High-Rate Diversity Across Time and Frequency Using Linear Dispersion

Jinsong Wu, *Member, IEEE*, and Steven D. Blostein, *Senior Member, IEEE*

Abstract—To improve performance of orthogonal frequency division multiplexing (OFDM) for fading channels, this paper proposes increasing frequency and time diversity using linear dispersion codes (LDC-OFDM). Methods of LDC-OFDM processing are proposed for both zero-padding (ZP) and cyclic-prefix (CP) type guard intervals. A two-step-estimation (TSE) decoding strategy is proposed that decouples symbol estimation from LDC decoding. This paper analyzes the upper bound diversity order of LDC-CP-OFDM, which is equal to the full diversity order available in the channels. A criterion for full frequency-time diversity design is derived, a rate-one code is provided and performance is examined through simulations. This paper also investigates LDC-CP-OFDM and LDC-ZP-OFDM performance under imperfect channel estimation and low complexity receiver structures, respectively. In addition, TSE is shown to have performance close to that of full complexity one-step estimation (OSE).

Index Terms—Linear dispersion codes, OFDM, diversity, COFDM, equalization, signal estimation, MMSE, SINR.

I. INTRODUCTION

IN recent years, multicarrier communications systems, especially those employing orthogonal frequency division multiplexing (OFDM) [1], have received increasing attention for high-data-rate communications in frequency selective fading environments [2]. In practical OFDM system design, it is important to notice that uncoded OFDM cannot provide the same order of diversity as uncoded single-carrier systems in severe frequency-selective fading environments, since the frequency responses of channel space branches differ from one another. One technique to mitigate the above problem is to combine interleaving and forward error correction across all subchannels at the price of reduced bandwidth efficiency, i.e., coded OFDM (COFDM) [3]–[5].

Coding rate is a critical issue related to bandwidth efficiency for high-data-rate transmission. In conventional COFDM, the coding rate is typically less than one, and achieving appropriate trade-offs between coding rate and error probability is critical to the system design. As a recent alternative to error control coding, linear constellation precoding has been combined with OFDM to maximize achievable frequency diversity and coding gain [6]. However, LCP-OFDM is not

able to exploit time diversity over different OFDM blocks in the channels.

Recently, Hassibi and Hochwald proposed a high-rate space-time coding framework, known as linear dispersion codes (LDC) [7], which can support arbitrary configurations of transmit and receive antennas. These LDC are designed to optimize the mutual information between the transmitted and received signals. This paper proposes and analyzes a new high-rate LDC approach to *jointly achieve both frequency diversity and time diversity* (LDC-OFDM).

The paper is organized as follows: LDC is defined in Section II. In Section III, the construction of an LDC-OFDM block is proposed. The proposed TSE based LDC-OFDM system is discussed in Section IV, and the proposed receiver structure is illustrated. An analytical discussion of diversity properties of LDC-OFDM is given in Section V, and then a rate-one full diversity LDC-OFDM design is provided. Performance analysis and comparison is presented in Section VI.

The following notation is used:$(\cdot)^{\dagger}$ denotes matrix pseudoinverse, $(\cdot)^{\mathcal{T}}$ matrix transpose, $(\cdot)^{\mathcal{H}}$ matrix transpose conjugate, $\mathbf{I}_K$ denotes identity matrix of size $K \times K$, $\mathbf{0}_{m \times n}$ denotes zero matrix of size $m \times n$, $\mathbf{A} \otimes \mathbf{B}$ denotes Kronecker (tensor) product of matrices $\mathbf{A}$ and $\mathbf{B}$, $C^{m \times n}$ denotes a complex matrix with dimensions $m \times n$, and $\mathbf{F}_M$ denotes the discrete Fourier transform (DFT) matrix, representing the M-point fast Fourier transform (FFT) with entries, $[\mathbf{F}_M]_{a,b} = \left(1/\sqrt{M}\right) \exp\left(-j2\pi(a-1)(b-1)/M\right)$.

II. LINEAR DISPERSION CODING AND ITS MATRIX FORM

A. Definition of Linear dispersion codes

Assume the data sequence has been modulated using complex-valued symbols chosen from an arbitrary, e.g. r-PSK or r-QAM, constellation. A linear dispersion code (LDC), $\mathbf{S}_{LDC}$, was first defined for multi-input, multi-output (MIMO) systems with M transmit antennas, N receive antennas, T channel uses and Q source constellation symbols as [7]

$$\mathbf{S}_{LDC} = \sum_{q=1}^{Q} \alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q \qquad (1)$$

where the LDC matrix is $\mathbf{S}_{LDC} \in C^{T \times M}$, $\mathbf{A}_q \in C^{T \times M}, \mathbf{B}_q \in C^{T \times M}, q = 1, ..., Q$ are called dispersion matrices, which transform data symbols into a space-time matrix. The constellation symbols are defined by $s_q = \alpha_q + j\beta_q, q = 1, ..., Q$.

This paper applies LDC to multicarrier systems, and the data symbol coding rate of LDC in such systems is defined as $R_{LDC}^{sym} = \frac{Q}{MT}$.
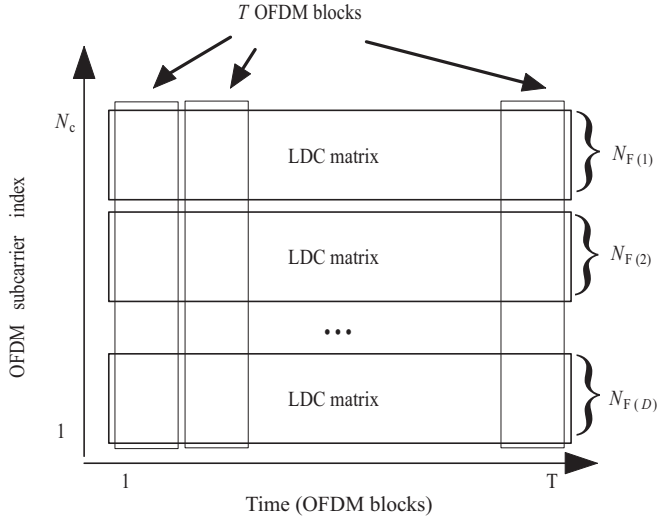
Fig. 1. A LDC-OFDM block in the time-frequency plane.

B. A subclass of LDC and corresponding matrix form

Without loss of generality, we consider a special subclass of dispersion matrices with the constraints

$$\mathbf{A}_q = \mathbf{B}_q, q = 1, ..., Q. \qquad (2)$$

Define the vec operation on $m \times n$ matrix $\mathbf{X}$ as

$$vec(\mathbf{X}) = \left[\left[[\mathbf{X}]_{:,1,} \right]^{\mathcal{T}}, ..., \left[[\mathbf{X}]_{:,n} \right]^{\mathcal{T}} \right]^{\mathcal{T}} \qquad (3)$$

where $[\mathbf{X}]_{:,i}$ is the ith column of $\mathbf{X}$.

Using (2) and (3), we transform (1) into

$$vec(\mathbf{S}_{LDC}) = \mathbf{G}_{LDC}\mathbf{s}, \qquad (4)$$

where

$$\mathbf{G}_{LDC} = [vec(\mathbf{A}_1), ..., vec(\mathbf{A}_Q)] \qquad (5)$$

and $\mathbf{s} = [s_1, ..., s_Q]^{\mathcal{T}}$. For general dispersion matrices, the following development holds but the definition of $\mathbf{G}_{LDC}$ changes.

