This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

High-Rate Codes Over Space, Time, and Frequency

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***Abstract*— This paper investigates increasing space, time, and frequency diversity through linear dispersion codes (LDC) in MIMO-OFDM wireless fading channels. Two new types of block-based high-rate space-time-frequency (STF) codes: (1) double linear dispersion space-time-frequency-coding (DLD-STFC), and (2) linear dispersion space-time-frequency-coding (LD-STFC) are proposed. In addition to double LDC encoding, DLD-STFC uses three-stage LDC decoding. The LD-STFC, on the other hand, requires only one LDC procedure across multiple OFDM subcarriers, OFDM blocks and multiple antennas. Both DLD-STFC and LD–STFC are backwards compatible to uncoded MIMO-OFDM systems. Comparison to an extension of a recently proposed LDC-OFDM to MIMO systems, called MIMO-LDC-OFDM, is made in which a single LDC-OFDM codeword is mapped to one transmit antenna. This paper discusses diversity properties of these STF block based designs. An error union bound analysis provides further insights, including more restrictive LDC code design criteria. Compared to other methods of similar complexity, simulations reveal that the bit error rate (BER) performance of full-rate DLD-STFC offers superior performance.**

I. Introduction

A challenging problem in wideband multiple transmit and receive antenna system (MIMO) design is to develop new coding and modulation methods to exploit all of the diversity available across space, time and frequency domains within reasonable computation complexity limits and maintain high bandwidth efficiency. Recently, investigations into space-time-frequency codes are appearing (STFC) [1] [2]. Existing block based STFC designs assume MIMO channel coefficients remain constant over one whole STFC codeword (over multiple orthogonal frequency division multiplexing or OFDM blocks), but may vary over different STFC codewords. In general, existing STFCs are not full-rate codes.

In this paper, we consider a STFC design with the following features: (1) support of arbitrary numbers of transmit antennas, (2) channel coefficients required only over a single OFDM block instead of over a whole STFC codeword, (3) provision of up to full-rate coding, (4) compatibility with uncoded MIMO-OFDM systems and (5) moderate computation complexity.

The key idea for our STFC design is to employ linear dispersion codes (LDC), as pioneered by Hassibi and Hochwald for space time coding over block flat fading channels [3]. Linear dispersion codes (LDC) possess coding rates of up to one, and can support any configuration of transmit and receive antennas. In the literature, maximum-likelihood (ML) or sub-optimal sphere decoding (SD) are primarily chosen LDC decoding methods [3]–[5]; both of these have high computational complexity.

In this paper, we propose and compare two block-based high-rate STFCs coding procedures with rates up to one - one termed double linear dispersion space-time-frequency-coding (DLD-STFC), and the other termed linear dispersion space-time-frequency-coding (LD-STFC). In these approaches, an STF block is formed only across a subset of subcarriers instead of across all available subcarriers. Both DLD-STFC and LD-STFC simultaneously exploit space, time and frequency diversity across multiple transmit antennas, OFDM blocks, and OFDM subcarriers. A challenging issue in DLD-STFC design is to use LDC, which is 2-D, in a 3-D code design. In DLD-STFC, two stages of encoding are used, each of which is a complete LDC coding system that processes all complex symbols within one DLD-STFC codeword space. In LD-STFC, only a single LDC procedure is used for one STF block, and to achieve performance comparable to DLD-STFC, LD-STFC uses larger LDC sizes, and thus may be of higher complexity than DLD-STFC. This paper also compares these to a system using a single LDC procedure applied only across frequency and time, termed MIMO-LDC-OFDM. This paper discusses the diversity properties of DLD-STFC and LD-STFC as well as the error union bound, which leads to novel code design criteria.

After introducing the LDC encoder in matrix form along with a MIMO-OFDM system model in Sections II and III, the systems are proposed in Section IV. Related diversity properties of STF block based designs are discussed in Section V. The LDC design criteria based on error union bound is analyzed in Section VI. System performance is discussed in Section VII. The following notation is used: $(\cdot)^\dagger$ denotes matrix pseudoinverse, $(\cdot)^T$ matrix transpose, $(\cdot)^H$ matrix transpose conjugate, and $C^{A\times B}$ denotes a complex matrix with dimensions $A\times B$.

