Exact Outage Probability for Equal Gain Combining With Cochannel Interference in Rayleigh Fading

Yi Song, *Student Member, IEEE*, Steven D. Blostein, *Senior Member, IEEE*, and Julian Cheng, *Member, IEEE*

***Abstract*—Equal gain combining (EGC) diversity has performance close to that of maximal ratio combining but at lower implementation complexity. We present a new outage performance analysis for EGC in mobile cellular radio systems that are limited by cochannel interference and undergo Rayleigh fading. We utilize a new model where interfering signals add in amplitude and phase across antenna array elements. In addition, the interfering signals may each have a different power. In comparing our analysis to an existing method, we find that: 1) as much as 1.5 dB difference in signal-to-interference ratio may exist at the same probability of outage and 2) the existing method can lead to overly optimistic outage performance prediction in certain situations.**

***Index Terms*—Cochannel interference (CCI), diversity methods, Rayleigh channels.**

I. Introduction

SPACE DIVERSITY can improve mobile radio system performance by weighting and combining the received signals from all antenna branches to combat fading and cochannel interference (CCI) [1]. Space diversity schemes include maximal ratio combining (MRC), equal gain combining (EGC), and selection diversity [2]. The EGC scheme has a performance close to that of MRC but with simpler implementation. We study the impact of CCI on EGC, since CCI is a major factor which limits frequency reuse in cellular systems. In particular, we focus on the outage probability, i.e., the probability of unsatisfactory reception in the intended coverage area.

In previous related work, Abu-Dayya and Beaulieu studied the outage probability for EGC with CCI in Nakagami-m fading [3], where the interfering signal components are added incoherently across antenna array elements. The analysis in [3] was restricted to the case of equal interferer powers. In real systems, however, the interferer powers may be distinct. More recently, Shah and Haimovich derived the signal-to-interference ratio (SIR) of EGC in CCI [4], where only the ratio of mean signal power to mean interference power was considered, and where the interferer powers were equally distributed.

In the following, we present an exact outage probability analysis of EGC in Rayleigh fading with CCI. A more accurate model for interference power calculation is presented in Section II in the sense that the coherence (amplitude and phase) of interferers is taken into account in the output power from the EGC-combined diversity branches. The model also takes in account pulse shape, random delays between interfering signals, intersymbol interference, as well as both equal and distinct interferer powers. In Section IV, we analytically compare the new outage analysis to that of [3] for the case of one interferer. In Section V, we present numerical outage calculations for varying numbers of antennas and interferers for both equal and unequal power distributions.

Manuscript received January 14, 2002; revised March 26, 2002; accepted May 8, 2002. The editor coordinating the review of this paper and approving it for publication is A. Svensson.

Y. Song and S. D. Blostein are with the Department of Electrical and Computer Engineering, Queen's University, Kingston, ON K7L 3N6, Canada (e-mail: sdb@ee.queensu.ca).

J. Cheng is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada.

Digital Object Identifier 10.1109/TWC.2003.816796

II. System Model

We assume that CCI is the limiting source of performance degradation [1]; therefore, for simplicity, we ignore the thermal noise in our system model and consider only an interference-limited environment. The transmitted signals from the desired and the ith interfering user are, respectively,

$$s_s(t) = \sqrt{P_s T} \sum_{m=-\infty}^{+\infty} a_s[m] h_T(t - mT)$$

and

$$s_i(t) = \sqrt{P_i T} \sum_{m=-\infty}^{+\infty} a_i[m] h_T(t - mT)$$

where $h_T(t)$ is transmitter pulse response, $1/T$ is the data transmission rate, and P_s and P_i are the transmitting powers of the desired and the ith interfering signals, respectively. The transmitter filter is assumed to have a square-root raised cosine frequency response with a rolloff factor ρ $(0 \leq \rho \leq 1)$[5]. The data symbols $a_s[m]$ and $a_i[m]$ are mutually independent with zero-mean and unit variance.