III. PROPOSED LDC-OFDM BLOCK CONSTRUCTION

Let there be N_C subcarriers in one OFDM block. One LDC-OFDM block, illustrated in Figure 1, consists of T adjacent OFDM blocks. An LDC-OFDM system includes D LDC codewords, each with LDC matrices occupying $N_{F(i)}$ subcarriers and T OFDM blocks $\in C^{T \times N_{F(i)}}, i = 1, ..., D$, with $\sum_{i=1}^{D} N_{F(i)} = N_C$. In OFDM systems, since the number of subcarriers is typically much larger than the number of antennas in space-time MIMO systems, the proposed LDC-OFDM system allows for freedom to choose larger dispersion matrices as well as exploits low correlation across OFDM subcarriers.

One LDC-OFDM block is organized into the matrix $\mathbf{S}_{LDC-OFDM}$ of size $N_C \times T$, $\mathbf{S}_{LDC-OFDM} = \left[\mathbf{s}_{OFDM}^{(1)}, ..., \mathbf{s}_{OFDM}^{(T)} \right]$, where $\mathbf{s}_{OFDM}^{(k)}$ is the k-th OFDM block symbol vector of size $1 \times N_C$, and represents the transmitted complex symbol vector before inverse Fourier transformation (IFFT) in the transmitter for the kth OFDM transmitted block. Elements $\mathbf{s}_{OFDM}^{(k)}$ consist of all the D row vectors $\mathbf{S}_{LDC(k,.)}^{(i)}, i = 1, ..., D$, where $\mathbf{S}_{LDC(k,.)}^{(i)} \in C^{1 \times N_{F(i)}}$ is the k-th row of the i-th LDC matrix codeword $\mathbf{S}_{LDC}^{(i)}$ in a single LDC-OFDM block. While $S_{LDC(k,.)}^{(i)}$ occupies $N_{F(i)}$ subcarriers, it is not necessary that these subcarriers be spectrally adjacent.

IV. TSE BASED LDC-OFDM

A. One step estimation

Previously, LDC has been proposed for MIMO systems [7], [8]. Here, the receiver combines LDC decoding and symbol estimation, which we denote as one-step-estimation (OSE). Mathematically, an OSE system may be formulated as

$$\mathbf{y} = \underline{\mathbf{H}}\mathbf{s} + \mathbf{v}, \qquad (6)$$

where $\mathbf{y}$, $\mathbf{s}$, and $\mathbf{v}$ are the received signal vector, source data symbol vector, and additive noise vector, respectively. The equivalent channel matrix, $\underline{\mathbf{H}}$, is a function of dispersion matrices and fading channel matrices for all channel uses. To decode a single system block, it is necessary to calculate the equivalent channel matrix $\underline{\mathbf{H}}$, which is typically large in size and results in high complexity, particularly for the case where an LDC block of symbols includes multiple LDC codewords. Current LDC decoding approaches use OSE and non-linear maximum-likelihood or sub-optimal sphere decoding [7], [8] and assume constant channel coefficients over the duration of one LDC codeword. Besides complexity, the OSE approach requires channel estimation that is a function of both fading gains and LDC coefficients to be decoupled and estimated, which is difficult to acheive.

B. Two step estimation and its necessary condition for LDC decoding

To simplify channel estimation and maintain diversity at reduced complexity, a two-step estimation (TSE) procedure is proposed for LDC-OFDM, permitting channel coefficients to change per OFDM block instead of per T OFDM blocks. This enables LDC decoding to be independent of the specific equalizers used, and in turn, enables wide applicability for enhancing different standards. One possible zero-forcing method to estimate the data symbol vector in (4) is via the Moore-Penrose pseudo-inverse of LDC encoding matrix $\mathbf{G}_{LDC}$, which is calculated and stored offline.

To remove dependence of LDC decoding on symbol estimation, LDC designs need to meet the following:

Correlation criterion: Denote the correlation matrix of $vec\left([\mathbf{S}_{LDC}]^{\mathcal{T}} \right)$ as $\mathbf{R}_{vec([\mathbf{S}_{LDC}]^{\mathcal{T}})}$. For the case that LD-coded symbols per channel use or per row of $\mathbf{S}_{LDC}$ are block-wise estimated, $\mathbf{S}_{LDC}$ needs to be row-wise uncorrelated. In other words, $\mathbf{R}_{vec([\mathbf{S}_{LDC}]^{\mathcal{T}})}$ needs to have the block diagonal form

$$\mathbf{R}_{vec([\mathbf{S}_{LDC}]^{\mathcal{T}})} = \begin{bmatrix} \mathbf{R}_{\mathbf{S}_{LDC(1,.)}} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{R}_{\mathbf{S}_{LDC(T,.)}} \end{bmatrix} \qquad (7)$$

where $\mathbf{R}_{\mathbf{S}_{LDC(k,\cdot)}} \in C^{M \times M}, k = 1..., T$ is the correlation matrix of the k-th row vector of $\mathbf{S}_{LDC}$, and $\mathbf{0}$s are $M \times M$ zero matrices. For the case that LD-coded symbols are estimated per element of $\mathbf{S}_{LDC}$, $\mathbf{S}_{LDC}$ needs to be element-wise uncorrelated. In other words, $\mathbf{R}_{vec([\mathbf{S}_{LDC}]^{\mathcal{T}})}$, needs to be diagonal, and more restrictive constraints are applied. The two steps are:

1) *Signal estimation per channel use:*
   Signals in each of T channel uses are estimated. No immediate signal detection is performed. (In different channel uses, channel matrices may be different);

2) *Data symbol estimation and detection per LDC block:*
   The data symbols corresponding to one LDC codeword are estimated. (In this step, channel knowledge is not required). Bit detection is then performed.

Unlike other estimation methods, the same core matrix-vector TSE processing may operate on different signal dimensions with different sized symbol blocks. The per-data-symbol complexity of encoding and decoding is constant and proportional to the LDC data symbol coding rate.

C. TSE based LDC-OFDM system

1) Wideband OFDM model: During transmission, for the k-th block of N_C IFFT transformed complex symbols, a block of P symbols (an OFDM block including guard interval) undergoes order L frequency selective, temporally flat Rayleigh fading with channel coefficients $\mathbf{h}^{(k)} = \left[h_0^{(k)}, ..., h_L^{(k)} \right]^{\mathcal{T}}$. Choosing $P \geq N_C + L$, the inter-block interference due to the previously transmitted block is eliminated by a guard interval of size $(P - N_C)$.

Denote $s_{OFDM(p)}^{(k)}, p = 1..., N_C$ as the LD-coded symbol transmitted on the p-th subcarrier during the k-th OFDM block. The receiver experiences additive complex Gaussian noise. Before transmission, a guard interval (e.g., cyclic prefix (CP)) is added to each OFDM block. After FFT processing, the received symbol is

$$x_p^{(k)} = \sqrt{\rho} H_p^{(k)} s_{OFDM(p)}^{(k)} + v_p^{(k)}, p = 1, ..., N_c \quad (8)$$

where $H_p^{(k)}$ is the p-th subcarrier channel gain during the k-th OFDM block, and
$H_p^{(k)} = \sum_{l=o}^{L} h_l^{(k)} e^{-j(2\pi/N_c)l(p-1)}$, or $H_p^{(k)} = [\mathbf{w}_p]^{\mathcal{T}} \mathbf{h}^{(k)}$,
where $\mathbf{w}_p = \left[1, \omega^{p-1}, \omega^{2(p-1)}, \cdots, \omega^{L(p-1)} \right]^{\mathcal{T}}$ and $\omega = e^{-j(2\pi/N_c)}$. The additive noise is circularly symmetric, zero-mean, complex Gaussian with variance N_0. It is assumed that the additive noise is statistically independent for different k, and ρ is the normalized signal to noise ratio (SNR).

