(\beta)\Big|_{L=1} = P_{\mathrm{OUT,SC}}(\beta)\Big|_{L=1} = \left(\frac{\beta_0}{\beta_0+\Lambda}\right)^M.$$

Therefore, the outage probabilities for CMC and SC are identical for $L = 1$.

When the number of interfering signals is greater than one ($L > 1$), it is proved in Appendix B that, for the special case of equal interferer powers, the outage probabilities for CMC are smaller than those of SC. For distinct interferer powers, the numerical results presented in Section V suggest that CMC still outperforms SC.

C. Outage Probability Comparison for EGC and SC

As shown in Section V, the relative outage performance of EGC and SC depends on factors such as the number of interferers and the interferer power distribution. More interestingly, SC can have better outage performance than EGC in the presence of one dominant interferer. An exact analytical outage comparison for EGC and SC in CCI is difficult. We will, however, use a geometric argument[3] to provide an intuitive explanation to this observation by considering a dual branch ($M = 2$) case. In SC, the SIR at the first antenna branch is

$$\mathrm{SIR_{SC,1}} = \frac{\alpha_{s,1}^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L}\frac{\alpha_{i,1}^2}{\Lambda_i}} = \frac{M\alpha_{s,1}^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L}\frac{M\alpha_{i,1}^2}{\Lambda_i}}. \quad (7)$$

Since $M\alpha_{i,1}^2$ is exponentially distributed with mean M, it is clear that $\xi_3 \triangleq \sum_{i=1}^{L}M\alpha_{i,1}^2/\Lambda_i$ in (7) and $\xi_2 \triangleq \sum_{i=1}^{L}\mu_i/\Lambda_i$ in (4) have the same distribution. Similarly, denoting $\xi_4 \triangleq \sum_{i=1}^{L}M\alpha_{i,2}^2/\Lambda_i$ in $\mathrm{SIR_{SC,2}}$, we express the respective outage probability for SC and EGC as that shown in (8) and (9) at the bottom of the page. Since $f_{\xi_2}(\xi) = f_{\xi_3}(\xi)$, in order to compare the outage probabilities in (8) and (9), one is only required to compare $\Pr(\alpha_{s,1} < \sqrt{\beta_0\xi/2}, \alpha_{s,2} < \sqrt{\beta_0\xi'/2})$ and $\Pr(\alpha_{s,1}+\alpha_{s,2} < \sqrt{\beta_0\xi})$.

Referring to Fig. 1, $\Pr(\alpha_{s,1} < \sqrt{\beta_0\xi/2}, \alpha_{s,2} < \sqrt{\beta_0\xi'/2})$ is obtained by integrating the joint pdf of $\alpha_{s,1}$ and $\alpha_{s,2}$ over the rectangular region and $\Pr(\alpha_{s,1}+\alpha_{s,2} < \sqrt{\beta_0\xi})$ is obtained by integrating the same pdf over the triangular region. For one interferer ($L = 1$), ξ_4 has an exponential distribution and takes small values of ξ' with high probability. A small value of ξ' will result, as shown in Fig. 1, a rectangular region (solid line) contained inside the triangular region, thus $\Pr(\alpha_{s,1} < \sqrt{\beta_0\xi/2}, \alpha_{s,2} < \sqrt{\beta_0\xi'/2}) < \Pr(\alpha_{s,1} + \alpha_{s,2} < \sqrt{\beta_0\xi})$. Therefore, $P_{\mathrm{OUT,SC}}(\beta) < P_{\mathrm{OUT,EGC}}(\beta)$ when $L = 1$. For large values of L, ξ_4 is chi-square distributed with its pdf pushed away from the origin and hence takes large values of ξ' with high probabilities. A large

[3] A similar, but not identical, geometric argument was used by Brennan [2].

$$P_{\mathrm{OUT,SC}}(\beta) = \int\left[\int \Pr\left(\alpha_{s,1} < \sqrt{\frac{\beta_0\xi}{2}}, \alpha_{s,2} < \sqrt{\frac{\beta_0\xi'}{2}}\right) f_{\xi_4}(\xi')\mathrm{d}\xi'\right] f_{\xi_3}(\xi)\mathrm{d}\xi \quad (8)$$

$$P_{\mathrm{OUT,EGC}}(\beta) = \int \Pr\left(\alpha_{s,1}+\alpha_{s,2} < \sqrt{\beta_0\xi}\right) f_{\xi_2}(\xi)\mathrm{d}\xi = \int\left[\int \Pr\left(\alpha_{s,1}+\alpha_{s,2} < \sqrt{\beta_0\xi}\right) f_{\xi_4}(\xi')\mathrm{d}\xi'\right] f_{\xi_2}(\xi)\mathrm{d}\xi \quad (9)$$

