Downlink Distributed Beamforming Through Relays with Imperfect CSI

Yi Zheng and Steven D. Blostein

Department of Electrical and Computer Engineering
Queen's University, Kingston, Ontario, K7L3N6, Canada

Abstract— **Future wireless systems face challenges in supporting high-rate multimedia streaming with wide coverage area and at low power. In this paper we study cooperative MIMO systems in the context of imperfect channel estimation with per-user quality-of-service constraints with the objective of minimizing the sum transmission power at the source and relays. Simulation results quantify the different performance tradeoffs obtained in incorporating imperfect CSI into the problem formulation as a function of channel estimation accuracy.**

I. Introduction

A variety of relaying technologies have been proposed. For example, diversity can be introduced through cooperation of spatially separated communication nodes, via relaying, to improve communications [1]. Relaying protocols include amplify and forward (AF), which has also been shown to achieve full diversity.

Recently, transmission techniques for broadcast channels have been extended to cooperative networks [2], where relays cooperatively transmit to a receiver using *distributed beamforming,* where amplitudes and phases of transmitted signals are coherently combined. In [3], a distributed beamforming system with a single transmitter and receiver and multiple relay nodes is considered, and second-order channel statistics are employed to optimize the distributed relay beamformer (DRBF). Single-antenna source-destination pairs that communicate peer-to-peer through a relay network are considered in [4], via a semi-definite programming (SDP) formulation and solved through semi-definite relaxation. In [5], an iterative algorithm jointly optimizes source and sum relay transmission power. Unfortunately, the requirement for accurate channel state information (CSI) and the distributed nature of wireless sensor/relay networks complicate transmit beamforming. Transceiver design that takes imperfect channel state information into account has also been studied [6] [7] [8]. In [9], a distributed beamforming scheme with two relays is proposed that has the advantages of limited feedback and improved diversity. While the problem of limited CSI feedback in multiple-input multiple output (MIMO) AF relay systems has been addressed in [10] using beamforming code books designed based on Grassmanian manifolds, perfect CSI is also assumed.

[1]This research has been supported by Defense R&D Canada through the Defense Research Program at the Communications Research Centre Canada.

Unlike [4], this paper considers basestation transmission of different streams to different users. We generalize the iterative algorithm in [5] to the case of imperfect channel estimation and study the performance tradeoffs as a function of channel estimation error that result from different training power levels. In Section II, we present the system model incorporating imperfect CSI. Linear precoder optimization assuming a given DRBF is developed in Section III, followed by DRBF optimization in Section IV. Numerical results are discussed in Section V.

II. System Model

Consider a broadcast channel as shown in Fig. 1. Data is transmitted from a source to multiple users through relays successively over two time slots. There is an N-antenna source, a network of n_R distributed single-antenna relays, and M single-antenna destinations, where $M \leq N$. The relays are backbone connected and serve as system infrastructure. Therefore, it is assumed that the channels from source to relays are estimated at the relays and reliably fed back to the source. The relays-destination channels are estimated at the destinations, fed back to the relays, who then forward back to the source. Since range extension is an intended application, there are no direct links between the source and destinations.

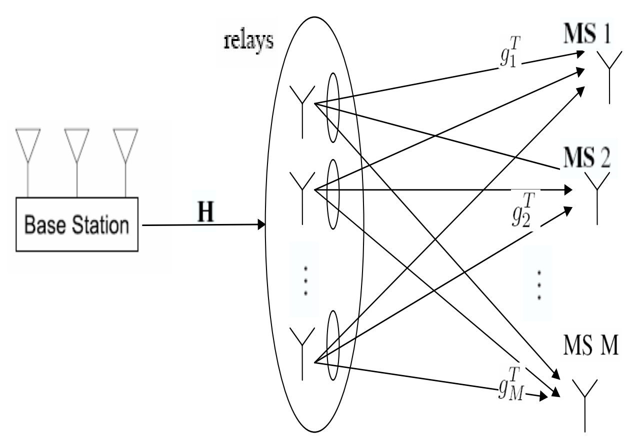

Fig. 1. Downlink distributed beamforming system.

A. Imperfect CSI model

We consider channel estimation from source to relays as well as from relays to single-antenna receivers, which is used to compute an SINR penalty.