The CP-OFDM system may also be written in block matrix form,

$$\mathbf{x}^{(k)} = \sqrt{\rho} \mathbf{D}_{\mathbf{H}}^{(k)} \mathbf{s}_{OFDM}^{(k)} + \mathbf{v}^{(k)} \quad (9)$$

where $\mathbf{x}^{(k)}$ and $\mathbf{v}^{(k)}$ are the frequency domain received signal and noise vectors, respectively, $\mathbf{D}_{\mathbf{H}}^{(k)} = \mathbf{F}_{N_C} \mathbf{H}^{(k)} [\mathbf{F}_{N_C}]^{\tilde{\mathcal{H}}} = diag(H_1^{(k)}, ..., H_{N_C}^{(k)})$, where $\left[\mathbf{H}^{(k)} \right]_{m,n} = h_{((m-n) \bmod N_C)}^{(k)}$.

When zero-padding (ZP) is used as the OFDM guard interval, orthogonality is destroyed, and the system model does
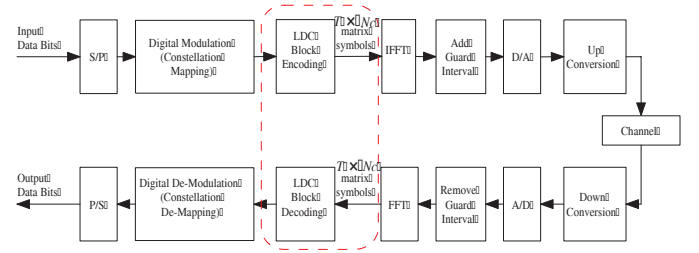

Fig. 2.   Proposed TSE based LDC-OFDM system. The dashed lines contain the enhancement over a conventional OFDM system.

not have the simple form shown in (8). However, the ZP-OFDM system model can be expressed in block matrix form in the time domain,

$$\mathbf{x}_{ZP\_OFDM}^{(k)} = \sqrt{\rho} \mathbf{H}_0^{(k)} [\mathbf{F}_{N_C}]^H \mathbf{s}_{OFDM}^{(k)} + \mathbf{v}_{ZP\_OFDM}^{(k)}, \quad (10)$$

with the k-th received ZP-OFDM block $\mathbf{x}_{ZP\_OFDM}^{(k)} \in C^{P \times 1}$, and the frequency selective channel matrix $\mathbf{H}_0^{(k)} \in C^{P \times N_C}$ corresponding to the k-th OFDM block. The Toeplitz channel matrix $\mathbf{H}_0^{(k)}$ is guaranteed to be invertible, regardless of the channel zero locations [9]. Zero-mean white additive complex Gaussian noise vector is represented by $\mathbf{v}_{ZP\_OFDM}^{(k)}$.

2) TSE based LDC-OFDM system: The proposed TSE LDC decoding procedure in Section IV-B is applied to the wideband OFDM channel described above. The differences between conventional OFDM and the proposed TSE based LDC-OFDM are indicated by the dashed lines in Figure 2. Note that the block size used in the proposed system differs from that of the conventional OFDM system.

3) LDC-OFDM receiver: The receiver for LDC-OFDM, illustrated in Figure 3, first estimates the signals in T OFDM blocks. Second, the estimated $S_{LDC-OFDM-block}$ is reorganized into D LDC blocks. The D LDC demodulators operate in parallel, followed by data bit detection. Denote the LDC encoding matrix of the i-th LDC matrix codeword $\mathbf{S}_{LDC}^{(i)} \in C^{T \times N_{F(i)}}$ as $\mathbf{G}_{LDC}^{(i)}$, which encodes source data symbol vector with zero mean, unit variance, $\mathbf{s}^{(i)} = \left[s_1^{(i)}, ..., s_{Q_i}^{(i)} \right]$, and Q_i is the number of source data symbols in $\mathbf{s}^{(i)}$. If $\mathbf{G}_{LDC}^{(i)} = \mathbf{G}_{LDC}, i = 1, ..., D$ are unitary matrices, the correlation matrices of $\mathbf{s}_{OFDM}^{(k)}, k = 1, ..., T$ are identity matrices. Note that in general, unitariness is not a necessary condition for $\mathbf{G}_{LDC}^{(i)}$.

Due to the independence of the two estimation steps, TSE LDC-OFDM systems possess a three-layered structure consisting of data, LDC, and OFDM layers, which provides flexibility and error protection for system design.

a) First estimation step - OFDM Demodulation: In the proposed TSE based LDC decoding strategy, LDC decoding is independent of OFDM signal estimation. Thus the proposed TSE based LDC-OFDM system is backwards-compatible with conventional OFDM systems.

In Section VI, minimum-mean-squared-error (MMSE) equalizers are chosen to investigate error performance. Assuming that OFDM symbols are normalized with unit variance
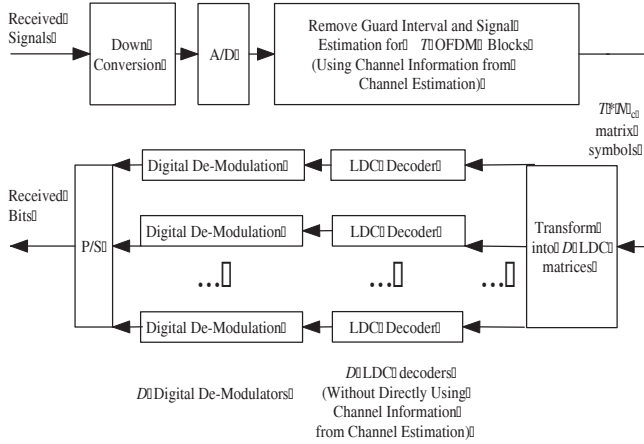
Fig. 3.   Proposed TSE based LDC-OFDM receiver structure.

and using (9) and (10), the respective equalizers are given by [9]

- case of CP-OFDM

$$\mathbf{G}_{OFDM\_CP\_MMSE}^{(k)} = \sqrt{\rho} \mathbf{C}_{\mathbf{s}_{OFDM}^{(k)}} \left(\mathbf{D}_{\mathbf{H}}^{(k)} \right)^{\mathcal{H}} \cdot$$
$$\left(\mathbf{I}_{N_C} + \rho \mathbf{D}_{\mathbf{H}}^{(k)} \mathbf{C}_{\mathbf{s}_{OFDM}^{(k)}} \left(\mathbf{D}_{\mathbf{H}}^{(k)} \right)^{\mathcal{H}} \right)^{-1} \quad (11)$$

and

$$\widehat{\mathbf{s}_{OFDM}^{(k)}} = \mathbf{G}_{OFDM\_CP\_MMSE}^{(k)} \mathbf{x}^{(k)}, \quad (12)$$

- case of ZP-OFDM

$$\mathbf{G}_{OFDM\_ZP\_MMSE}^{(k)} = \sqrt{\rho} \mathbf{F}_{N_C} \mathbf{C}_{\mathbf{s}_{OFDM}^{(k)}} \left(\mathbf{H}_0^{(k)} \right)^{\mathcal{H}} \cdot$$
$$\left(\mathbf{I}_P + \rho \mathbf{H}_0^{(k)} \left[\mathbf{F}_{N_C} \right]^{\mathcal{H}} \mathbf{C}_{\mathbf{s}_{OFDM}^{(k)}} \mathbf{F}_{N_C} \left(\mathbf{H}_0^{(k)} \right)^{\mathcal{H}} \right)^{-1} \quad (13)$$

and

$$\widehat{\mathbf{s}_{OFDM}^{(k)}} = \mathbf{G}_{OFDM\_ZP\_MMSE}^{(k)} \mathbf{x}_{ZP\_OFDM}^{(k)}, \quad (14)$$

where $k = 1, ..., T$, $\mathbf{C}_{\mathbf{s}_{OFDM}^{(k)}}$ is the covariance matrix of the k-th OFDM block symbols, $\mathbf{C}_{\mathbf{s}_{OFDM}^{(k)}}$ can be derived using $\mathbf{G}_{LDC}^{(i)}, i = 1, ..., D$.