The baseband received signal vector at an M-element receiver antenna array is

$$\boldsymbol{r}(t) = \sqrt{P_s T}\boldsymbol{c}_s \sum_{m=-\infty}^{+\infty} a_s[m] h_T(t - mT) + \sum_{i=1}^{L} \sqrt{P_i T}\boldsymbol{c}_i \sum_{m=-\infty}^{+\infty} a_i[m] h_T(t - mT - \tau_i)$$

where L is the number of interfering signals, the random delay τ_i is assumed to be uniformly distributed over the interval $[0, T)$, and $\boldsymbol{c}_s$ and $\boldsymbol{c}_i$ are the channel vectors for the desired and the ith interfering user, respectively. All channel vectors are assumed to be quasistatic (constant over a time frame [4]) and to have uncorrelated realizations in different frames. We further assume independent Rayleigh fading among diversity branches, i.e., the elements of $\boldsymbol{c}_s$ and $\boldsymbol{c}_i$ are independent identically distributed (i.i.d.) complex Gaussian random variables (RVs) with zero-mean and unit variance. Therefore, the channel covariance

1536-1276/03$17.00 © 2003 IEEE

matrix becomes $E\left[\boldsymbol{c}_s\boldsymbol{c}_s^H\right] = E\left[\boldsymbol{c}_i\boldsymbol{c}_i^H\right] = \boldsymbol{I}$, where the superscript H denotes the Hermitian operation and $\boldsymbol{I}$ denotes an $M \times M$ identity matrix. Finally, we assume that $\boldsymbol{c}_s$ and $\boldsymbol{c}_i$s are mutually independent.

Assuming perfect synchronization for the desired user, sampling the output of the receiver matched filter at $t = nT$, we obtain

$$\boldsymbol{r}[n] = \sqrt{P_sT}\boldsymbol{c}_s a_s[n] + \sum_{i=1}^{L}\sqrt{P_iT}\boldsymbol{c}_i z_i[n]$$

where

$$z_i[n] = \sum_{m=-\infty}^{+\infty} a_i[m]h(nT - mT - \tau_i)$$

and where $h(t)$ is a Nyquist pulse. It can be shown [6] that $E\{z_i[k]\} = 0, E\left\{|z_i[k]|^2\right\} = 1-(\rho/4)$, and $E\{z_i[k]z_j^*[k]\} = 0$ for $i \neq j$, where $*$ denotes the conjugate operation.

We express, component-wise, the desired and the interfering channel vectors as $\boldsymbol{c}_s = \left[\alpha_{s,1}e^{j\theta_{s,1}} \cdots \alpha_{s,M}e^{j\theta_{s,M}}\right]^T$ and $\boldsymbol{c}_i = \left[\alpha_{i,1}e^{j\theta_{i,1}} \cdots \alpha_{i,M}e^{j\theta_{i,M}}\right]^T$, where the superscript T denotes the transpose operation. The phase for the desired user channel $\theta_{s,j}$ and the phase for the interfering user channel $\theta_{i,j}$ are uniformly distributed over $[0, 2\pi)$. The fading amplitudes $\alpha_{s,j}$ and $\alpha_{i,j}$ are Rayleigh-distributed as

$$f_\alpha(\alpha) = 2\alpha e^{-\alpha^2}, \quad \alpha \geq 0.$$

III. Outage Probability of EGC With CCI

In EGC, the outputs of all the branches are cophased (with respect to the desired user signal) and weighted equally. The combining weight vector of an equal gain combiner is $\boldsymbol{w} = \left[e^{j\theta_{s,1}} \cdots e^{j\theta_{s,M}}\right]^T$, and the output of the combiner becomes

$$\begin{aligned}\boldsymbol{w}^H\boldsymbol{r}[n] =& \sqrt{P_sT}\left(\boldsymbol{w}^H\boldsymbol{c}_s\right)a_s[n] + \sum_{i=1}^{L}\sqrt{P_iT}\left(\boldsymbol{w}^H\boldsymbol{c}_i\right)z_i[n]\\ =&\sqrt{P_sT}\left(\sum_{j=1}^{M}\alpha_{s,j}\right)a_s[n]\\ &+\sum_{i=1}^{L}\sqrt{P_iT}\left(\sum_{j=1}^{M}\underbrace{\alpha_{i,j}e^{j(\theta_{i,j}-\theta_{s,j})}}_{g_{i,j}}\right)z_i[n]. \end{aligned} \quad (1)$$

It can be shown that $(\theta_{i,j}-\theta_{s,j}) \bmod 2\pi$ is uniformly distributed over $[0, 2\pi)$ and is independent of $\alpha_{i,j}$. Since $\alpha_{i,j}$ is Rayleigh-distributed, $g_{i,j}$ is complex Gaussian with zero-mean and unit variance.