1) Channel estimation from source to relays: Adapting the model in [8] [11] [12], the channels from the source to the relays can be decomposed into an estimate, $\hat{\mathbf{H}}$, and an estimation error term, $\mathbf{E_H}$ via $\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E_H}$. As the source has multiple co-located antennas, we assume the correlation matrix for source transmission to be known and given by $\mathbf{R}_T$. Since the relays are distributed, the correlation matrix for relay transmission is assumed to be $\mathbf{I}_{n_R}$, where $\mathbf{I}_{n_R}$ is the $n_R \times n_R$ identity matrix. For channel estimation from source to relays, the relays jointly receive the $n_R \times N$ matrix

$$\mathbf{Y}_{tr} = \mathbf{H}\mathbf{S}_{tr} + \mathbf{N}_{tr} \tag{1}$$

where $\mathbf{S}_{tr}$ is the transmitted $N \times N$ training signal matrix and $\mathbf{N}_{tr}$ is the collection of noise vectors. Equivalently,

$$\mathbf{Y}_{tr} = \mathbf{H}_w \mathbf{R}_T^{1/2} \mathbf{S}_{tr} + \mathbf{N}_{tr} \tag{2}$$

where $\mathbf{H}_w$ represents a spatially white matrix whose entries are independent and identically distributed (i.i.d) $N_c(0,1)$. Let P_{tr} denote the total source training power, i.e., $\text{Tr}(\mathbf{S}_{tr}\mathbf{S}_{tr}^H) = P_{tr}$. To obtain orthogonality, $\mathbf{S}_{tr} = \mathbf{R}_T^{-1/2}\mathbf{S}_0$ is chosen, where $\mathbf{S}_0$ is a unitary matrix scaled by $\sqrt{P_{tr}/\text{Tr}(\mathbf{R}_T^{-1})}$. The spatially white channel matrix $\mathbf{H}_w$ can be shown to be [6] [13]

$$\mathbf{H}_w = \frac{1}{1+\sigma_{ce}^2}\mathbf{Y}_{tr}\mathbf{S}_0^{-1} + \frac{1}{\sqrt{1+\sigma_{ce}^2}}\mathbf{E}_w \tag{3}$$

where $\mathbf{E}_w$ are i.i.d $N_c(0, \sigma_{ce}^2)$. This results in channel estimation error matrix

$$\mathbf{E_H} = \frac{1}{\sqrt{1+\sigma_{ce}^2}}\mathbf{E}_w. \tag{4}$$

In (4), since the relays are distributed, the entries of $\mathbf{E_H}$ are modeled as i.i.d. $N_c(0, \frac{\sigma_{ce}^2}{1+\sigma_{ce}^2})$. For the case of transmit antennas, n_T time slots are required for training.

2) Channel estimation from relays to destinations: Similarly, the channel gain from the ith relay to the jth destination's receiver can be represented as

$$g_{j,i} = \hat{g}_{j,i} + e_{j,i}, \quad j = 1, \ldots, M, \quad , i = 1, \ldots, n_R. \tag{5}$$

The availability of $\hat{g}_{j,i}$ is assumed using single input single output (SISO) channel estimation and the distribution of the estimation error $e_{j,i}$ is assumed to be $N_c(0, \sigma_{e_{j,i}}^2)$. For the n_R relays, n_R time slots are required for channel training and estimation from relays to destinations. Mutual independence of noise across the different relays is also assumed.

B. Cooperative system model with imperfect CSI

The $N \times 1$ vector $\mathbf{h}_r$ represents the link from the source to the rth relay, $1 \leq r \leq n_R$, which receives symbols

$$x_r = \mathbf{h}_r^T \sum_{i=1}^{M} \mathbf{t}_i s_i + \nu_r \tag{6}$$

where $\mathbf{t}_i$ denotes $N \times 1$ transmit beamforming vector corresponding to signal s_i intended for the ith destination, $1 \leq i \leq M$.

To model distributed beamforming, the ith relay multiplies its received signal by complex coefficient w_i. Signals transmitted from all relays to the destinations is represented by

$$\mathbf{u} = \mathbf{W}^H \mathbf{x} \tag{7}$$

where diagonal DBRF matrix $\mathbf{W} = \text{diag}(w_1, w_2, \ldots, w_{n_R})$ and $\mathbf{x} = [x_1 \; x_2 \; \ldots \; x_{n_R}]^T$. Using (6), the received signal at the ith destination is

$$\begin{aligned} y_i &= \mathbf{g}_i^T \mathbf{u} + n_i \\ &= \underbrace{\mathbf{g}_i^T \mathbf{W}^H \mathbf{H} \mathbf{t}_i s_i}_{Desired\ Signal} + \underbrace{\mathbf{g}_i^T \mathbf{W}^H \mathbf{H} \sum_{j=1, j\neq i} \mathbf{t}_j s_j}_{interference} + \underbrace{\mathbf{g}_i^T \mathbf{W}^H \boldsymbol{\nu} + n_i}_{noise} \end{aligned} \tag{8}$$

where $n_R \times N$ matrix $\mathbf{H} = [\mathbf{h}_1 \ldots \mathbf{h}_{n_R}]^T$ is the combined source-to-relay channel, $1 \times n_R$ row vector $\mathbf{g}_i^T$ represents the channel from relays to ith destination, and $n_R \times 1$ vector $\boldsymbol{\nu} = [\nu_1, \ldots, \nu_{n_R}]^T$ represents noise at the relays.