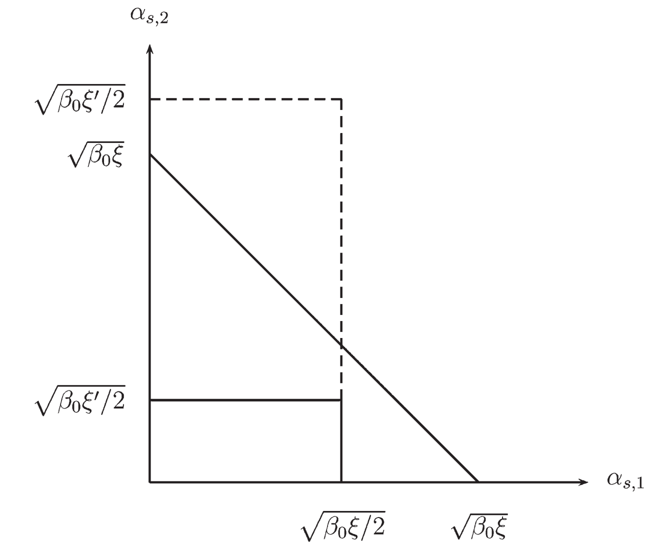

Fig. 1. Regions of integration for the conditional outage probability of SC and EGC.

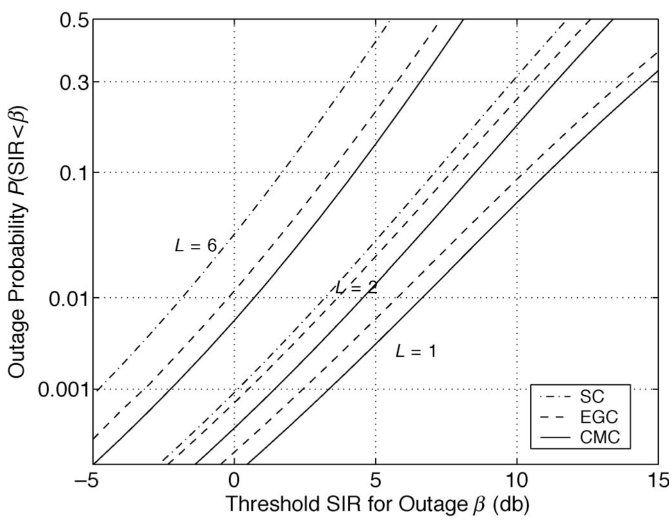

Fig. 2. Analytical outage probability of CMC, EGC, and SC for an interference-limited environment: equal interferer powers ($\Lambda = 10$ dB), $M = 4$ antennas, and $L = 1$, 2, and 6 interferers.

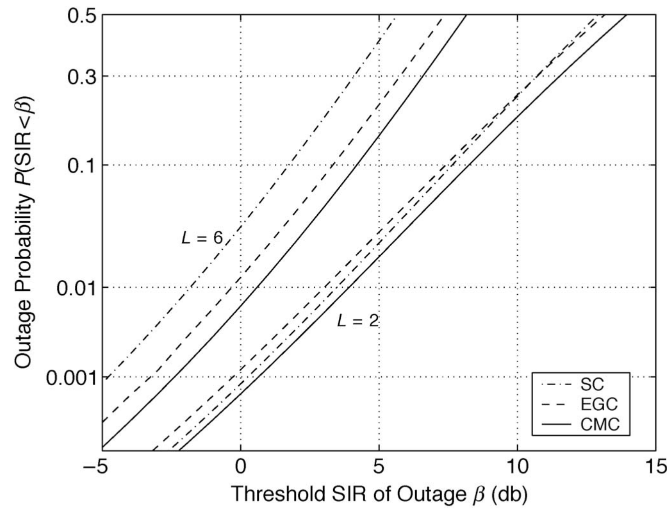

Fig. 3. Analytical outage probability of CMC, EGC, and SC for an interference-limited environment: distinct interferer powers ($\Lambda_{\text{avg}} = 10$ dB), $M = 4$ antennas, and $L = 2$ and 6 interferers. The interference power vectors for $L = 2$ and $L = 6$ are, respectively, [0.1, 0.9] and [0.05, 0.1, 0.15, 0.22, 0.23, 0.25].