Since $z_i[n]$ and $z_j[n]$ are uncorrelated for $i \neq j$, the total interference power at the combiner output is obtained by adding interference powers from different interferers. For each interferer, interference from different antennas can combine either incoherently [see [3], (8b)] or coherently. In the incoherent case, to compute the ith interferer's power, the channel amplitude of each diversity branch is first squared, and all branches are then summed, i.e., $\sum_{j=1}^{M}\alpha_{i,j}^2$. If the interfering signals arriving at different antennas are mutually uncorrelated, the incoherent calculation is exact. However, these interfering signals are, in general, correlated, thus, the incoherent calculation is only an approximation. In the *coherent* interference power calculation, *phasor* addition of each interfering signal is employed, i.e., $\alpha_{i,j}e^{j(\theta_{i,j}-\theta_{s,j})}$ are added first, and then squared, i.e.,

$$\left|\sum_{j=1}^{M} g_{i,j}\right|^2$$

in (1). Coherent interference diversity combining is an improved model of EGC over incoherent interference combining, since coherent combining is performed at the receiver, and, therefore, interfering signals add as complex-valued quantities. Numerical results in Section V demonstrate cases where the two interference power calculation methods lead to significantly different outage probabilities.

The instantaneous SIR at the output of the equal gain combiner, assuming coherent interference power calculation over the diversity branches, is

$$\mathrm{SIR} = \frac{P_s\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L}P_i\left|\sum_{j=1}^{M}g_{i,j}\right|^2} = \frac{\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2}{\left(1-\frac{\rho}{4}\right)\sum_{i=1}^{L}\frac{\mu_i}{\Lambda_i}} \quad (2)$$

where $\Lambda_i \triangleq P_s/P_i$, for $i = 1,\ldots,L$, is the power ratio of the desired signal to the ith interfering signal, and we have also denoted

$$\mu_i \triangleq \left|\sum_{j=1}^{M} g_{i,j}\right|^2.$$

Here, $g_{i,1},\ldots,g_{i,M}$ are i.i.d. complex Gaussian RVs with zero-mean and unit variance, thus, $\sum_{j=1}^{M} g_{i,j}$ is a complex Gaussian RV with mean zero and variance M. It can be shown that μ_i is exponentially distributed with mean M[5]. We further note that the denominator and the numerator in (2) are independent. This is due to the independence assumption between the channel vectors for the desired and the interfering users. This independence property simplifies the ensuing outage probability analyzes.

Letting $X \triangleq \sum_{j=1}^{M}\alpha_{s,j}$ and $U \triangleq \sum_{i=1}^{L}(\mu_i/\Lambda_i)$ in (2), the output SIR of EGC can be rewritten as $\mathrm{SIR} = (X^2/(1-(\rho/4))U)$. The outage probability, which is defined as the probability that the instantaneous SIR is less than a certain outage threshold β, is expressed as

$$\begin{aligned}P_{\mathrm{OUT}}(\beta) =& \Pr(\mathrm{SIR} < \beta)\\ =& E_U\left\{\Pr\left(X < \sqrt{\beta_1 U}\,\middle|\, U\right)\right\}\\ =& E_U\left\{\Pr\left(X < \sqrt{\beta_1 U}\right)\right\}\end{aligned} \quad (3)$$

where $\beta_1 = (1-(\rho/4))\beta$ and the last equality comes from the fact that X and U are independent.

Computation of the outage probability in (3) requires the knowledge of the cumulative distribution function (cdf) of X. We recall that X is a sum of M i.i.d. Rayleigh RVs and no known closed-form expression exists except for $M = 2$. In [7], Beaulieu derived an infinite series for the cdf of a sum of independent RVs, and in [8], an alternative derivation was given which provided insights into the uses and limitations of the Beaulieu series. We write the conditional outage probability in (3) as [8]

$$\Pr\left(X < \sqrt{\beta_1 U}\right) = \frac{1}{2} - \sum_{\substack{n=1 \\ n\text{odd}}}^{+\infty} \frac{2\,\Im\left\{e^{-jn\omega_0\sqrt{\beta_1 U}}\;\phi_X(n\omega_0)\right\}}{n\pi} + \Delta$$

where $\omega_0 = (2\pi/T)$, T is a parameter that controls the accuracy [7], $\phi_X(\omega)$ is the characteristic function of X, $\Im(z)$ denotes the imaginary part of z, and Δ is an error term which tends to zero for large T. Assuming T is large, we omit the error term Δ in the following analysis. It can be shown that the characteristic function of X is [5, eq. (2–1-133)][1]

$$\phi_X(\omega) = \left[{}_1F_1\left(1; \frac{1}{2}; -\frac{\omega^2}{4}\right) + j\frac{\sqrt{\pi}}{2}\omega e^{-\omega^2/4}\right]^M \qquad (4)$$

where ${}_1F_1(\cdot;\cdot;\cdot)$ is the degenerate hypergeometric function. The outage probability in (3) now can be expressed as

$$P_{\text{OUT}}(\beta) = \frac{1}{2} - \sum_{\substack{n=1 \\ n\text{odd}}}^{+\infty} \frac{2\,\Im\left\{E_U\left\{e^{-jn\omega_0\sqrt{\beta_1 U}}\right\}\;\phi_X(n\omega_0)\right\}}{n\pi}. \qquad (5)$$