It is easy to show that if $\mathbf{G}_{LDC}^{(i)}, i = 1, ..., D$ is unitary, (13) requires more computation than (11) when matrix dimensions are high, since $\mathbf{D}_{\mathbf{H}}^{(k)}$ is diagonal. Due to the layering in TSE based LDC-OFDM, complexity of ZP-OFDM may be reduced. An approximate solution can be obtained via [9]: Denote $\mathbf{F}_{ZP} = \begin{bmatrix} \mathbf{F}_{N_C} & \mathbf{0}_{(P-N_C) \times N_C} \end{bmatrix}^{\mathcal{H}}$, $\mathbf{U} = [\mathbf{F}_P \ \mathbf{F}_{ZP}]^{\dagger}$, and $\mathbf{D}_{\mathbf{H}_P^{(k)}} = \mathbf{F}_P C_P^{(\mathbf{h}^{(k)})} [\mathbf{F}_P]^{\mathcal{H}}$, where $C_P^{(\mathbf{h}^{(k)})}$ is a $P \times P$ circulant matrix, and $C_P^{(\mathbf{h}^{(k)})} = Circ(h_0^{(k)}, 0, ..., h_L^{(k)}, ..., h_1^{(k)})$. Then the low complexity MMSE ZP-OFDM equalizer corresponding to (13) is given as

$$\mathbf{G}_{OFDM\_ZP\_FAST\_MMSE}^{(k)} =$$
$$\mathbf{U} \left[\mathbf{D}_{\mathbf{H}_P^{(k)}} \right]^{\mathcal{H}} \left[\frac{P}{N_C} \mathbf{I}_P + \rho \mathbf{D}_{\mathbf{H}_P^{(k)}} \left[\mathbf{D}_{\mathbf{H}_P^{(k)}} \right]^{\mathcal{H}} \right]^{-1}. \quad (15)$$

Note that since $\mathbf{U}$ may be pre-computed and $\mathbf{D}_{\mathbf{H}_P^{(k)}}$ is diagonal, matrix inversion is simplified.

b) Second estimation step - LDC-OFDM Block Demodulation: Reorganizing the estimation results of the first estimation step into D estimated LDC matrix codewords, $\widehat{\mathbf{S}_{LDC}^{(i)}}, i = 1, ..., D$, the estimated data symbol vectors corresponding to D LDC blocks are

$$\widehat{\mathbf{s}^{(i)}} = \left[\mathbf{G}_{LDC}^{(i)} \right]^{\dagger} vec \left(\widehat{\mathbf{S}_{LDC}^{(i)}} \right). \quad (16)$$

D. Practical Issues

Since LDC encoding involves a complex transformation (containing both phase and amplitude rotation), LDC-OFDM systems may be more sensitive to channel estimation errors than conventional OFDM systems. The performance of LDC-OFDM under channel estimation error is discussed in Section VI-C2. Also, low peak-to-average power ratio (PAPR) is critical to OFDM systems. In [10], it is shown that LDC-OFDM and OFDM systems have similar PAPRs and LDC-OFDM systems may decrease BER without increasing PAPR. We remark that the unitary property of the chosen G_{LDC} is a possible explanation [10]. A promising method to reduce the PAPR in LDC-OFDM is to apply clustering in OFDM as suggested in [6], [11].

V. DIVERSITY ANALYSIS OF LDC-OFDM

The following diversity analysis procedure is adapted from a spectral decomposition approach originally applied to single-input single-output (SISO) communications systems with a linear model. The application of the resulting rank and product criteria is not new. For example, diversity analysis of frequency flat fading channels appears in [12], [13] and a diversity analysis of time selective fading channels appears in [14]. This method is adapted to analyze the diversity properties of joint time-varying frequency selective channels. As will be shown, the analysis leads to a general design criterion for full joint frequency and time diversity, and is applied to high-rate full diversity designs in such channels.

The orthogonality property of CP-OFDM makes the analysis of LDC-OFDM tractable. In the following, LDC-CP-OFDM is therefore assumed. Without loss of generality, we consider a single time-frequency (TF) block, i.e., we consider a single $T \times N_{F(i)}$ block $\mathbf{C}^{(i)}, i = 1, ..., D$ in a LDC-OFDM codeword. The block $\mathbf{C}^{(i)}$ is created after encoding all the i-th LDC codewords within a LDC-OFDM codeword. Denote subcarrier indices chosen for TF block $\mathbf{C}^{(i)}, i = 1, ..., D$ as $\{p_{n_{F(i)}}^{(k)}, n_{F(i)} = 1, ..., N_{F(i)}, i = 1, ..., D, k = 1, ..., T\}$. Denote the block components

$$\mathbf{C}^{(i)} = \begin{bmatrix} c_{p_{1_{(i)}}}^{(1)} & c_{p_{2_{(i)}}}^{(1)} & \cdots & c_{p_{N_{F(i)}}}^{(1)} \\ c_{p_{1_{(i)}}}^{(2)} & c_{p_{2_{(i)}}}^{(2)} & \cdots & c_{p_{N_{F(i)}}}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p_{1_{(i)}}}^{(T)} & c_{p_{2_{(i)}}}^{(T)} & \cdots & c_{p_{N_{F(i)}}}^{(T)} \end{bmatrix}.$$

The transmission of a generic LDC block $\mathbf{C}^{(i)}$ is expressed as

$$\mathbf{r}^{(i)} = \sqrt{\rho} \mathbf{M}^{(i)} \mathbf{H}^{(i)} + \mathbf{v}^{(i)}, \quad (17)$$

where received signal vector $\mathbf{r}^{(i)}$ and noise vector $\mathbf{v}^{(i)}$ are of size $N_{F(i)}T \times 1$, the i-th LDC symbol matrix

$$\mathbf{M}^{(i)} = diag(c_{p_{1_{(i)}}}^{(1)}, ..., c_{p_{N_{F(i)}}}^{(1)}, ..., c_{p_{1_{(i)}}}^{(T)}, ..., c_{p_{N_{F(i)}}}^{(T)}), \quad (18)$$

is of size $N_{F(i)}T \times N_{F(i)}T$, $c_{p_{n_{F(i)}}}^{(k)}$ is the channel symbol of the k-th OFDM block, $p_{n_{F}}$-th subcarrier, and i-th LDC codeword, $n_{F(i)} = 1, ..., N_{F(i)}$, and $i = 1, ..., D$. The channel

$$\mathbf{H}^{(i)} = \begin{bmatrix} H_{p_{1_{(i)}}}^{(1)}, H_{p_{2_{(i)}}}^{(1)}, ..., H_{p_{N_{F(i)}}}^{(1)}, \\ ..., H_{p_{1_{(i)}}}^{(T)}, H_{p_{2_{(i)}}}^{(T)}, ..., H_{p_{N_{F(i)}}}^{(T)} \end{bmatrix}^{T} \quad (19)$$

is of size $N_{F(i)}T \times 1$, where

$$H_{p_{n_{F(i)}}}^{(k)} = \left[\mathbf{w}_{p_{n_{F(i)}}}\right]^{T} \mathbf{h}^{(k)}, \quad (20)$$

is the path gain of the k-th OFDM block and $p_{n_{F(i)}}$-th subcarrier for block $\mathbf{C}^{(i)}$. The quantities $\mathbf{w}_p$ and $\mathbf{h}^{(k)}$ have been defined in Section IV-C1.