value of ξ' will result, as the dashed line shown in Fig. 1, in $\Pr(\alpha_{s,1} < \sqrt{\beta_0\xi/2}, \alpha_{s,2} < \sqrt{\beta_0\xi'/2}) > \Pr(\alpha_{s,1} + \alpha_{s,2} < \sqrt{\beta_0\xi})$ for Rayleigh-distributed $\alpha_{s,1}$ and $\alpha_{s,2}$ [2]. Therefore, $P_{\text{OUT,SC}}(\beta) > P_{\text{OUT,EGC}}(\beta)$ for large values of L.

V. Numerical Results

In this section, we make quantitative comparisons of the outage probabilities for CMC, EGC, and SC with CCI in a Rayleigh fading channel. In obtaining all the numerical results without loss of generality, we set $\rho = 0$.

Fig. 2 plots the outage probabilities for CMC, EGC, and SC versus the outage threshold β with four diversity branches ($M = 4$) and equal interferer powers ($\Lambda = 10$ dB) for $L = 1$, 2, and 6 interferers. Fig. 2 confirms, as argued in Section IV-A, that the outage probabilities for CMC are smaller than those of EGC in all cases considered. The improvement of CMC over EGC is approximately 1 dB over a wide range of outage probability levels and numbers of interferers. As expected, Fig. 2 also indicates that SC and CMC have the same outage performance when the system has one interferer. However, the outage performance of SC degrades quickly (with respect to that of CMC) when the number of interferers increases.

Fig. 3 shows the effect of interferer power distribution on the outage probabilities for CMC, EGC, and SC. We first define the ratio of the desired signal power to the average interference power as $\Lambda_{\text{avg}}(\text{dB}) = 10\log_{10}(P_s/(1/L)\sum_{i=1}^{L} P_i)$. Denoting the normalized interference power vector by $\boldsymbol{q} = [q_1, q_2, \ldots, q_L]$, where $\sum_{i=1}^{L} q_i = 1$, we can calculate the power ratio $\Lambda_i(\text{dB}) = P_s/P_i(\text{dB}) = \Lambda_{\text{avg}}(\text{dB}) - 10\log_{10}(Lq_i)$. With four diversity branches ($M = 4$), Fig. 3 compares the outage probabilities for two interferers ($L = 2$) with a highly unbalanced interference power vector [0.1, 0.9] and for six interferers ($L = 6$) with a more evenly distributed interference power vector [0.05, 0.1, 0.15, 0.22, 0.23, 0.25]. In both cases, as expected, CMC outperforms both EGC and SC. The relative performance for EGC and SC, however, depends on the interferer power distribution. With six interferers, Fig. 3 shows that EGC outperforms SC in a scenario that approximates the equal interferer power case studied in Fig. 2. For two interferers, however, EGC is inferior to SC. This can be explained by noting that the interference power vector [0.1, 0.9] represents the case of a strong dominant interferer, a scenario where the outage performance of SC is almost equivalent to that of CMC.

In the presence of noise, we use Monte Carlo simulation to evaluate the outage probabilities. Assuming all antenna branches have the same noise powers, Figs. 4 and 5 show the outage probabilities of CMC,[4] EGC, and SC at different SNRs for four antennas ($M = 4$), one interferer ($L = 1$), and

[4]In this case, CMC is equivalent to MRC.