We recall that U is a weighted sum of L i.i.d. exponential RVs. The probability density function (pdf) of U, in the case of equal interferer powers, $\Lambda_1 = \cdots = \Lambda_L = \Lambda$, is given by [5, eq. (14–4-13)]

$$f_U(u) = \frac{1}{(L-1)!\,\left(\frac{M}{\Lambda}\right)^L} u^{L-1} e^{-(\Lambda/M)u}, \quad u \geq 0 \qquad (6a)$$

and in the case of distinct interferer powers, $\Lambda_i \neq \Lambda_j$ for $i \neq j$, is given by [5, eq. (14–5-26)]

$$f_U(u) = \sum_{k=1}^{L} \frac{\Lambda_k}{M}\pi_k e^{-(\Lambda_k/M)u}, \quad u \geq 0 \qquad (6b)$$

where $\pi_k = \prod_{\substack{i=1 \\ i\neq k}}^{L} \Lambda_i/(\Lambda_i - \Lambda_k)$.

[1]In [5, eq. (2–1-133)], a minor typo needs to be corrected, i.e., $j\sqrt{\pi/2}v\sigma^2 e^{-v^2\sigma^2/2}$ should be $j\sqrt{\pi/2}v\sigma e^{-v^2\sigma^2/2}$.

By using [9, eq. 3.952(7), (8)], the case of equal interferer powers can be shown in (7a), at the bottom of the page, and, in the case of distinct interferer powers

$$\begin{aligned} E_U\left\{e^{-jn\omega_0\sqrt{\beta_1 U}}\right\} &= \sum_{k=1}^{L} \pi_k e^{-(n^2\omega_0^2/4\Lambda_k)\beta_1 M}\left\{{}_1F_1\left(-\frac{1}{2}; \frac{1}{2}; \frac{n^2\omega_0^2}{4\Lambda_k}\beta_1 M\right) - j\frac{\sqrt{\pi}}{2}n\omega_0\sqrt{\frac{\beta_1 M}{\Lambda_k}}\right\} \\ &= \sum_{k=1}^{L} \pi_k e^{-(n^2\omega_0^2/8\Lambda_k)\beta_1 M} D_{-2}\left(jn\omega_0\sqrt{\frac{\beta_1 M}{2\Lambda_k}}\right) \end{aligned} \qquad (7b)$$

where $D_p(z)$ is the parabolic cylinder function [9]. Substitution of (4) and (7) into (5) yields the outage probability of EGC for both equal and distinct interferer powers.

The exact outage probability can be derived for $M = 2$. The cdf of a sum of two i.i.d Rayleigh RVs is known [10], and the conditional outage probability for $M = 2$ is

$$\Pr\left(X < \sqrt{\beta_1 U}\right) = 1 - e^{-\beta_1 U} - \sqrt{\frac{\pi}{2}\beta_1 U} \times e^{-(1/2)\beta_1 U}\left[1 - \text{erfc}\left(\sqrt{\frac{\beta_1 U}{2}}\right)\right]. \qquad (8)$$

Averaging (8) with respect to (6a) or (6b) gives the outage probability. In the case of equal interferer powers, using [9, eq. 3.478(1), 6.286(1)]

$$\begin{aligned} P_{\text{OUT}}(\beta) = &1 - \left(\frac{\Lambda}{2\beta_1 + \Lambda}\right)^L - \frac{\Gamma\left(L + \frac{1}{2}\right)}{\Gamma(L)} \\ &\times \sqrt{\frac{\pi\beta_1}{\beta_1 + \Lambda}}\left(\frac{\Lambda}{\beta_1 + \Lambda}\right)^L + \frac{2L}{2L+1}\left(\frac{\Lambda}{\beta_1}\right)^L \\ &\times {}_2F_1\left(L + \frac{1}{2}, L+1; L + \frac{3}{2}; -\frac{\Lambda + \beta_1}{\beta_1}\right) \end{aligned}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the hypergeometric function [9], and in the case of distinct interferer powers, using [9, eq. 3.478(1), 6.292], is shown in the equation, at the bottom of the next page.