Consider a pair of matrices $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ corresponding to two different time-frequency (TF) blocks $C^{(i)}$ and $\tilde{C}^{(i)}$. Then the upper bound pairwise error probability (PEP) [15] between $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ is

$$P\left(\mathbf{M}^{(i)} \rightarrow \tilde{\mathbf{M}}^{(i)}\right) \leq \begin{pmatrix} 2r-1 \\ r \end{pmatrix} \left(\prod_{a=1}^{r} \gamma_a\right)^{-1} (\rho)^{-r} \quad (21)$$

where r is the rank of

$$\boldsymbol{\Lambda}^{(i)} \triangleq \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \mathbf{R}_{H^{(i)}} \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^{\mathcal{H}},$$

and $\mathbf{R}_{H^{(i)}} = E\left\{\mathbf{H}^{(i)} \left[\mathbf{H}^{(i)}\right]^{\mathcal{H}}\right\}$ is the correlation matrix of vector $\mathbf{H}^{(i)}$, $\gamma_a, a = 1, ..., r$ are the non-zero eigenvalues of $\boldsymbol{\Lambda}^{(i)}$.

Then the corresponding rank and product criteria are

1) Rank criterion: the minimum rank of $\boldsymbol{\Lambda}^{(i)}$ over all pairs of different matrices $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ and should be as large as possible.
2) Product criterion: the minimum value of the product $\prod_{a=1}^{r} \gamma_a$ over all pairs of different $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ should be maximized.

Basically, the key term $\boldsymbol{\Lambda}^{(i)}$ in the spectral decomposition consists of two terms: the difference of channel symbol matrices and the channel correlation matrix. The system performance, in terms of PEP, is impacted by the interaction of both terms. This implies that the optimal diversity system design is determined by this interaction.

Next, we derive the matrix form of $\mathbf{R}_{\mathbf{H}^{(i)}}$. Denote

$$\mathbf{W}^{(i)} = \left[\mathbf{w}_{p_{1_{(i)}}}, \cdots, \mathbf{w}_{p_{N_{F(i)}}}\right]^{T} \quad (22)$$

and

$$\mathbf{h} = \left[\left[\mathbf{h}^{(1)}\right]^{T}, \cdots, \left[\mathbf{h}^{(T)}\right]^{T}\right]. \quad (23)$$

From (19),

$$\mathbf{H}^{(i)} = \left(\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right) \mathbf{h}. \quad (24)$$

Substituting into the correlation matrix,

$$\begin{aligned} \mathbf{R}_{\mathbf{H}^{(i)}} &= E\left\{\left(\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right) \mathbf{h} \left[\left(\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right) \mathbf{h}\right]^{\mathcal{H}}\right\} \\ &= \left[\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right] E\left\{\mathbf{h}\left[\mathbf{h}\right]^{\mathcal{H}}\right\} \left[\mathbf{I}_T \otimes \left[\mathbf{W}^{(i)}\right]^{\mathcal{H}}\right], \\ &= \left[\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right] \boldsymbol{\Phi} \left[\mathbf{I}_T \otimes \left[\mathbf{W}^{(i)}\right]^{\mathcal{H}}\right] \end{aligned} \quad (25)$$

where $\boldsymbol{\Phi} = E\left\{\mathbf{h}\left[\mathbf{h}\right]^{\mathcal{H}}\right\}$.

Using the well-known property,

$$rank\left(\mathbf{AB}\right) \leq \min\left\{rank\left(\mathbf{A}\right), rank\left(\mathbf{B}\right)\right\}, \quad (26)$$

$$\begin{aligned} rank\left(\boldsymbol{\Lambda}^{(i)}\right) &\leq \\ \min&\left\{rank\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right), rank\left(\mathbf{R}_{\mathbf{H}^{(i)}}\right)\right\}. \end{aligned} \quad (27)$$

To maximize the right-hand side of (27), it is clear that the maximum of rank of $\boldsymbol{\Phi}$ in the second term is $T(L+1)$. To maximize the rank of $\mathbf{R}_{H^{(i)}}$, we need to maximize the rank of matrix $\mathbf{W}^{(i)}$ of size $N_{F(i)} \times (L+1)$. Thus, we need to choose $N_{F(i)} \geq L+1$. When $p_{n_F}^{(i)} = p_1^{(i)} + b(n_F - 1), n_F = 1, ..., N_{F(i)}, N_{F(i)} \geq L+1$, where $p_{n_F}^{(i)} \leq N_C$ and b is a positive integer, $\mathbf{W}^{(i)}$ could achieve maximum rank $L+1$. Then $\mathbf{R}_{H^{(i)}}$ has the potential to achieve the maximum rank of $T(L+1)$, only if $rank(\boldsymbol{\Phi}) = T(L+1)$. That is to say, channels need to be full rank jointly in frequency and time domains. For a description on how to choose interval b, see [6] and [16].

Since $\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}$ is of size $N_{F(i)}T \times N_{F(i)}T$,

$$rank\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \leq N_{F(i)}T, \quad (28)$$

and $N_{F(i)} \geq L+1$. Note that $\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}$ is a diagonal matrix. Thus, the sufficient and necessary condition for maximizing the rank of $\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}$ is that all the diagonal elements are non-zero, which is summarized as

Theorem 1: 1) If the correlation matrix $\mathbf{R}_{\mathbf{H}^{(i)}}$ of channel vector $\mathbf{H}^{(i)}$ is full rank $T(L+1)$, the necessary condition that the frequency-time (FT) block $\mathbf{C}^{(i)}$ of LDC-OFDM achieves full joint frequency-time diversity order, i.e. $rank(\boldsymbol{\Lambda}_{(i)}) = T(L+1)$, is that the frequency dimension size of the FT block $\mathbf{C}^{(i)}$ satisfies $N_{F(i)} \geq L+1$.

2) The sufficient condition that the frequency-time (FT) block $\mathbf{C}^{(i)}$ of LDC-OFDM achieves available joint frequency-time diversity order, $rank(\mathbf{R}_{\mathbf{H}^{(i)}})$, is that any two elements $c_{p_{n_{F(i)}}}^{(k)}$ and $\widetilde{c_{p_{n_{F(i)}}}^{(k)}}$, of any two different blocks, $\mathbf{C}^{(i)}$ and $\widetilde{\mathbf{C}}^{(i)}$ are different. Mathematically, the sufficient condition is

$$c_{p_{n_{F(i)}}}^{(k)} - \widetilde{c_{p_{n_{F(i)}}}^{(k)}} \neq 0, \quad (29)$$

where $n_{F(i)} = 1, ..., N_{F(i)}, k = 1, ..., T$;

3) If both $N_{F(i)} = L+1$ and $rank(\mathbf{R}_{\mathbf{H}^{(i)}}) = T(L+1)$ are satisfied, the condition (29) is the sufficient and necessary condition that the frequency-time (FT) block $\mathbf{C}^{(i)}$ of LDC-OFDM achieves joint full frequency-time diversity order, $rank(\boldsymbol{\Lambda}_{(i)}) = T(L+1)$;

4) The related product criterion of design is that the minimum of products

$$\prod_{k=1}^{\mathcal{T}} \prod_{a=1}^{N_{F(i)}} \left| c_{p_{a(i)}}^{(k)} - \widetilde{c_{p_{a(i)}}^{(k)}} \right|^2$$

taken over distinct FT blocks $\mathbf{C}^{(i)}$ and $\widetilde{\mathbf{C}^{(i)}}$ must be maximized. ∎

A proof is provided in the Appendix A.