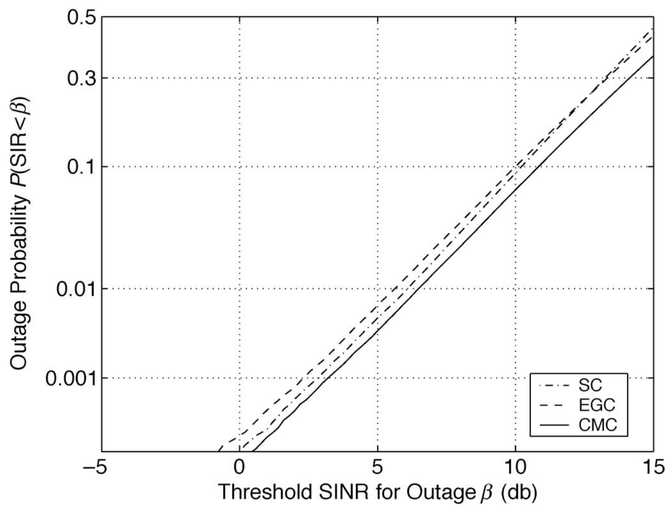

Fig. 4. Monte Carlo simulated outage probability of CMC, EGC, and SC for a finite SNR: $M = 4$ antennas, $L = 1$ interferer, $\Lambda = 10$ dB, and $\mathrm{SNR} = 20$ dB.

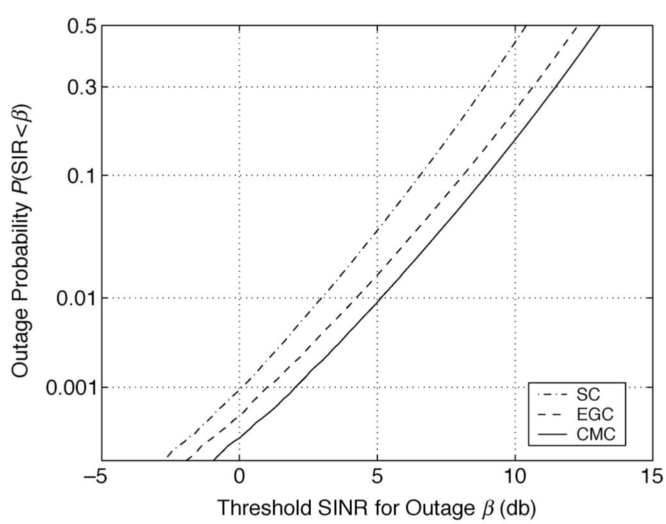

Fig. 5. Monte Carlo simulated outage probability of CMC, EGC, and SC for a finite SNR: $M = 4$ antennas, $L = 1$ interferer, $\Lambda = 10$ dB, and $\mathrm{SNR} = 10$ dB.

$\Lambda = 10$ dB. We observe that, for one interferer, as expected, SC outperforms EGC at high SNRs but may not outperform EGC in lower SNRs. To investigate how closely the analysis of an interference-limited environment holds for finite SNRs, we compare the $L = 1$ curves in Fig. 2 to those in Figs. 4 and 5. We note that the performance of CMC and EGC is within 2 dB at low SNR (10 dB) and almost the same as that predicted by the interference-limited analysis at higher SNR (20 dB). The interference-limited analysis of SC is less accurate for finite SNRs.

VI. Conclusion

In this paper, we have analytically compared the outage performance of CMC, EGC, and SC for an interference-limited environment in flat Rayleigh fading. We have shown that CMC has a lower outage probability than that of EGC and that CMC has no greater outage probability than that of SC. The relative outage performance between EGC and SC, however, depends on the number of interferers and interferer power distribution. For finite SNRs, the simulation results show that the relative performance between EGC and SC is SNR dependent.

Appendix A

In (2), let $X \triangleq \sum_{j=1}^{M} \alpha_{s,j}^2$ and $U \triangleq \sum_{i=1}^{L} \eta_i/\Lambda_i$, where $\alpha_{s,j}$ is Rayleigh distributed and η_i is exponentially distributed with unit mean. Since the denominator and the numerator in (2) are independent, we have

$$P_{\mathrm{OUT,CMC}}(\beta) = \Pr\left\{\frac{X}{U} < \beta_0\right\} = \int_0^{\infty} f_X(x) \int_{\frac{x}{\beta_0}}^{\infty} f_U(u)\mathrm{d}u\mathrm{d}x \tag{10}$$

where the pdf of X is $f_X(x) = [1/\Gamma(M)]x^{M-1}e^{-x},\ x > 0$ since X is chi-square distributed [12]. The pdf of U is [12, eqs. 14-4-13, and 14-5-26]

$$f_U(u) = \begin{cases} \dfrac{\Lambda^L}{(L-1)!} u^{L-1} e^{-\Lambda u},\ u > 0 \\ \quad \text{equal interferer powers} \\ \sum_{k=1}^{L} \Lambda_k \pi_k e^{-\Lambda_k u},\ u > 0 \\ \quad \text{distinct interferer powers.} \end{cases}$$