III. Transmit precoder optimization

Based on the channel estimation model, the received signal at the ith destination is

$$y_j = \underbrace{\hat{\mathbf{g}}_j^T \mathbf{W}^H \hat{\mathbf{H}} \mathbf{t}_j s_j}_{desired\ signal} + \underbrace{\hat{\mathbf{g}}_j^T \mathbf{W}^H \hat{\mathbf{H}} \sum_{i=1, i\neq j}^{M} \mathbf{t}_i s_i}_{Interference} + \underbrace{\xi_j}_{noise} \tag{9}$$

where the noise term

$$\begin{aligned} \xi_j = {} & \hat{\mathbf{g}}_j^T \mathbf{W}^H \mathbf{E}_H \sum_{i=1}^{M} \mathbf{t}_i s_i + \\ & \mathbf{e}_{\mathbf{g}_j}^T \mathbf{W}^H (\hat{\mathbf{H}} + \mathbf{E_H}) \sum_{i=1}^{M} \mathbf{t}_i s_i + (\hat{\mathbf{g}}_j^T + \mathbf{e}_{\mathbf{g}_j}^T) \mathbf{W}^H \boldsymbol{\nu}_{\boldsymbol{r}} + n_j \end{aligned} \tag{10}$$

where $\mathbf{e}_{\mathbf{g}_j}^T = [e_{j,1}, e_{j,2}, \ldots, e_{j,M}]$. First, the optimal minimum source power transmit precoder is found under the constraints that for $1 \leq k \leq M$, the kth destination node's quality of service (QoS), expressed in terms of its signal-to-interference plus noise ratio, SINR_k, which is kept above pre-defined threshold γ_k for a fixed DRBF. This leads to the following optimization problem:

$$\min_{\mathbf{t}_1, \mathbf{t}_2, \ldots, \mathbf{t}_M} \; P_T \text{ s.t. } \text{SINR}_j \geq \gamma_j, \;\; \text{for } \; k = 1, 2, \ldots, M \tag{11}$$

where P_T is the transmission power at the source,

$$\text{SINR}_j = \frac{P_s^j}{P_I^j + P_n^j} \qquad (12)$$

and s.t. stands for *subject to*. In (12), P_s^k, P_I^k and P_n^k denote desired signal power, interference power, and noise power at the kth destination, respectively. Average transmission power P_T is given by

$$\begin{aligned} P_T &= E\{(\sum_{i=1}^{M}\mathbf{t}_i s_i)^H(\sum_{j=1}^{M}\mathbf{t}_j s_j)\} \qquad (13)\\ &= \sum_{i=1}^{M}\mathbf{t}_i^H\mathbf{t}_i = \sum_{i=1}^{M}\text{Tr}\{\mathbf{t}_i\mathbf{t}_i^H\} \end{aligned}$$

where Tr$(\cdot)$ stands for trace$(\cdot)$. In (12),

$$\begin{aligned} P_s^j &= \mathbf{t}_j^H\hat{\mathbf{H}}^H\mathbf{W}\mathbf{R}_{\hat{\mathbf{g}}}^j\mathbf{W}^H\hat{\mathbf{H}}\mathbf{t}_j, \qquad (14)\\ P_I^j &= \sum_{i=1,i\neq j}^{M}\mathbf{t}_i^H\hat{\mathbf{H}}^H\mathbf{W}\mathbf{R}_{\hat{\mathbf{g}}}^j\mathbf{W}^H\hat{\mathbf{H}}\mathbf{t}_i, \qquad (15)\\ P_n^j &= P_{n_1}^j + P_{n_2}^j + P_{n_3}^j + P_{n_4}^j, \qquad (16)\\ P_{n_1}^j &= \sum_{i=1}^{M}\frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\text{Tr}(\mathbf{W}\mathbf{R}_{\hat{\mathbf{g}}}^j\mathbf{W}^H)\mathbf{t}_i^H\mathbf{t}_i,\\ P_{n_2}^j &= \sum_{i=1}^{M}\mathbf{t}_i^H\hat{\mathbf{H}}^H\mathbf{W}\mathbf{R}_{\mathbf{e}_g}^j\mathbf{W}^H\hat{\mathbf{H}}\mathbf{t}_i,\\ P_{n_3}^j &= \sum_{i=1}^{M}\frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\text{Tr}(\mathbf{W}\mathbf{R}_{\mathbf{e}_g}^j\mathbf{W}^H)\mathbf{t}_i^H\mathbf{t}_i,\\ P_{n_4}^j &= \sigma_\nu^2\mathbf{W}^H\left(\mathbf{R}_{\hat{\mathbf{g}}}^j + \mathbf{R}_{\mathbf{e}_g}^j\right)\mathbf{W} + \sigma_n^2, \qquad (17) \end{aligned}$$