The result of Theorem 1 is somewhat surprising in that the design criterion is different from that for space-time rapid fading channels [17]. If $c_{p_{n_{F(i)}}}^{(k)}, n_{F(i)} = 1, ..., N_{F(i)}$ were space-time coded signals, and $n_{F(i)}$ were to refer to the indices of transmit antennas, then the design criterion (distance criterion) would be

$$c_{p_{1(i)}}^{(k)} c_{p_{2(i)}}^{(k)} ... c_{p_{N_{F(i)}}}^{(k)} \neq \widetilde{c_{p_{1(i)}}^{(k)}} \widetilde{c_{p_{2(i)}}^{(k)}} ... \widetilde{c_{p_{N_{F(i)}}}^{(k)}}.$$

Denote

$$\left| c_{(i)}^{k} - \widetilde{c_{(i)}^{k}} \right|^2 = \sum_{a=1}^{N_{F(i)}} \left[c_{p_{a(i)}}^{(k)} - \widetilde{c_{p_{a(i)}}^{(k)}} \right]^2$$

and the corresponding product criterion for the space-time codes would be that the minimum of products

$$\prod_{k=1}^{\mathcal{T}} \left| c_{(i)}^{k} - \widetilde{c_{(i)}^{k}} \right|^2$$

taken over distinct codewords $\mathbf{C}^{(i)}$ and $\widetilde{\mathbf{C}^{(i)}}$ be maximized.

Clearly there are differences between design criteria of frequency-time and space-time codes for 2-D rapid fading channels, since in the space-time case, the received signals at each antenna are superpositions of signals from multiple transmit antennas. However, there is no such superposition of parallel signals at the receiver in frequency-time. The differences in design criteria can lead to different matrix codeword designs, implying that the best design in space-time is not necessarily the best design in frequency-time, especially in channels undergoing rapid fading. In particular, the above analysis reveals that instead of a block using all available subcarriers, a more efficient frequency-time (FT) block design would utilize a much smaller block to achieve diversity order up to $T(L+1)$. By achieving this increased diversity order, a Rayleigh fading frequency selective channel may approach the diversity order of a Gaussian channel [18].

The necessary condition that a FT block design achieves a certain diversity order is that the rank of the channel correlation matrix be equal to the diversity order of the FT block. The diversity order actually achieved is based on the chosen LDC design. Originally, Hassibi and Hochwald did not consider diversity order as a design criterion [7]. In [8], Heath and Paulraj consider both capacity and error probability as criteria, but channel coefficients are assumed constant over time within an entire LDC codeword. The analysis in this section, however, considers correlation across parallel frequency channels (OFDM subcarriers) as well as across time channel uses (OFDM blocks).

An important special case of FT-block design is $T = 1$, with upper bound diversity order $L + 1$, known as linear constellation precoding OFDM, or LCP-OFDM [6]. The diversity order of LCP-OFDM is therefore always no larger that of full frequency-time diversity LDC-OFDM (of order $T(L+1)$ with $T > 1$) in dynamic frequency-selective fading channels.

A rate-one LDC-OFDM design that achieves full joint frequency-time (FT) diversity under arbitrary frequency-time correlation can be obtained by considering time varying block frequency selective channels of full rank $\mathbf{R}_{\mathbf{H}^{(i)}}$, and a LDC-CP-OFDM system with T OFDM blocks and N_C subcarriers per OFDM block. Full-diversity frequency-time (FT) sub-blocks $\mathbf{C}^{(i)}$ of frequency dimension N_F and time dimension T, $i = 1, ..., D$, are constructed from a vector of $Q = N_F T$ source symbols with rectangular QAM (or PAM, BPSK, QPSK) constellations, where $N_F \geqslant L + 1$. The vector is encoded via linear constellation precoding [6], [19] whereby a $Q \times Q$ LDC encoding matrix

$$\mathbf{G}_{LDC} = \Theta = \begin{bmatrix} 1 & \alpha_1 & \cdots & \alpha_1^{Q-1} \\ 1 & \alpha_2 & \cdots & \alpha_1^{Q-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_Q & \cdots & \alpha_Q^{Q-1} \end{bmatrix}$$

where α_q, $q = 1, ..., Q$, are defined in [6], [19]. The above FT-LDC design achieves full joint frequency-time diversity with maximal achievable diversity order $T(L + 1)$. The diversity order can be established by using results found in [6], [19], showing that $\left[vec \left(\mathbf{C}^{(i)} - \widetilde{\mathbf{C}^{(i)}} \right) \right]_{q,1} \neq 0$ where $q = 1, ..., TN_F$. It is clear that $\left[vec \left(\mathbf{C}^{(i)} - \widetilde{\mathbf{C}^{(i)}} \right) \right] = \left[\Theta \left(\mathbf{s}^{(i)} - \widetilde{\mathbf{s}^{(i)}} \right) \right]$. Note that a rotation matrix of full signal space diversity, Θ, enables $\left[\Theta \left(\mathbf{s}^{(i)} - \widetilde{\mathbf{s}^{(i)}} \right) \right]_{q,1} \neq 0$ for any $\mathbf{s}^{(i)} \neq \widetilde{\mathbf{s}^{(i)}}$ [6], [19], where $\mathbf{s}^{(i)}$ and $\widetilde{\mathbf{s}^{(i)}}$ denote a pair of source symbol vectors of rectangular QAM (or PAM, or BPSK, or QPSK). The above design is denoted as LCP-LDC. As discussed in Section V, the best frequency-time diversity code design may not necessarily correspond to the best space-time diversity code design. Thus it is possible that LCP-LDC has performance inferior to other powerful space-time code designs for multiple antenna systems.

VI. SYSTEM PERFORMANCE COMPARISON

A. Simulation setup

Perfect channel state information (CSI), including amplitude and phase, is assumed at the receiver but not at the transmitter. In all simulations, the LCP-LDC design of size $N_F \times T$ is chosen and the corresponding LDC encoding matrices $\mathbf{G}_{LDC}$, defined in (5) are unitary. This code therefore meets the correlation criterion of Section IV-B. The D LDC demodulators each decode $T \times N_{F(i)}$ LDC matrices. In particular, we set $N_{F(i)} = N_F = T, i = 1, ..., D$. $N_C = 16$ OFDM subcarriers are chosen. An evenly and maximally spaced subcarrier mapping with respect to the subcarrier indices is used within LDC codewords. Data symbols use 4-QAM modulation. The frequency-selective Rayleigh fading channel has 4 paths with uniform power delay profile. The channel is assumed to be constant over an integer number of OFDM blocks, and independent and identically-distributed between blocks. We denote this interval of blocks as the channel change interval (CCI).
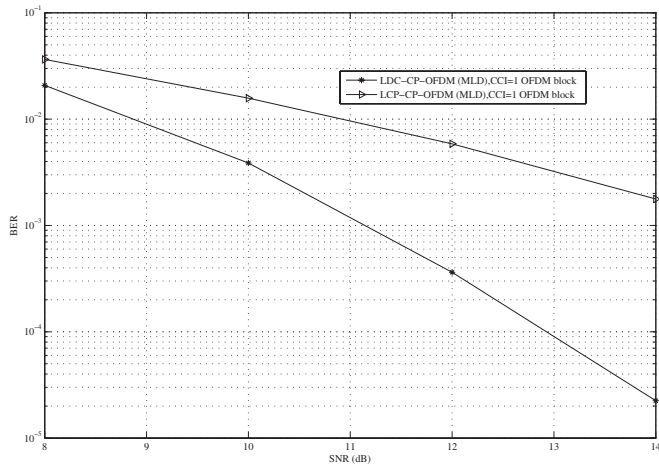
Fig. 4. BER Performance of LDC-CP-OFDM vs. LCP-CP-OFDM with MLD, $N_C = 16$, $N_F = 4$, $T = 4$, $L = 3$.
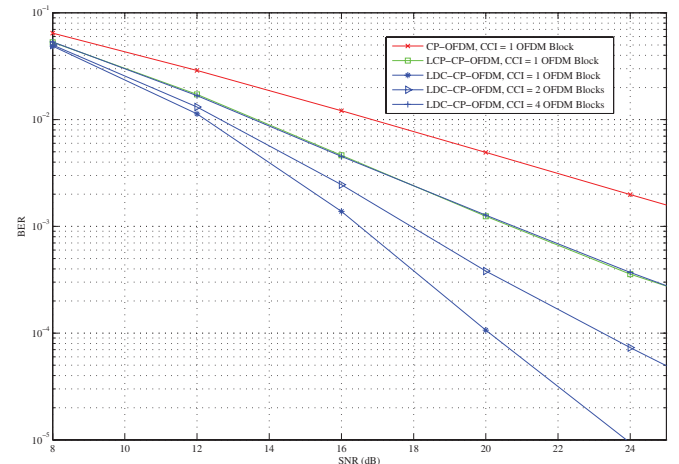

Fig. 5. BER Performance of CP based Systems under different channel dynamics, $N_C = 16$, $N_F = 4$, $T = 4$, $L = 3$.