IV. Coherent and Incoherent Interference Calculation: Case of $L = 1$

We compare the two calculation methods analytically for the case of one interferer. With the incoherent interference power

$$\begin{aligned} E_U\left\{e^{-jn\omega_0\sqrt{\beta_1 U}}\right\} =&e^{-(n^2\omega_0^2/4\Lambda)\beta_1 M}\left\{{}_1F_1\left(\frac{1}{2} - L; \frac{1}{2}; \frac{n^2\omega_0^2}{4\Lambda}\beta_1 M\right) - j\frac{\Gamma\left(L + \frac{1}{2}\right)}{\Gamma(L)}n\omega_0\sqrt{\frac{\beta_1 M}{\Lambda}}\,{}_1F_1\left(1 - L; \frac{3}{2}; \frac{n^2\omega_0^2}{4\Lambda}\beta_1 M\right)\right\} \\ =&\frac{2^L}{\sqrt{\pi}}\,\Gamma\left(L + \frac{1}{2}\right)e^{-(n^2\omega_0^2/8\Lambda)\beta_1 M} D_{-2L}\left(jn\omega_0\sqrt{\frac{\beta_1 M}{2\Lambda}}\right) \end{aligned} \qquad (7a)$$

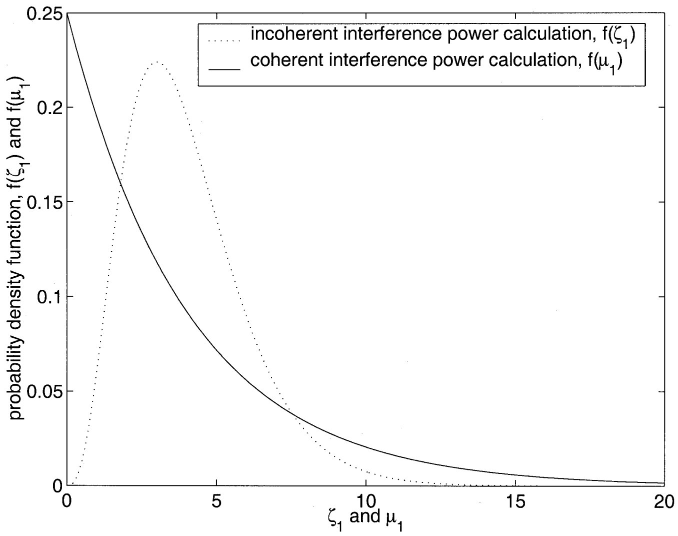

Fig. 1. Pdfs of μ_1 and ζ_1 for $L = 1$ interferer and $M = 4$ antennas.

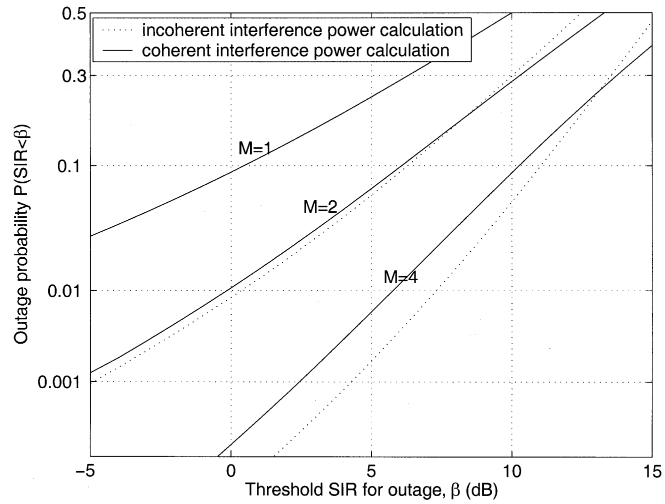

Fig. 2. Outage probability comparison of coherent and incoherent interference power calculation with one interferer ($L = 1$) and equal interferer powers ($\Lambda = 10$ dB) for $M = 1,\ 2,$ and 4 antennas.

calculation, the SIR at the EGC output becomes

$$\mathrm{SIR}_{\text{incoherent}} = \frac{\frac{\Lambda_1}{1-\frac{\rho}{4}}\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2}{\sum_{j=1}^{M}\alpha_{s,j}^2} = \frac{\kappa}{\zeta_1} \tag{9}$$

where we have denoted $\Lambda_1/(1-(\rho/4))\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2$ by κ and $\sum_{j=1}^{M}\alpha_{s,j}^2$ by ζ_1 in (9). It can be shown straightforwardly that ζ_1 has a chi-square distribution with $2M$ degrees of freedom.