II. LDC Encoding

Assume that an uncorrelated data sequence has been modulated using complex-valued source data symbols chosen from an arbitrary, e.g. r-PSK or r-QAM, constellation. A $T\times M$ LDC matrix codeword, $\mathbf{S}_{LDC}$, is transmitted from M transmit channels and occupies T channel uses and encodes Q source data symbols. Denote the LDC codeword matrix as $\mathbf{S}_{LDC}\in C^{T\times M}$, and $\mathbf{A}_q\in C^{T\times M}, \mathbf{B}_q\in C^{T\times M}, q=1,...,Q$ are called dispersion matrices.

Just as in [6], we consider the case $\mathbf{A}_q=\mathbf{B}_q, q=1,...,Q$. We have the matrix LDC encoding equation,

$$vec(\mathbf{S}_{LDC})=\mathbf{G}_{LDC}\mathbf{s}, \tag{1}$$

where $\mathbf{s}=\begin{bmatrix} s_1 & \cdots & s_Q\end{bmatrix}^T$ is the source complex symbol vector,

$$\mathbf{G}_{LDC}=[vec(\mathbf{A}_1),...,vec(\mathbf{A}_Q)] \tag{2}$$

is the LDC encoding matrix. To estimate the data symbol vector in (1), we may calculate the Moore-Penrose pseudo-inverse of LDC encoding matrix G_{LDC} offline and store the result.

III. MIMO-OFDM system model

Consider a MIMO-OFDM system with N_T transmit antennas, N_R receive antennas and in a block of N_C OFDM subcarriers per antenna. The channel between the m-th transmit antenna and n-th receive antenna in the k-th OFDM block experiences frequency-selective, temporally flat Rayleigh fading with channel coefficients $\mathbf{h}_{m,n}^{(k)}=\left[h_{m,n(0)}^{(k)},...,h_{m,n(L)}^{(k)}\right]^T, m=1,...,N_T, n=1,...,N_R$, where $L=\max\{L_{m,n}, m=1,...,N_T, n=1,...,N_R\}$, $L_{m,n}$ is the frequency-selective channel order of the path between the m-th transmit antenna and n-th receive antenna. We assume constant channel coefficients within one OFDM block but statistically independent among different OFDM blocks.

Denote $x_{m,p}^{(k)}$ as the channel symbol transmitted on the p-th subcarrier from the m-th transmit antenna during the k-th OFDM block. The channel symbols $\left\{x_{m,p}^{(k)}, m = 1, ...N_T, p = 1, ..., N_C\right\}$ are transmitted on N_C subcarriers in parallel by N_T transmit antennas.

Each receive antenna signal experiences additive complex Gaussian noise. At the transmitter, a cyclic prefix (CP) guard interval is appended to each OFDM block. After CP is removed, the received channel symbol sample $y_{n,p}^{(k)}$ at the n-th receive antenna, is

$$y_{n,p}^{(k)} = \sqrt{\frac{\rho}{N_T}} \sum_{m=1}^{N_T} H_{m,n,p}^{(k)} x_{m,p}^{(k)} + v_{n,p}^{(k)}, \tag{3}$$

where $n = 1, ..., N_R$, $p = 1, ..., N_c$, $H_{m,n,p}^{(k)}$ is the p-th subcarrier channel gain from the m-th transmit antenna and n-th receive antenna during the k-th OFDM block,

$$H_{m,n,p}^{(k)} = \sum_{l=o}^{L} h_{m,n(l)}^{(k)} e^{-j(2\pi/N_c)l(p-1)}, \tag{4}$$

and the additive noise is circularly symmetric, zero-mean, complex Gaussian with variance N_0. We assume the additive noise to be statistically independent for different n, p, and k. The normalizaton $\sqrt{\frac{\rho}{N_T}}$ ensures that the signal-to-noise-ratio (SNR) at each receive antenna ρ is independent of N_T.

IV. Proposed systems

A. DLD-STFC codeword construction

Double linear dispersion space-time-frequency-coded (DLD-STFC) block coding across N_T transmit antennas and T OFDM blocks in time is performed in two stages. Each stage is a complete LDC coding procedure itself and processes all complex symbols within the space of one DLD-STFC codeword. The first encoding stage is the frequency-time LDC (FT-LDC), performed across frequency (OFDM subcarriers) and time (OFDM blocks), enabling frequency and time diversity. The second encoding stage is the space-time (ST-LDC), performed across space (transmit antennas) and time (OFDM blocks), enabling space and time diversity.