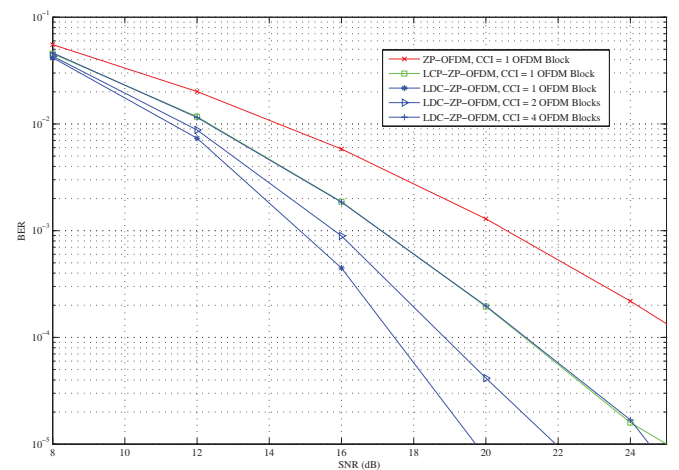Fig. 6. BER Performance of ZP based Systems under different channel dynamics, $N_C = 16$, $N_F = 4$, $T = 4$, $L = 3$.

B. Comparison of LDC-CP-OFDM and LCP-CP-OFDM using maximum likelihood decoding (MLD)

Linear constellation precoded CP-OFDM (LCP-CP-OFDM) with subcarrier grouping has been proposed as a non-redundancy approach to improve BER performance [6]. Although LCP-CP-OFDM achieves both maximum frequency selective diversity gain and coding gain, it cannot exploit time diversity over OFDM blocks. Using MLD, we investigate the performance limitations of LDC-CP-OFDM. For a fair comparison, all parameters of LCP-CP-OFDM are chosen the same as those of LDC-CP-OFDM. Thus the available diversity in the channels is the same for both systems. In Figure 4, we observe that LDC-CP-OFDM, which achieves full joint frequency and time diversity, significantly outperforms LCP-CP-OFDM under frequency-domain MLD in rapid fading channels ($CCI = 1$).

C. Comparison of Two-Step-Estimation (TSE) OFDM Systems LDC-OFDM

1) Comparison of TSE based LDC-OFDM, LCP-OFDM, and OFDM under different channel dynamics: To the best of the authors' knowledge, the combination of LCP and ZP-OFDM has not been proposed and investigated previously, while LCP-CP-OFDM has been studied in [6]. Figures 5 and 6 compare the performances of LDC-OFDM, LCP-OFDM, OFDM with cyclic-prefix and zero-padding, respectively, under different CCIs. Note that different CCIs represent different degrees of temporal channel correlation. We note that the performance curves of LDC-OFDM under $CCI = 4$ and LCP-OFDM under $CCI = 1$ overlap, which demonstrates that LDC-OFDM achieves full frequency diversity gain even without time diversity in the channel.

As discussed in Section V, the diversity order of LDC-OFDM is achievable only if the channel provides corresponding diversity. This agrees with the simulation results of LDC-CP-OFDM, where performance is improved by faster channel dynamics, and, as expected, is especially notable in high SNR regions. Also, LDC-OFDM outperforms LCP-OFDM under $CCI = 1$, i.e., when channels have less temporal correlation

over time, which corroborates with the analysis in Section V. Overall, LDC-OFDM systems exhibit noticeable advantages over LCP-OFDM systems in channel environments that vary over OFDM blocks.

2) Effect of LDC-CP-OFDM under estimated CSI: To consider the more realistic scenario of imperfect channel state information (CSI) at the receiver, Figure 7 quantifies the performance degradation of LDC-CP-OFDM performance under estimated CSI obtained through standard pilot-based MMSE channel estimation [20]. The pilot sequence is chosen randomly from a sequence of 4-QAM symbols. Linear MMSE estimation is used to determine the gains in the frequency domain. The results show that both full-rate LDC-CP-OFDM and CP-OFDM degrade under estimated CSI: at a BER of 10^{-2}, the degradation is 0.63dB and 0.35dB, respectively. As expected, LDC-CP-OFDM is more sensitive to channel estimation errors. However, LDC-CP-OFDM still outperforms CP-OFDM under frequency domain MMSE channel estimation, due to its large compensating diversity gain.

3) Performance of LDC-ZP-OFDM using low complexity MMSE receivers: In Figure 8, TSE LDC-ZP-OFDM
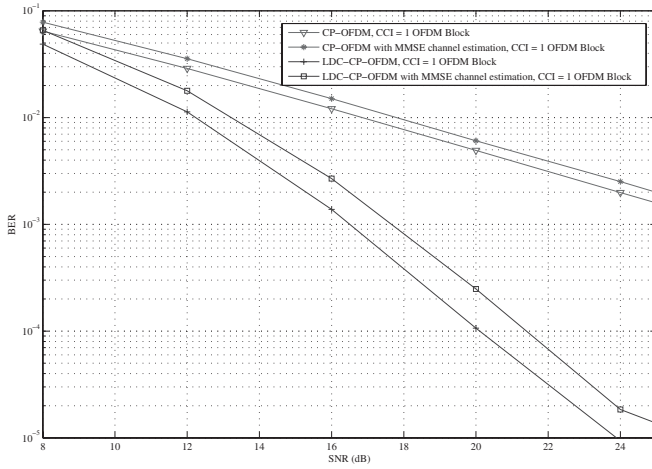
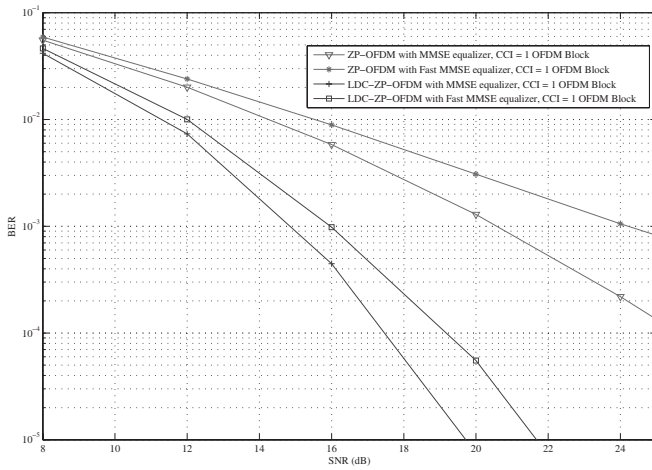Fig. 7.   Effects of LDC-CP-OFDM under estimated channel information, $N_C = 16$, $N_F = 4$, $T = 4$, $L = 3$.

Fig. 9.   BER Performance of LDC-OFDM (OSE) vs. LDC-OFDM (TSE), $N_C = 16$, $N_F = 4$, $T = 4$, $L = 3$.