$\mathbf{R}_{\hat{\mathbf{g}}}^j = E(\hat{\mathbf{g}}_j^*\hat{\mathbf{g}}_j^T)$, and $\mathbf{R}_{\mathbf{e}_g}^j = E(\mathbf{e}_{\mathbf{g}_j}\mathbf{e}_{\mathbf{g}_j}^H)$. Using the above expressions, the precoder optimization of (11) becomes

$$\begin{aligned} &\min_{\mathbf{t}_1,\mathbf{t}_2,\ldots,\mathbf{t}_M} \quad \textstyle\sum_{j=1}^{M}\mathbf{t}_j^H\mathbf{t}_j \quad \text{s.t.}\\ &\mathbf{t}_j^H\mathbf{Q}_{j,j}\mathbf{t}_j \geq \gamma \quad \textstyle\sum_{i=1,i\neq j}^{M} \quad \mathbf{t}_i^H\mathbf{Q}_{i,j}\mathbf{t}_i + \gamma P_{n_4}^j, \qquad (18) \end{aligned}$$

where

$$\begin{aligned} \mathbf{Q}_{j,j} &= \hat{\mathbf{H}}^H\mathbf{W}\mathbf{R}_{\hat{\mathbf{g}}}^j\mathbf{W}^H\hat{\mathbf{H}} - \gamma\hat{\mathbf{H}}^H\mathbf{W}\mathbf{R}_{\mathbf{e}_g}^j\mathbf{W}^H\hat{\mathbf{H}} -\\ &\gamma\frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\text{Tr}(\mathbf{W}\mathbf{R}_{\hat{\mathbf{g}}}^j\mathbf{W}^H)\mathbf{I} - \gamma\frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\text{Tr}(\mathbf{W}\mathbf{R}_{\mathbf{e}_g}^j\mathbf{W}^H)\mathbf{I}, \qquad (19) \end{aligned}$$

and

$$\begin{aligned} \mathbf{Q}_{i,j} &= \mathbf{H}^H\mathbf{W}\mathbf{R}_{\hat{\mathbf{g}}}^j\mathbf{W}^H\hat{\mathbf{H}} + \frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\text{Tr}(\mathbf{W}\mathbf{R}_{\hat{\mathbf{g}}}^j\mathbf{W}^H) +\\ &\hat{\mathbf{H}}^H\mathbf{W}\mathbf{R}_{\mathbf{e}_g}^j\mathbf{W}^H\hat{\mathbf{H}} + \frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\text{Tr}(\mathbf{W}\mathbf{R}_{\mathbf{e}_g}^j\mathbf{W}^H),\ i\neq j. \qquad (20) \end{aligned}$$

In the above, $\mathbf{R}_{\hat{\mathbf{g}}}^j = E(\hat{\mathbf{g}}_j^*\hat{\mathbf{g}}_j^T)$, $\mathbf{R}_{\mathbf{e}_g}^j = E(\mathbf{e}_{\mathbf{g}_j}\mathbf{e}_{\mathbf{g}_j}^H)$. Using the semi-definite relaxation technique similar to [3] [4], we instead solve a relaxed version of (18). Defining matrices

$$\mathbf{T}_i = \mathbf{t}_i\mathbf{t}_i{}^H, \quad i = 1,2,\ldots,M, \qquad (21)$$

and dropping the rank-one constraints of the $\mathbf{T}_i$, problem (18) transforms to

$$\begin{aligned} &\min_{\mathbf{T}_1,\ldots,\mathbf{T}_M}\sum_{j=1}^{M}\text{Tr}(\mathbf{T}_j) \ \text{ s.t.}\\ &\text{Tr}(\mathbf{T}_j\mathbf{Q}_{j,j}) \geq \gamma\sum_{i=1,i\neq j}^{M}\text{Tr}\quad(\mathbf{T}_i\mathbf{Q}_{i,j}) + \gamma P_{n_4}^j. \qquad (22) \end{aligned}$$

Using arguments similar to those found in [14], it can be proven that there always exists at least one solution to (22) with rank$(\mathbf{T}_j) = 1, j = 1,\ldots,M$. A full proof is based on the method in [14].