In the FT-LDC stage, there are D LDC matrix codewords. The d-th matrix codeword is of size $T \times N_F^{(d)}, d = 1, ..., D$, and D is a multiple of N_T. We group D LDC matrix codewords into N_T sub-groups. The m-th subgroup, which is allocated to the m-th antenna, has $D_m = D/N_T, m = 1, ..., N_T$ LDC matrix codewords. The i-th LDC codeword of the m-th subgroup in the FT-LDC stage is of size $T \times N_{F(m,i)}, i = 1, ..., D_m, m = 1, ..., N_T$, where $i = d (\mathrm{mod}\ D_m)$. Notation $N_{F(m,i)}$ differs from $N_F^{(d)}$ in subscript $i = 1, ..., D_m$, the local index of FT-LDC for each transmit antenna, and superscript $d = 1, ..., D$ which stands for the global index for all D LDC codewords. We choose LDC codewords in the FT-LDC stage with size constraints

$$N_{F(m,i)} = N_{F(i)}, \tag{5}$$

$$\sum_{i=1}^{D_m} N_{F(m,i)} = N_C, \tag{6}$$

$$\sum_{d=1}^{D} N_F^{(d)} = N_T N_C, \tag{7}$$

where $i = 1, ..., D_m, m = 1, ..., N_T$. The size of DLD-STFC codeword is $N_T N_C T$ symbols, and one DLD-STFC codeword consists of D_m independent STF blocks, each of size $N_T N_{F(i)} T, i = 1, ..., D_m$. Constraint (5) specifies that the i-th LDC codewords of subgroups $m = 1, ..., N_T$, are of the same matrix size. Further, we propose that the i-th LDC codewords of all the m-th subgroups, where $m = 1, ..., N_T$, use the same LDC dispersion matrices and share the same subcarrier mappings, i.e., the same subcarrier indices of OFDM. Thus the FT-LDC coded symbols with the same subcarrier index among different transmit antennas share similar frequency-time diversity properties. The D LDC encoders of FT-LDC encode $Q_d, d = 1, ..., D$ data symbols in parallel. Each codeword is mapped to N_T transmit antennas and T OFDM blocks. Consequently, a three-dimensional array, $\mathbf{U}_{k,m,p}, k = 1, ..., T, m = 1, ..., N_T, p = 1, ...N_c,$, is created. In the FT-LDC stage, LDC symbol coding rate could be less than or equal to one.

In the ST-LDC stage, the signals from the FT-LDC stage are encoded per subcarrier. Thus there are N_C LDC encoders in this stage. Notationally, define the space time symbol matrix having been encoded in FT-LDC stage for the p-th OFDM subcarrier as $\mathbf{U}_p$, and $[\mathbf{U}_p]_{k,m} = \mathbf{U}_{k,m,p}, k = 1, ..., T, m = 1, ..., N_T, p = 1, ...N_C$. Denote $\mathbf{u_p^{vec}} = \mathbf{vec}(\mathbf{U_p})$, which is the source signal sequence of the p-th LDC codeword to be encoded in the ST-LDC stage, where $p = 1, ...N_C$. This stage enables space and time diversity. In this stage, LDC symbol coding rate is required to be one or full-rate.

B. LD-STFC codeword construction

We also propose an alternative LDC system with a single combined STFC stage, termed LD-STFC. This comprises only one complete LD coding procedure, and one LDC codeword is applied across multiple OFDM blocks and multiple antennas.

In one LD-STFC codeword, there are D LDC matrix codewords. The i-th matrix codeword is of size $T \times N_{LD}^{(i)}, i = 1, ..., D$, and $N_{LD}^{(i)}$ is a multiple of N_T. We set constraint

$$N_C = \frac{1}{N_T} \sum_{i=1}^{D} N_{LD}^{(i)}. \tag{8}$$

We partition the i-th LDC codeword into N_T matrix blocks, each of which is of size $T \times N_{LD(m,i)}$, and

$$N_{LD(m,i)} = \frac{1}{N_T} N_{LD}^{(i)}. \tag{9}$$

We map each $T \times N_{LD(m,i)}$ block into the m-th transmit antenna, where T denotes the number of OFDM blocks. Thus each LDC codeword is across multiple space (antennas), time (OFDM blocks) and frequency (OFDM subcarriers). The size of an LD-STFC codeword is $N_T N_C T$ symbols, and one LD-STFC codeword consists of D STF blocks, each with size $N_T N_{LD(m,i)} T, i = 1, ..., D$.