OSE involves matrix inversion of $O\left((N_C T)^3\right)$, while TSE involves matrix inversion of $O\left(T(N_C)^3\right)$ as well as D matrix-vector multiplications of $O\left((N_F T)^2\right)$, for both CP and ZP cases.

From Fig. 9, it can be seen that the performances of OSE and TSE for both LDC-ZP-OFDM and LDC-CP-OFDM are similar. We reiterate that the chosen LDC encoding matrices $\mathbf{G}_{LDC}^{(i)}, i = 1, ..., D$ are unitary, which result in uncorrelated LDC-coded symbols. In the unitary case, however, TSE two estimation stages are decoupled. Thus, with the proper choice of LDC coding matrices, TSE performance may approach that of OSE at much lower complexity. While attractive, not all LDC-OFDM designs can effectively use a TSE procedure. In general, where LDC encoders produce correlated coded symbols, a TSE procedure leads to performance loss.

Fig. 8.   BER Performance of LDC-ZP-OFDM (TSE) under MMSE vs. low complexity MMSE receivers, $N_C = 16$, $N_F = 4$, $T = 4$, $L = 3$.

VII. CONCLUSION

Inspired by a space-time processing technique, LDC have been applied to improve OFDM performance. These LD codes can be advantageously combined with OFDM transmission to enable simple decoding of large LDC matrices. To this end, a novel two-step estimation (TSE) LDC decoding strategy is proposed for a special subclass of LDC matrices with the constraint in Eq. (2), which can decouple CSI and LDC decoding. With high spectral efficiency at a cost of increased decoding delay, the proposed LDC-OFDM system can achieve superior BER performance by exploiting available frequency and time diversity in dynamic frequency selective channels. Insight into LDC performance is obtained by deriving the upper bound time and frequency diversity order that LDC-OFDM can achieve. This paper also provides a criterion and example design of a full diversity time-frequency LDC-OFDM. At lower complexity, the TSE LDC-OFDM system has performance that in certain cases, may be close to that of OSE LDC-OFDM. Simulations reveal that performance losses under imperfect CSI as well as with low complexity receivers can be modest.

performance is compared using both conventional and low complexity (approximate) MMSE equalization described in Section IV-C3, Eq. (15). From Figure 8, it can be seen that in the low to medium SNR range, the performances of LDC-ZP-OFDM under MMSE vs. low complexity MMSE equalizers are reasonably close. At a BER of 10^{-3}, performance degrades by 1.2 dB. We remark that according to our experience, the amount of performance degradation may vary by employing different LDC encoding matrices. At increasing SNR, the performance loss using a low complexity approach becomes more significant:, increasing to 1.8 dB at a BER of 10^{-4}. The above comparisons illustrate the performance and complexity tradeoffs of the layered TSE structure.

D. Comparison of OSE and TSE LDC-OFDM

As discussed in Sections IV-A and IV-B, OSE has higher complexity than TSE, requiring one large system of linear equations for the whole LDC-OFDM block operating on $N_C T$ subcarriers and $D = N_C/N_F$ LDC codewords. Due to space limitations, we omit a detailed description of the OSE LDC-OFDM transceiver system. For linear MMSE estimation,
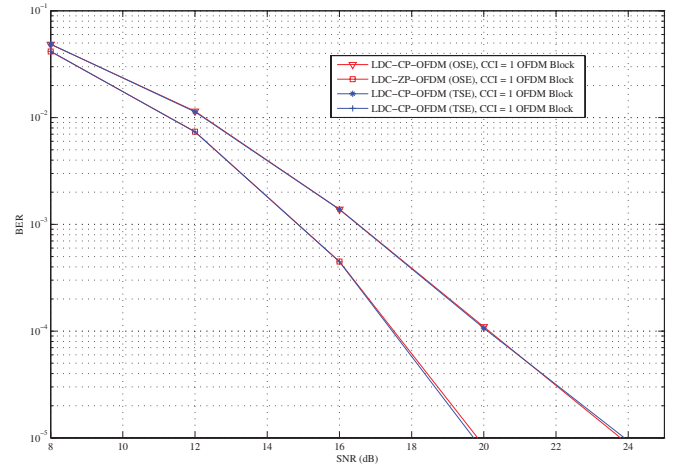
APPENDIX

Part 1) of Theorem 1 has been discussed in previous parts of this section. Since part 4) is a straightforward result of 2) or 3), only 2) and 3) are shown. Note that $\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)$ is of size $TN_{F(i)} \times TN_{F(i)}$. Therefore the condition (29) ensures

$$rank\left(\left[\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)\right]\right) = TN_{F(i)}.$$

Accordingly,

$$
\begin{aligned}
&rank(\mathbf{\Lambda}_{(i)}) \\
&= rank\left(\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)\mathbf{R}_{\mathbf{H}^{(i)}}\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^{H}\right) \\
&= rank\left(\mathbf{R}_{\mathbf{H}^{(i)}}\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^{H}\right) \\
&= rank\left(\mathbf{R}_{\mathbf{H}^{(i)}}\right)
\end{aligned}
$$

proving statement 2). If both $N_{F(i)} = L + 1$ and $rank(\mathbf{R}_{\mathbf{H}^{(i)}}) = T(L + 1)$ hold, to achieve $rank(\mathbf{\Lambda}_{(i)}) = T(L + 1)$, it is necessary to have

$$
\begin{aligned}
&T(L + 1) \\
&= rank\left(\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)\mathbf{R}_{\mathbf{H}^{(i)}}\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^{H}\right) \\
&\leq rank\left(\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)\right)
\end{aligned}
$$

However, $\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)$ is of size $T(L + 1) \times T(L + 1)$ in this case. Hence, (29) is a necessary condition. Conversely, using 2) of Theorem 1, $rank(\mathbf{\Lambda}_{(i)}) = rank\left(\mathbf{R}_{\mathbf{H}^{(i)}}\right) = T(L + 1)$, proving 3).

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Jinsong Wu received the B.Eng. degree from Changsha Railway University (now part of Central South University), Changsha, China, in 1990, the M.A.Sc. degree from Dalhousie University, Halifax, NS, Canada, in 2001, the Ph.D. degree from Queen's University, Kingston, ON, Canada, in 2006, all in Electrical and Computer Engineering.

He has held engineering positions in the areas of information technology and telecommunications with Stone in China, Great-Wall Computer in China, Nortel Networks in Canada, Philips Research North America in USA, and Sprint-Nextel in USA. His current research interests lie in general communications theory and signal processing, space-time-frequency processing and coding, cooperative communications, network protocols with applications. He is Technical Program Committee member, IEEE International Conference on Communications 2009.

Steven D. Blostein (M'83, SM'96) received the B.S. degree in Electrical Engineering from Cornell University, Ithaca, NY, in 1983, and the M.S. and Ph.D. degrees in Electrical and Computer Engineering from the University of Illinois, Urbana-Champaign, in 1985 and 1988, respectively. He has been on the faculty at Queen's University since 1988 and currently holds the position of Professor and Head of the Department of Electrical and Computer Engineering. From 1999-2003, he was the leader of the Multi-Rate Wireless Data Access Major Project sponsored by the Canadian Institute for Telecommunications Research. His current interests lie in the application of signal processing to wireless communications systems, including smart antennas, MIMO systems, and space-time-frequency processing for MIMO-OFDM systems. He has been a member of the Samsung 4G Wireless Forum. He served as Chair of IEEE Kingston Section (1994), Chair of the Biennial Symposium on Communications (2000,06,08), Associate Editor for IEEE TRANSACTIONS ON IMAGE PROCESSING (1996-2000), and Publications Chair for IEEE ICASSP 2004. He is currently serving as Associate Editor of IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He is a registered Professional Engineer in Ontario.