IV. Distributed relay beamforming (DRBF)

Given the source precoder obtained above, we next consider minimum power DRBF to multiple destinations.

A. Multi-Destination Relay Power Minimization

Under imperfect CSI, we express the received signal as

$$y_j = \underbrace{\mathbf{w}^H\text{diag}(\hat{\mathbf{g}}_j^T)\hat{\mathbf{H}}\mathbf{t}_j s_j}_{desired\ \ signal} + \underbrace{\mathbf{w}^H\text{diag}(\hat{\mathbf{g}}_j^T)\hat{\mathbf{H}}\sum_{i=1,i\neq j}^{M}\mathbf{t}_i s_i}_{Interference} + \underbrace{\varsigma_j}_{noise} \qquad (23)$$

where noise term

$$\begin{aligned} \varsigma_j \ &= \mathbf{w}^H\text{diag}(\hat{\mathbf{g}}_j^T)\mathbf{E}_\mathbf{H}\textstyle\sum_{i=1}^{M}\mathbf{t}_i s_i + \mathbf{w}^H\text{diag}(\hat{\mathbf{g}}^T + \mathbf{e}_{\mathbf{g}_j}^T)\boldsymbol{\nu}, \qquad (24)\\ &+\mathbf{w}^H\text{diag}(\mathbf{e}_{\mathbf{g}_j}^T)(\hat{\mathbf{H}} + \mathbf{E}_\mathbf{H})\textstyle\sum_{i=1}^{M}\mathbf{t}_i s_i + n_j, \end{aligned}$$

and DRBF column vector $\mathbf{w} = diag\{\mathbf{W}\}$. We aim to solve:

$$\begin{aligned} &\min_\mathbf{w} \quad P_R \quad \text{s.t.}\\ &\text{SINR}_k \quad \geq \quad \gamma_k \ \text{ for } \ k = 1,\ldots,M \qquad (25) \end{aligned}$$

where the average sum transmission power at the relays

$$\begin{aligned} P_R &= E\{\mathbf{u}^H\mathbf{u}\}\\ &= \text{Tr}\{\mathbf{W}^H E\{\mathbf{x}\mathbf{x}^H\}\mathbf{W}\}\\ &= \mathbf{w}^H\mathbf{D}\mathbf{w}, \qquad (26) \end{aligned}$$

$$\begin{aligned} \mathbf{D} \ &\triangleq \text{diag}([\mathbf{R}_x]_{1,1}, [\mathbf{R}_x]_{2,2}, \ldots, [\mathbf{R}_x]_{n_R,n_R}), \ \text{with}\\ \mathbf{R}_x \ &\triangleq \hat{\mathbf{H}}^H(\sum_{k=1}^{M}\mathbf{t}_k\mathbf{t}_k^H)\hat{\mathbf{H}}^H + \frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\text{Tr}(\sum_{k=1}^{M}\mathbf{t}_k\mathbf{t}_k^H)\mathbf{I}_{n_R}\\ &+\sigma_\nu^2\mathbf{I}_{n_R}. \end{aligned}$$

Manipulating the SINR constraints, (25) can be expressed as

$$\begin{aligned} &\min_\mathbf{w} \quad \mathbf{w^H D w} \qquad (27)\\ &s.t. \ \ \mathbf{w^H}\left(\hat{\mathbf{U}}_\mathbf{j} - \gamma_j\hat{\mathbf{V}}_\mathbf{j}\right)\mathbf{w} \geq \gamma_j\sigma_n^2\\ &\text{for} \quad j = 1,2,\ldots,M, \ \text{ where} \end{aligned}$$

$$\hat{\mathbf{U}}_j = \text{diag}(\hat{\mathbf{g}}_j^T)\hat{\mathbf{H}}\mathbf{t_j}\mathbf{t_j}^\mathbf{H}\hat{\mathbf{H}}^\mathbf{H}\text{diag}(\hat{\mathbf{g}}_\mathbf{j}^*), \text{ and} \qquad (28)$$