C. DLD-STFC system receiver

In a DLD-STFC receiver, signal reception involves three estimation steps: (1) estimation of MIMO-OFDM signals for one entire DLD-STFC block, i.e., T OFDM blocks transmitted from N_T antennas; (2) estimation of the source symbols of the ST-LDC; (3) estimation of the source symbols of the FT-LDC. After the third estimation step, data bit detection is performed.

Denote the d-th data source symbol vector with zero-mean, unit variance for the d-th LDC codeword of the FT-LDC stage as $\mathbf{s^{(d)}} = \left[\begin{array}{cccc} s_1^{(d)} & s_2^{(d)} & \cdots & s_{Q_d}^{(d)} \end{array}\right]$ where $d = 1, ..., D$, and Q_d denotes the number of data source symbols encoded in the d-th LDC codeword $\mathbf{S}_{FT\text{-}LDC}^{(d)}$ of the FT-LDC stage and $\widehat{\mathbf{s}^{(d)}}$ is the corresponding estimated data source symbol vector.

In addition, denote the estimate of $\mathbf{S}_{FT\_LDC}^{(d)}$ as $\widehat{\mathbf{S}_{FT\_LDC}^{(d)}}$, the estimate of $\mathbf{u}_p^{vec}$ as $\widehat{\mathbf{u}_p^{vec}}$, and the estimated $\mathbf{S}_{ST\_LDC}^{(p)}$ as $\widehat{\mathbf{S}_{ST\_LDC}^{(p)}}$. Denote the LDC encoding matrices needed to obtain $\mathbf{S}_{FT\_LDC}^{(d)}$ and $\mathbf{S}_{ST\_LDC}^{(p)}$ as $\mathbf{G}_{FT\_LDC}^{(d)}$ and $\mathbf{G}_{ST\_LDC}^{(p)}$, respectively.

For simplicity of discussion, we consider the case that $\mathbf{G}_{FT\_LDC}^{(d)} = \mathbf{G}_{FT\_LDC}$, $\mathbf{G}_{ST\_LDC}^{(p)} = \mathbf{G}_{ST\_LDC}$, $d = 1, ..., D$, $p = 1, ..., N_C$ are all unitary matrices and $Q_d = Q, d = 1, ..., D$. The covariance matrices of MIMO-OFDM channel symbols are then identity matrices. This can also be generalized to the case of non-identically distributed uncorrelated symbols.

1) First estimation step - MIMO-OFDM signal estimation: In the proposed DLD-STFC decoding algorithm, LDC decoding is independent of MIMO-OFDM signal estimation. Thus the proposed DLD-STFC system could be made backwards-compatible with uncoded MIMO-OFDM systems. The advantage arising from DLD-STFC decoding is that it is not required that channel coefficients remain constant over multiple OFDM blocks. In the simulations, minimum-mean-squared-error (MMSE) equalizers are chosen to investigate error performance.

2) Second estimation step - ST-LDC block signal estimation: Reorganizing the results of the MIMO OFDM estimation into N_C estimated LDC matrix codewords $\widehat{\mathbf{S}_{ST\_LDC}^{(p)}}$, the estimates are

$$\widehat{\mathbf{u}_p^{vec}} = \left[\mathbf{G}_{ST\_LDC}^{(p)}\right]^{\dagger} vec\left(\widehat{\mathbf{S}_{ST\_LDC}^{(p)}}\right), \tag{10}$$

where $p = 1, ..., N_C$.

3) Third estimation step - FT-LDC block signal estimation: Reorganizing the estimation results of step 2 into D estimated LDC matrix codewords $\widehat{\mathbf{S}_{FT\_LDC}^{(d)}}, d = 1, ..., D$ of the FT-LDC stage, we obtain

$$\widehat{\mathbf{s}^{(d)}} = \left[\mathbf{G}_{FT\_LDC}^{(d)}\right]^{\dagger} vec\left(\widehat{\mathbf{S}_{FT\_LDC}^{(d)}}\right), \tag{11}$$

where $d = 1, ..., D$.