Hence, using [14, 3.351-2], we have

$$\int_{\frac{x}{\beta_0}}^{\infty} f_U(u)\mathrm{d}u = \begin{cases} \sum\limits_{k=0}^{L-1} \frac{1}{k!}\left(\dfrac{\Lambda}{\beta_0}\right)^k x^k e^{-\frac{\Lambda}{\beta_0}x} & \text{equal interferer powers} \\ \sum\limits_{k=1}^{L} \pi_k e^{-\frac{\Lambda_k}{\beta_0}x} & \text{distinct interferer powers.} \end{cases} \tag{11}$$

Substituting (11) into (10) and using [14, 3.351-3], we obtain (3).

Appendix B

In this appendix, we prove $P_{\mathrm{OUT,CMC}}(\beta) < P_{\mathrm{OUT,SC}}(\beta)$ for equal interferer powers, i.e., from (3) and (5)

$$\left(\frac{\beta_0}{\beta_0+\Lambda}\right)^M \sum_{k=0}^{L-1} \frac{(k+M-1)!}{k!(M-1)!}\left(\frac{\Lambda}{\beta_0+\Lambda}\right)^k < \left[1 - \left(\frac{\Lambda}{\beta_0+\Lambda}\right)^L\right]^M \tag{12}$$

for $L > 1$ and $M > 1$. Before proving this result, we first introduce a useful lemma.

Lemma 1: For positive integers $L > 1$ and $M > 1$,

$$\left(1 + x + \cdots + x^{L-1}\right)^M = \sum_{k=0}^{L-1} \frac{(M+k-1)!}{(M-1)!k!} x^k + \text{higher order terms.} \quad (13)$$

Proof: We prove Lemma 1 by induction. It can be shown easily that (13) is true for $M = 2$. Now assuming (13) holds, we need to show that the expression holds for $M + 1$, i.e.,

$$\left(1 + x + \cdots + x^{L-1}\right)^{M+1} = \sum_{k=0}^{L-1} \frac{(M+k)!}{M!k!} x^k + \text{higher order terms.} \quad (14)$$

To do so, we expand the left side of (14) as

$$\begin{aligned}\left(1 + x + \cdots + x^{L-1}\right)^{M+1} &= \left(1 + x + \cdots + x^{L-1}\right) \\ &\times \left(1 + Mx + \cdots + \frac{(M+k-1)!}{(M-1)!k!} x^k + \cdots \right. \\ &\left. + \frac{(M+L-2)!}{(M-1)!(L-1)!} x^{L-1} + \text{higher order terms}\right) \\ &= 1 + (M+1)x + \sum_{j=0}^{2} \frac{(M+j-1)!}{(M-1)!j!} x^2 + \cdots \\ &+ \sum_{j=0}^{L-1} \frac{(M+j-1)!}{(M-1)!j!} x^{L-1} + \text{higher order terms.}\end{aligned}$$

Applying the identity $\sum_{j=0}^{k}((M+j-1)!/(M-1)!j!) = ((M+k)!/M!k!)$ [15, p. 212] in the above expression, we have (14). Therefore, by induction, Lemma 1 holds. ■

From Lemma 1, it follows that for $x > 0$, $M > 1$, and $L > 1$

$$\left(1 + x + \cdots + x^{L-1}\right)^M > \sum_{k=0}^{L-1} \frac{(M+k-1)!}{(M-1)!k!} x^k. \quad (15)$$

Denoting $(\Lambda/\beta_0 + \Lambda)$ by x, for the case of equal interferer powers, we rewrite (3) and (5) as

$$P_{\mathrm{OUT,CMC}}(x) = (1-x)^M \sum_{k=0}^{L-1} \frac{(M+k-1)!}{(M-1)!k!} x^k \quad (16)$$

$$\begin{aligned}P_{\mathrm{OUT,SC}}(x) &= \left(1 - x^L\right)^M \\ &= (1-x)^M \left(1 + x + \ldots + x^{L-1}\right)^M \quad (17)\end{aligned}$$

where $0 < x < 1$ (since both β_0 and Λ are positive). Comparing (16) with (17) and using (15), we obtain the inequality in (12).

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