$$
\begin{aligned}
\hat{\mathbf{V}}_j = &\operatorname{diag}(\hat{\mathbf{g}}_j^T)\hat{\mathbf{H}}(\sum_{i=1,i\neq j}^{M}\mathbf{t}_i\mathbf{t}_i^H)\hat{\mathbf{H}}^H\operatorname{diag}(\hat{\mathbf{g}}_j^*) \\
&+\frac{\sigma_{ce}^2}{1+\sigma_{ce}^2}\mathbf{w}^H\operatorname{diag}(\hat{\mathbf{g}}^T)\hat{\mathbf{H}}(\sum_{i=1,i\neq j}^{M}\mathbf{t}_i\mathbf{t}_i^H)\hat{\mathbf{H}}^H\operatorname{diag}(\hat{\mathbf{g}}_j^*)\mathbf{w} \\
&+\sigma_\nu^2\mathbf{w}^H(\operatorname{diag}(\hat{\mathbf{g}}_j^T)\operatorname{diag}(\hat{\mathbf{g}}_j^*)+\mathbf{R}_{\mathbf{e}_g}^j)\mathbf{w} \\
&+\mathbf{w}^H\operatorname{diag}(\hat{\mathbf{H}}(\sum_{i=1}^{M}\mathbf{t}_i\mathbf{t}_i^H)\hat{\mathbf{H}}^H))\mathbf{R}_{\mathbf{e}_g}^j\mathbf{w} \\
&+\frac{\sigma_g^2\sigma_{ce}^2}{1+\sigma_{ce}^2}\mathbf{w}^H\operatorname{diag}(\sum_{i=1}^{M}\mathbf{t}_i\mathbf{t}_i^H)\mathbf{R}_{\mathbf{e}_g}^j\mathbf{w}.
\end{aligned}
\tag{29}
$$

We remark that with the above definitions, problem (27) has been transformed into one equivalent to Eq. (17) in [4]. However, for the multiple antenna source considered here, the above matrices differ considerably from their counterparts $\mathbf{F_k}, k=1,2,\ldots,M$ in [4]. As these problems are equivalent, the discussion surrounding [4], Eq. (18) applies and semi-definite relaxation can be employed. In particular, the relaxed problem achieves the optimal rank-1 solution for the case of $M \leq 3$ users [15]. That is, by defining $\mathbf{Z} \triangleq \mathbf{w}\mathbf{w}^H$, and dropping constraint rank$(\mathbf{Z})=1$, (27) becomes

$$
\begin{aligned}
\min_{\mathbf{Z}} \quad & \operatorname{Tr} \quad (\mathbf{ZD}) \text{ s.t.} \\
& \operatorname{Tr} \quad (\mathbf{Z}(\mathbf{E}_k-\gamma_k\mathbf{F}_k)) \geq \gamma_k\sigma_n^2 \text{ and } \mathbf{Z}\succeq 0 \\
& \quad\quad \text{for } k=1,2,\ldots,M.
\end{aligned}
\tag{30}
$$

B. Joint Precoding and Relay Beamforming

To achieve both low-power precoding and relay beamforming, we propose the following alternation algorithm:

1. Initialize DRBF vector $\mathbf{w} = c\mathbf{v}$, where constant c is chosen large relative to σ_n^2, and where $\mathbf{v} = [e^{j\theta_1}, e^{j\theta_2}, \ldots e^{j\theta_{n_R}}]^T$, and $\theta_i, 1 \leq i \leq n_R$, are i.i.d. uniform random variables over $[0, 2\pi]$.

2. Check feasibility of the constraints of (22). If infeasible, loosen the SINR constraints and go back to Step 1.

3. Solve (22) using SDP to optimize the precoder with the current relay weights fixed. Use the rank-one matrices to obtain the precoder vectors.

4. Solve (30) using SDP. If there is a rank-one solution, e.g., when $M \leq 3$, the DBRF vector is the eigenvector corresponding to the largest eigenvalue of the rank-one matrix. Otherwise apply the randomization method in [16].

5. If the relay sum power is sufficiently close to a fixed point, or else if a predetermined number of iterations is exceeded, then stop. Otherwise go back to Step 3.

We remark that since transmission powers of the source and relays are both lower-bounded, and that in each of Steps 3 and 4 power is non-increasing, it can be proven that the power values converge to a fixed point. This is apparent since the optimizations in Steps 3 and 4 each have the same SINR constraints. A proof is omitted due to lack of space. However, due to non-convex of the optimization problem, this fixed point may not be the globally optimal point. Also, while power values reach a fixed point, the precoder and DRBF vectors themselves may fluctuate during the iterations.

V. Numerical results

We investigate the performance of the proposed algorithm under interference, noise, and imperfect CSI with different levels of channel estimation quality. As explained earlier, CSI is assumed to be available to the BS. The channel coefficient matrix $\mathbf{H}$ and vector $\mathbf{g}_k$ are assumed to be independent where $\mathbf{H}$ represents the set of n_R distributed channels from the source to the relays and $n_R \times 1$ vector $\mathbf{g}_k$ represents the n_R channels from the relays to the kth destination. Channel vectors $\mathbf{g}_k$ are also assumed to be mutually independent.