With a coherent interference power calculation, (2) becomes

$$\mathrm{SIR}_{\text{coherent}} = \frac{\frac{\Lambda_1}{1-\frac{\rho}{4}}\left(\sum_{j=1}^{M}\alpha_{s,j}\right)^2}{\mu_1} = \frac{\kappa}{\mu_1} \tag{10}$$

where μ_1 is exponentially distributed with mean M. We note that the *numerators* in (9) and (10) are identical. For a given outage threshold β, the outage probabilities for incoherent and coherent interference power calculation are, respectively, given by $\Pr(\zeta_1 > (\kappa/\beta))$ and $\Pr(\mu_1 > (\kappa/\beta))$. For practical applications, the low outage probability region is of interest, i.e., small values of β, and it is sufficient to compare the tail probabilities of ζ_1 and μ_1. Fig. 1 plots the pdfs of ζ_1 (incoherent calculation method) and μ_1 (coherent method) for four antennas and one interferer. By comparing tails, it is clear that in the low outage probability region, the coherent interference power calculation yields the higher outage probability. Our results in Section V also agree with this analysis.

V. Numerical Results

For the coherent interference combining model, the outage probabilities in (5) are used with T in a range of 40–80. It was found that typically 64 or 128 terms in the series of (5) enable an accuracy of 10^{-8} to be achieved. For the incoherent interference power calculation, the outage probabilities are obtained from [3, eq. (16)] by specializing the Nakagami-m fading to Rayleigh fading.[2] Unless otherwise stated, a coherent interference power calculation is assumed and the rolloff factor is chosen as $\rho = 0$. All outage probabilities are plotted on normal probability paper by the method in [11, Appendix 2B].

Figs. 2 and 3 compare outage probabilities using the incoherent and coherent interference power calculations. Fig. 2 plots the outage probabilities for $L = 1$ interferer and for $M = 1,\ 2,$ and 4 antennas under equal interferer powers. As shown, for the trivial case of $M = 1$, as expected, both calculation methods give the same outage probability. When $M > 1$ antennas and for low outage probabilities, the coherent interference power calculation method predicts higher outage probability. In other words, an outage analysis using incoherent interference power combining over the diversity branches can underestimate the outage probability. For example, for $M = 4$, at a 0.01 outage probability level, the incoherent interference power calculation overestimates the output SIR by about 1.5 dB. Similar observations can be made in Fig. 3 for a fixed number of antennas ($M = 4$) and different numbers of interferers, where we note that the outage performance difference increases between these two interference power calculation methods as the number of interferers decreases.

[2]In [3], a minor typo in (16a) needs to be corrected, i.e., A_n should be A_n^L.

$$P_{\text{OUT}}(\beta) = \sum_{k=1}^{L}\frac{\Lambda_k}{2}\pi_k\left[\frac{2\beta_1}{\Lambda_k(\Lambda_k+\beta_1)} - \pi\sqrt{\beta_1}(\Lambda_k+\beta_1)^{-3/2} + 2\sqrt{\beta_1}(\Lambda_k+\beta_1)^{-3/2}\arctan\left(\sqrt{1+\frac{\Lambda_k}{\beta_1}}\right)\right]$$

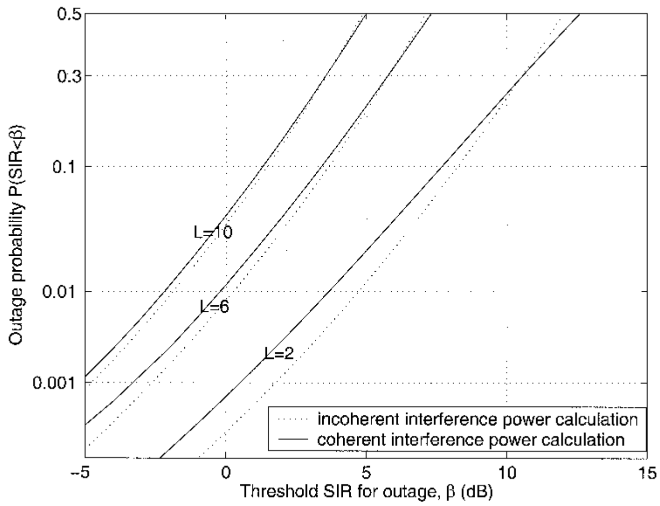

Fig. 3. Outage probability comparison of coherent and incoherent interference power calculation with four antennas ($M = 4$) and equal interferer powers ($\Lambda = 10$ dB) for $L = 2,\ 6,$ and 10 interferers.