D. Symbol coding rate for DLD-STFC, LD-STFC and MIMO-LDC-OFDM systems

For DLD-STFC, assume that the d-th LDC matrix codeword of the FT-LDC stage is encoded using Q_d complex source symbols. For LD-STFC, assume that the d-th LDC matrix codeword is also encoded using Q_d complex source symbols. We also consider a third system with only FT-LDC stage (each LDC codeword is not across multiple transmit antennas but transmitted on one antenna), termed MIMO-LDC-OFDM, i.e. straightforwardly applying LDC-OFDM as proposed in [6] to each antenna of a MIMO system.

We define the symbol coding rate of each of the three systems as

$$R = \frac{\sum_{d=1}^{D} Q_d}{N_T N_C T}. \tag{12}$$

E. Complexity issues

DLD-STFC and LD-STFC require coding matrices with the property that STFC codeword symbols are uncorrelated. Thus, to reduce computation, both DLD-STFC and LDSTFC receivers may advantageously employ multiple successive estimation stages instead of single-stage maximum likelihood or sphere decoding detectors. In principle, it is possible to utilize a single STF block across all transmit antennas, subcarriers and OFDM blocks, and a full rate STFC design would need codeword matrices of size $N_T N_C T \times N_T N_C T$, which leads to extremely high computation complexity. The complexity of proposed systems with smaller STF blocks is reduced.

V. Diversity properties

Note that since both DLD-STFC and LD-STFC are STF block based designs, an analysis of pairwise error probability can be employed to determine diversity order. Both methods involve LDC coding within either a $T \times N_{F(i)} N_T$ block or a $T \times N_{LD(m,i)} N_T$ block. A unified notation $N_{freq(i)}$ is used for both $N_{F(i)}$ and $N_{LD(m,i)}$. For DLD-STFC, this block is created after encoding the i-th FT-LDC codeword on all transmit antennas for the corresponding ST-LDC codewords. For LD-STFC, this block is created after encoding the i-th LDC codeword across all transmit antennas and OFDM blocks.

Su and Liu [7] recently analyzed the diversity of STFC based on a block of size $T \times N_C N_T$ (in [7] N denotes the number of subcarriers). In contrast, the number of subcarriers used for each STF block in DLD-STFC and LD-STFC is typically much less than N_C. Following the strategy of [7] but for STF block sizes of $T \times N_{freq(i)} N_T$, it can be shown that the upper bound of diversity order,

$$\min\left\{N_{freq(i)} N_R T, T \sum_{m=1}^{N_T} \sum_{n=1}^{N_R} (L_{m,n} + 1)\right\},$$

could be equal to the upper bound of the diversity order for a STF block of size $T \times N_C N_T$. Due to space limitations, detailed derivations are omitted. Thus, the smaller block size STFC design may in fact achieve high performance with lower complexity. The necessary condition for this to occur is that the rank of the channel correlation matrix be less than or equal to the diversity order of one STF block.

However, the actual diversity order achieved is based on the specific LDC design chosen. In the original LDC paper [3], diversity order is not optimized. In [4], both capacity and error probability are used as criteria but the diversity analysis is based on quasi-static flat fading space-time channels. The proposed LD-STFC includes only one LDC procedure operating in 3-D STF space. The diversity order of this LDC thus determines the diversity order of LD-STFC. In contrast, DLD-STFC includes two complete LDC procedures, operating over different 2-D planes in STF. If FT-LDC and ST-LDC procedures achieve full diversity order, then DLD-STFC can achieve diversity order up to full diversity order. In addition, in DLD-STFC, source symbols for ST-LDC are coded FT-LDC symbols. Thus time dependency is already included, and therefore the upper bound of the additional maximal diversity order for ST-LDC is N_T instead of $N_T T$. As mentioned earlier, DLD-STFC operates on much smaller 2-D FT-LDC and ST-LDC blocks instead of larger 3-D STF blocks.