In the scenario considered, the cooperative system has N=6 source antennas, n_R=12 relays and M=4 destinations. The source antenna array is assumed to have spatial correlation corresponding to a uniform linear array (ULA) with correlation between adjacent source antennas of 0.5. Since the relays are part of the infrastructure of the system, the source to relay distances are fixed and equal. For imperfect channel estimation, we set $\sigma_{ce}^2 = \operatorname{Tr}(\mathbf{R}_T^{-1})/P_{tr} = 0.01$, with training power to noise ratio P_{tr}/σ_n^2 set to 29.7dB.

For channel estimation from relays to destinations, three cases are considered: P_{tr}/σ_n^2 is chosen to be 20 dB (*high* training SNR), 15 dB (*medium* training SNR) and 10 dB *low* training SNR), corresponding to users at the nominal distance of 250m. The CSI error variances, quantified by the diagonal elements of the $n_R \times n_R$ matrices $\mathbf{R}_{\mathbf{e}_g}^j$ for destinations $j=1,\ldots M$, are determined according to path loss from uniformly distributed users ranging from 250 to 750 meters according to log-distance model

$$
PL = PL_0 + 10\gamma\log_{10}\frac{d}{d_0} \tag{31}
$$

where path loss exponent $\gamma = 2.5$, and reference distance $d_0 = 500$m, and the effects of fast fading are averaged out. Reference power PL_0 is irrelevant due to normalization. These additional path loss effects result in CSI estimation error variances in the ranges [20, 31.9], [15, 26.9] and [10, 21.9] dB, corresponding to the cases of *high*, *medium* and *low* training-power-to-noise ratios, respectively. Zero mean independent Gaussian noise is then added to the channel estimates with the above CSI error variances.

Fig. 2 compares source transmission power (normalized to σ_n^2) (i) for perfect CSI, (ii) the proposed formulation that takes imperfectly estimated CSI into account for the three different channel training SNRs, as well as (iii) a system with imperfectly estimated CSI, for the case of *high* training power, and with optimization that ignores CSI uncertainly. As shown, for the proposed method, as channel estimation training SNR degrades from high to low, power consumption at the source increases. When channel estimation is perfect, in (19) and (20), $\mathbf{Q}_{j,j}$ is positive definite. As CSI estimation quality degrades, eigenvalues of $\mathbf{Q}_{j,j}$ decrease while eigenvalues of $\mathbf{Q}_{i,j}$ increase, tightening the constraints, until they become infeasible. The SINR threshold γ has a similar effect on the feasibility of the constraints. In the situation where channel estimation error is not taken into account, these constraints may not be guaranteed.

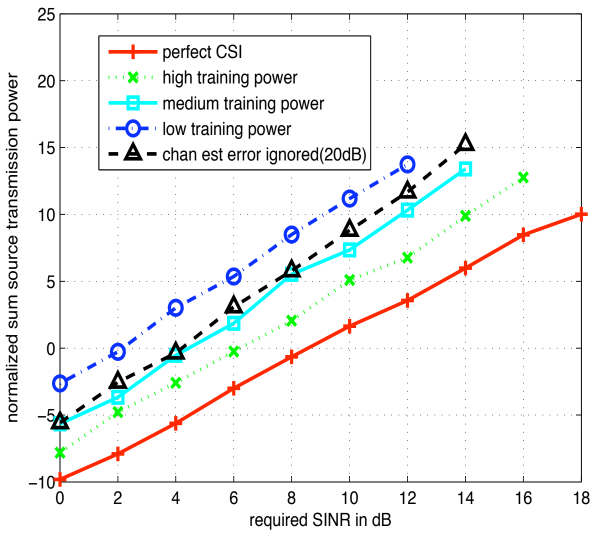

Fig. 2. Source sum transmission power versus SINR threshold: effect of imperfect CSI and effect of not taking channel estimation error into account.

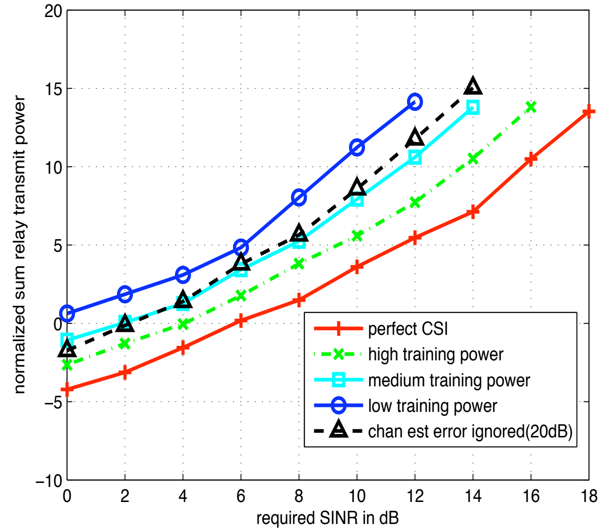

Fig. 3. Relay sum transmission power versus SINR threshold: effect of imperfect CSI and effect of not taking channel estimation error into account.