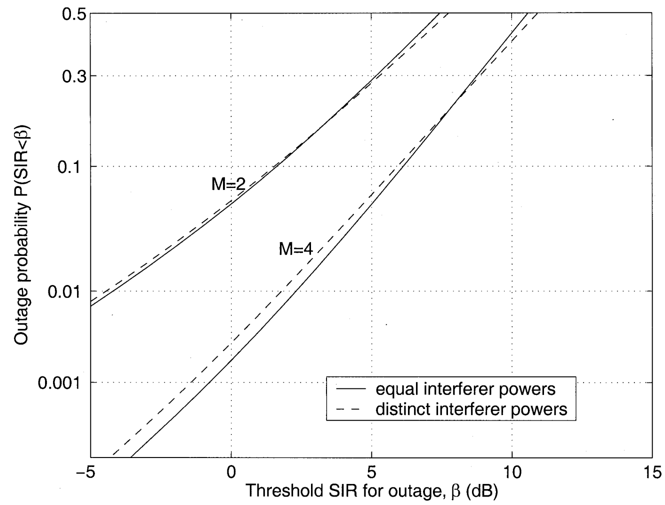

Fig. 5. Outage probability for equal ($\Lambda = 10$ dB) and distinct ($\Lambda_{\text{avg}} = 10$ dB) interference power distributions with $L = 3$ interferers, and $M = 2$ and $M = 4$ antennas. The interference power vector for $L = 3$ is [0.1,0.2,0.7].

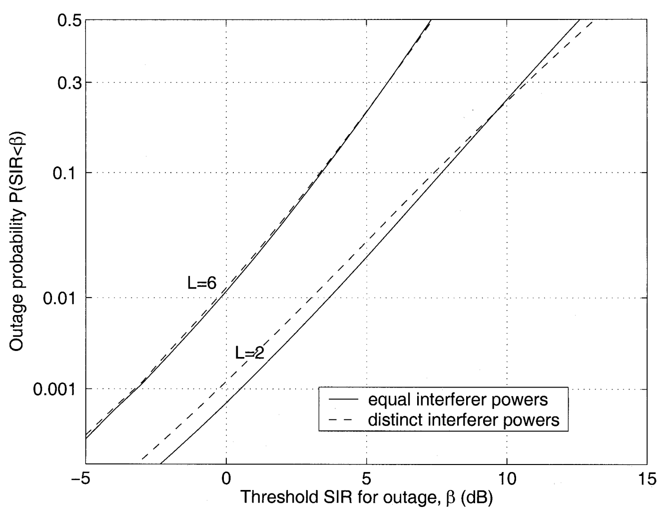

Fig. 4. Outage probability for equal ($\Lambda = 10$ dB) and distinct ($\Lambda_{\text{avg}} = 10$ dB) interference power distributions with four antennas ($M = 4$), and $L = 2$ and $L = 6$ interferers. The interference power vectors for $L = 2$ and $L = 6$ are, respectively, [0.1,0.9] and [0.05,0.1,0.15,0.22,0.23,0.25].

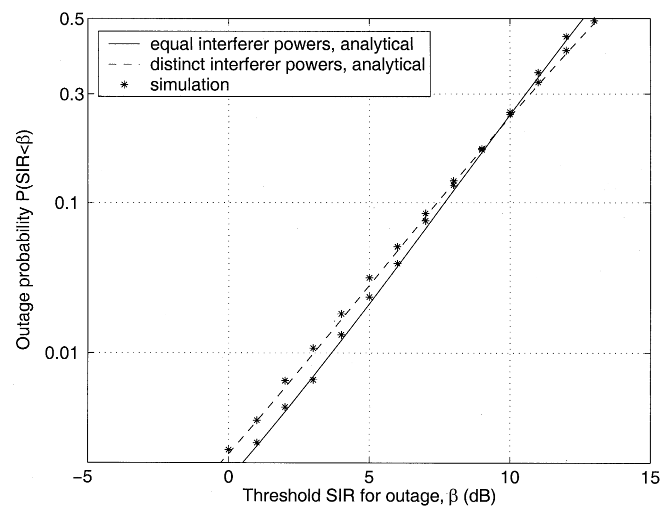

Fig. 6. Analytical outage probability using coherent interference power calculation and Monte Carlo simulated outage probability for equal ($\Lambda = 10$ dB) and distinct ($\Lambda_{\text{avg}} = 10$ dB) interference power distributions with $L = 2$ interferers and $M = 4$ antennas. The interference power vector for $L = 2$ is [0.1,0.9].