VI. Design criteria based on union bound

The error union bound (EUB), an upper bound on the average error probability, is an average of the pairwise error probabilities between all pairs of codewords. Based on EUB, we analyze an LDC coding stage across multiple transmit antennas, i.e., the ST-LDC stage of DLD-STFC and the STF stage of LD-STFC. In [8], space time codes were analyzed based on union bound, where channel gains are assumed constant over time during the entire space time codewords. We provide a union bound analysis for MIMO OFDM channels whose gains may vary over the time duration of an LDC codeword, e.g., over different OFDM blocks. Basically, the EUB can be written as

$$P_U = \sum_{a=1}^{N_B} p_a \sum_{b \neq a}^{N_B} PEP_{ab} \leq (N_B - 1) \max_{ab} PEP_{ab} \tag{13}$$

where p_a is the probability that LDC codeword $\mathbf{X}^{(a)}$ was transmitted, PEP_{ab} is the probability that receiver decides $\mathbf{X}^{(b)}$ when $\mathbf{X}^{(a)}$ is actually transmitted, and N_B is LDC code book size.

We write a unified system equation for one STF block as

$$\mathbf{R}_U = \mathbf{H}_U \sum_{q=1}^{Q} vec(\mathbf{A}_q) s_q + \mathbf{V}_U \tag{14}$$

where $\mathbf{R}_U$ and $\mathbf{V}_U$ are the received signal and additive noise vectors, respectively, $\mathbf{A}_q, q = 1, ..., Q$ are linear dispersion matrices, $s_q, q = 1, ..., Q$ are source symbols for this LDC coding procedure. $\mathbf{H}_U$, denotes the channel matrix corresponding to different code mappings.

For simplicity, in block $C^{(i)}$, consider the case that the subcarrier indices chosen from all the OFDM blocks are the same, and denote subcarrier indexes chosen $\{p^{(m)}_{n_{F(i)}}, n_{F(i)} = 1_{(i)}, ..., N_{freq(i)}, i = 1, ..., D_m, m = 1, ..., N_T\}$, where $i = 1, ..., D_m$ for DLD-STFC or $i = 1, ..., D$ for LD-STFC.

For LD-STFC, $\mathbf{H}_U = \mathbf{H}^{(i)}_{LD\_STFC}$, and

$$\mathbf{H}^{(i)}_{LD\_STFC} = \begin{bmatrix} \mathbf{H}^{(i)}_{LD\_STFC(1,1)} & \cdots & \mathbf{H}^{(i)}_{LD\_STFC(N_T,1)} \\ \vdots & \ddots & \vdots \\ \mathbf{H}^{(i)}_{LD\_STFC(1,N_R)} & \cdots & \mathbf{H}^{(i)}_{LD\_STFC(N_T,N_R)} \end{bmatrix}$$

where

$$\mathbf{H}^{(i)}_{LD\_STFC(m,n)} = diag(H^{(1)}_{m,n,p^{(m)}_{1_{(m,i)}}}, ..., H^{(T)}_{m,n,p^{(m)}_{1_{(m,i)}}}, ..., H^{(1)}_{m,n,p^{(m)}_{N_{LD(m,i)}}}, ..., H^{(T)}_{mn,p^{(m)}_{N_{LD(m,i)}}})$$

and $p^{(m)}_{n_{F(i)}}, n_{F(i)} = 1_{(m,i)}, ..., N_{LD(m,i)}$ For the ST-LDC stage of DLD-STFC, $\mathbf{H}_U = \mathbf{H}^{(p_{n_{F(i)}})}_{DLD\_STFC\_ST}$, with

$$\mathbf{H}^{(p_{n_{F(i)}})}_{DLD\_STFC\_ST} = \begin{bmatrix} \mathbf{H}^{(p_{n_{F(i)}})}_{DLD\_STFC\_ST(1,1)} & \cdots & \mathbf{H}^{(p_{n_{F(i)}})}_{DLD\_STFC\_ST(N_T,1)} \\ \vdots & \ddots & \vdots \\ \mathbf{H}^{(p_{n_{F(i)}})}_{DLD\_STFC\_ST(1,N_R)} & \cdots & \mathbf{H}^{(p_{n_{F(i)}})}_{DLD\_STFC\_ST(N_T,N_R)} \end{bmatrix}$$

where

$$\mathbf{H}^{(p_{n_{F(i)}})}_{DLD\_STFC\_ST(m,n)} = diag(H^{(1)}_{m,n,p^{(m)}_{n_{F(i)}}}, \cdots, H^{(T)}_{m,n,p^{(m)}_{n_{F(i)}}}),$$

and $p^{(m)}_{n_{F(i)}}, n_{F(i)} = 1_{(i)}, \cdots, N_{F(i)}$.