Fig. 3 shows the sum relay powers corresponding to the above cases. As expected, as channel estimation quality degrades, relay power consumption increases. Here, feasibility of the constraints depends on eigenvalues of $\mathbf{U}_k - \gamma_k \mathbf{V}_k$ and similar behavior is observed with regard to the effects of taking imperfect CSI into account. As shown by these two figures, ignoring the effects of imperfect CSI can result in a loss of performance of approximately 5dB over the range of target SINR constraints from 0 to 14 dB. At lower quality CSI, the problem becomes infeasible.

VI. Conclusions

Wireless transmission from a multiple-antenna base station to multiple single-antenna destinations through a network of single-antenna relays is investigated using iterative optimization of precoding and distributed relay beamforming, in the sense of minimizing source and relay power. Interference, noise, and level of CSI training power are taken into account to ensure solution feasibility. At lower CSI quality, ignoring effects of imperfect CSI can be catastrophic. As future work, performance tradeoffs could be investigated in comparison with more conservative robust beamforming formulations.

References

[1] J. N. Laneman, D. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Sept. 2004.

[2] Y. Jing and B. Hassibi, "Network beamforming using relays with perfect channel information," *ICASSP*, pp. 473–476, Apr. 2007.

[3] V. H. Nassab, S. Shahbazpanahi, A. Grami, and Z. Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. Signal Processing*, vol. 56, no. 9, pp. 4306–4316, Sept. 2008.

[4] S. Fazeli-Dehkordy, S. Shahbazpanahi, and S. Gazor, "Multiple peer-to-peer communications using a network of relays," *IEEE Trans. Signal Processing*, vol. 57, no. 4, pp. 3053–3062, Aug. 2009.

[5] Y. Zheng and S. Blostein, "Downlink distributed beamforming through relay networks," *Proc. IEEE Globecom, Honolulu, Hawii,*, 2009.

[6] S. Serbetli and A. Yener, "MMSE transmitter design for correlated MIMO systems with imperfect channel estimates: power allocation trade-offs," *IEEE Trans. Wireless Commun.*, vol. 5, no. 8, pp. 2295–2304, Oct. 2006.

[7] T. Weber, A. Sklavos, and M. Meurer, "Imperfect channel state information in MIMO transmission," *IEEE Trans. Commun.*, vol. 54, pp. 543–552, Mar. 2006.

[8] M. Ding and S. D. Blostein, "MIMO LMMSE transceiver design with imperfect CSI at both ends," *IEEE Trans. Signal Processing*, vol. 57, pp. 1141–1150, Mar. 2009.

[9] Z. Ding, W. H. Chin, and K. K. Leung, "Distributed beamforming and power allocation for cooperative networks," *IEEE Trans. Inform. Theory*, vol. 7, pp. 1817–1822, May 2008.

[10] B. Khoshnevis, W. Yu, and R. Adve, "Grassmannian beamforming for MIMO amplify-and-forward relaying," *IEEE Journal on Selected Areas Communication*, vol. 26, no. 8, pp. 1397–1407, Oct. 2008.

[11] X. Zhang, D. P. Palomar, and B. Ottersten, "Statistically robust design of linear MIMO transceivers," *IEEE Trans. Signal Processing*, vol. 56, pp. 3678–3689, Aug. 2008.

[12] L. Musavian, M. R. Nakhai, M. Dohler, and A. H. Aghvami, "Channel uncertainty on the mutual information of MIMO fading channels," *IEEE Trans. Veh. Technol.*, vol. 56, pp. 2798–2806, Sept. 2007.

[13] T. Yoo and A. Goldsmith, "Capacity and power allocation for fading MIMO channels with channel estimation error," *IEEE Trans. Inform. Theory*, vol. 52, pp. 2203–2214, May 2006.

[14] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," *Handbook of Antennas in Wireless Communications, CRC Press*, 2001.

[15] Y. Huang and S. Zhang, "Complex matrix decomposition and quadratic programming," *Mathematics of Operations Research*, vol. 32, no. 3, pp. 758–768, Aug. 2007.

[16] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Signal Processing*, vol. 5, pp. 2239–2251, June 2006.