To study outage for unequal interferer power distributions, we define

$$\Lambda_{\text{avg}}(\text{dB}) = 10\log_{10}\frac{P_s}{\frac{1}{L}\sum_{i=1}^{L}P_i}$$

and vector $q = [q_1, q_2, \ldots, q_L]$ of normalized interference powers, where $\sum_{i=1}^{L} q_i = 1$. Power ratio

$$\Lambda_i(\text{dB}) = \frac{P_s}{P_i}(\text{dB}) = \Lambda_{\text{avg}}(\text{dB}) - 10\log_{10}(Lq_i).$$

Assuming $M = 4$ antennas, Fig. 4 depicts how unequal power among interferers increases outage probability for two cases: $L = 2$ interferers with a highly unbalanced interference power vector $q = [0.1, 0.9]$, and for $L = 6$ interferers with a more evenly distributed $q = [0.05, 0.1, 0.15, 0.22, 0.23, 0.25]$. It is obvious that the outage probability difference between equal and distinct interferer powers becomes large for a highly unbalanced interference power distribution. In Fig. 5, it is shown that outage probability discrepancies between equal and distinct interferer power distribution become larger as the number of antennas increases. Both Figs. 4 and 5 suggest that the case of equal interferer powers tends to give lower outage probabilities than the case of distinct interferer powers.

In Fig. 6, we compare a Monte Carlo simulation of the outage probability using (2) with the analytical outage probability using the new coherent interference power calculation method for the cases of both equal and distinct interferer powers. As shown, the analysis and simulation results agree closely.

VI. Conclusion

In this letter, we have computed the exact outage probability for EGC with multiple cochannel interferers in an interference-limited environment over flat Rayleigh fading. The new analysis method assumes that the interferers add in amplitude and phase, rather than incoherently. Numerical results show that the previous incoherent interference power combination model may lead to optimistic outage probability estimates.

ACKNOWLEDGMENT

The authors wish to thank Prof. N. C. Beaulieu from the University of Alberta for sharing his insights on Beaulieu's series and the anonymous reviewers for their useful suggestions.

REFERENCES

[1] G. L. Stüber, *Principle of Mobile Communication*. Norwell, MA: Kluwer, 1996.

[2] D. G. Brennan, "Linear diversity combining techniques," *Proc. IRE*, vol. 47, pp. 1075–1102, June 1959.

[3] A. A. Abu-Dayya and N. C. Beaulieu, "Outage probabilities of diversity cellular systems with cochannel interference in Nakagami fading," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 343–355, Nov. 1992.

[4] A. Shah and A. M. Haimovich, "Performance analysis of maximal ratio combining and comparison with optimum combining for mobile radio communications with cochannel interference," *IEEE Trans. Veh. Technol.*, vol. 49, pp. 1454–1463, July 2000.

[5] J. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

[6] N. C. Beaulieu and A. A. Abu-Dayya, "Bandwidth efficient QPSK in cochannel interference and fading," *IEEE Trans. Commun.*, vol. 43, pp. 2464–2474, Sept. 1995.

[7] N. C. Beaulieu, "An infinite series for the computation of the complementary probability distribution function of a sum of independent random variables and its application to the sum of Rayleigh random variables," *IEEE Trans. Commun.*, vol. 38, pp. 1463–1474, Sept. 1990.

[8] C. Tellambura and A. Annamalai, "Further results on the Beaulieu series," *IEEE Trans. Commun.*, vol. 48, pp. 1774–1777, Nov. 2000.

[9] I. Gradshteyn and I. Ryzhik, *Tables of Integrals, Series, and Products*, 5th ed. San Diego, CA: Academic, 1994.

[10] S. W. Halpern, "The effect of having unequal branch gains in practical predetection diversity systems for mobile radio," *IEEE Trans. Veh. Technol.*, vol. VT-26, pp. 94–105, Feb. 1977.

[11] S. M. Kay, *Fundamental Statistical Signal Processing: Detection Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1998.