Denote the channel weighted inner product between two dispersion matrices as

$$\begin{aligned} \Omega_{p,q} &= \langle vec(\mathbf{A}_p), vec(\mathbf{A}_q)\rangle_{\mathbf{H}_u} \\ &= \frac{1}{2}\left(\begin{array}{l} Tr\left[[vec(\mathbf{A}_p)]^H [\mathbf{H}_U]^H \mathbf{H}_U vec(\mathbf{A}_q)\right] + \\ Tr\left[[vec(\mathbf{A}_q)]^H [\mathbf{H}_U]^H \mathbf{H}_U vec(\mathbf{A}_p)\right] \end{array}\right) \\ &= \mathrm{Re}Tr\left([vec(\mathbf{A}_p)]^H [\mathbf{H}_U]^H \mathbf{H}_U vec(\mathbf{A}_q)\right) \\ &= \mathrm{Re}Tr\left(\mathbf{H}_U vec(\mathbf{A}_p) [vec(\mathbf{A}_q)]^H [\mathbf{H}_U]^H\right) \end{aligned} \tag{15}$$

and

$$\Omega_{q,q} = \|\mathbf{H}_U vec(\mathbf{A}_q)\|_F^2 \geq 0 \tag{16}$$

where $p,q = 1, ..., Q$ Denote squared pairwise Euclidean distance between two received codewords $\mathbf{X}^{(a)}$ and $\mathbf{X}^{(b)}$ and for the given channel $\mathbf{H}_U$ as

$$\begin{aligned} \mathcal{D}_{a,b} &= \left\|\mathbf{H}_U\left(\mathbf{X}^{(a)} - \mathbf{X}^{(b)}\right)\right\|_F^2 \\ &= \left\|\sum_{q=1}^{Q}\left[[\mathbf{H}_U vec(\mathbf{A}_q)]\left(s_q^{(a)} - s_q^{(b)}\right)\right]\right\|_F^2 \\ &= \sum_{q}^{Q}\left[\Omega_{q,q}\left|e_q^{(a,b)}\right|^2\right] + 2\sum_{q=1}^{Q}\sum_{p<q}^{Q}\left[\Omega_{q,p} e_q^{(a,b)} e_p^{(a,b)}\right] \end{aligned} \tag{17}$$

where

$$e_q^{(a,b)} = s_q^{(a)} - s_q^{(b)}$$

is the difference between source symbol sequences (a) and (b) at the q-th position. Assume codeword $\mathbf{X}$ has unit energy, i.e., $\|\mathbf{X}\|_F^2 = 1$.

The pairwise error probability conditioned on channel $\mathbf{H}_U$ is

$$PEP_{ab|\mathbf{H}_U} = Q\left(\sqrt{\frac{\eta}{2}\mathcal{D}_{ab}}\right) \tag{18}$$

where η denotes signal to noise ratio (SNR).

The EUB conditioned on channel $\mathbf{H}_U$ is

$$P_{U|\mathbf{H}_U} = \sum_{a=1}^{N} p_a \sum_{b \neq a}^{N} Q\left(\sqrt{\frac{\eta}{2}\mathcal{D}_{ab}}\right) \tag{19}$$

Denote

$$\Delta_1^{(a,b)} = \frac{\eta}{2}\sum_{q}^{Q}\left[\Omega_{q,q}\left|e_q^{(a,b)}\right|^2\right] \tag{20}$$

and

$$\Delta_2^{(a,b)} = \frac{\eta}{2} 2\sum_{q=1}^{Q}\sum_{p<q}^{Q}\left[\Omega_{q,p} e_q^{(a,b)} e_p^{(a,b)}\right]. \tag{21}$$

Using (15), (16), (17), (19), (20) and (21), we get

$$P_{U|\mathbf{H}_U} = \sum_{a=1}^{N} p_a \sum_{b \neq a}^{N} Q\left(\sqrt{\Delta_1^{(a,b)} + \Delta_2^{(a,b)}}\right) \tag{22}$$

If all source symbols are equally likely, i.e. $p_a = \frac{1}{N}$ for all a, the following two lemmas from [8], apply. Note that while our definitions of $\Omega_{p,q}$ and $\Omega_{q,q}$ differ from those of [8], the results apply here with only minor modifications, so the proofs are omitted.

Lemma 1: [8] By carefully selecting terms over a and b, we can always pair up terms in the expression for $P_{U|H}$ as follows

$$q = \frac{g}{N}\left[Q\left(\sqrt{\Delta_1 + \Delta_2}\right) + Q\left(\sqrt{\Delta_1 - \Delta_2}\right)\right] \tag{23}$$

where $\Delta_1 \geq \Delta_2 \geq 0$ and g is an integer denoting the number of such pairs.

Lemma 2: [8] For a given Δ_1, the pair $q = \frac{g}{N}\left[Q\left(\sqrt{\Delta_1 + \Delta_2}\right) + Q\left(\sqrt{\Delta_1 - \Delta_2}\right)\right]$ in the above is minimized if and only if $\Delta_2 = 0$

In the context of linear dispersion codes in 2-D rapid fading channels, we have the following optimal design criterion based on EUB conditioned on channel realization $\mathbf{H}_U$:

Proposition 1: Consider LDC with $T \times M$ dispersion matrices $\mathbf{A}_q, q = 1, ..., Q$ used for real and imaginary parts of source symbols,

$$\begin{aligned} \mathbf{A}_q [\mathbf{A}_q]^H &= I_T, \quad \text{if} \quad \mathrm{T} \leq \mathrm{M} \\ [\mathbf{A}_q]^H \mathbf{A}_q &= I_M, \quad \text{if} \quad \mathrm{T} \geq \mathrm{M} \end{aligned}$$

Union bound $P_{U|\mathbf{H}_U}$ achieves a minimum iff the matrices satisfy the following condition

$$\Omega_{p,q} = \mathrm{Re}Tr\left(\left[vec(\mathbf{A}_p)\right]^H \left[\mathbf{H}_U\right]^H \mathbf{H}_U vec(\mathbf{A}_q)\right) = 0 \quad (24)$$

for any $1 \leq p \neq q \leq Q$.

Proposition 1 is equivalent to requiring $vec(\mathbf{A}_p)$ and $vec(\mathbf{A}_p)$ to be pairwise orthogonal for any weight $\Theta = [\mathbf{H}_U]^H \mathbf{H}_U$. Note that for quasi-static channels, the result is of the form [8]

$$\Omega_{p,q} = \mathrm{Re}Tr\left(\left[(\mathbf{A}_p)\right]^H \left[\mathbf{H}\right]^H \mathbf{H}(\mathbf{A}_q)\right) = 0. \quad (25)$$

Although our results are extensions to 2-D time-varying channels, the induction argument in the proof is quite similar and uses Lemma 2.

Based on average channel $\mathbf{H}_U$, we also have the following suboptimal criterion for unknown channels to transmitters.

Theorem 1: Consider LDC with $T \times M$ dispersion matrices and $\mathbf{A}_q, q = 1, ..., Q$ corresponding to real and imaginary parts of source symbols, satisfying

$$\mathbf{A}_q \left[\mathbf{A}_q\right]^H = I_T, \ \text{if} \quad \mathrm{T} \leq \mathrm{M}$$
$$\left[\mathbf{A}_q\right]^H \mathbf{A}_q = I_M, \ \text{if} \quad \mathrm{T} \geq \mathrm{M}$$

The union bound P_U based on averaged channel realizations is approximately minimized if

$$Tr\left[vec(\mathbf{A}_p)\left[vec(\mathbf{A}_q)\right]^H\right] = 0 \quad (26)$$

for any $1 \leq p \neq q \leq Q$.

The proof of Theorem 1 is omitted due to space limitations. Theorem 1 provides a new EUB design criterion for LDC. In [9], it is shown that uniform linear dispersion codes (U-LDC) meet this criterion.

VII. Performance

A. Simulation setup

Perfect channel knowledge (amplitude and phase) is assumed at the receiver but not at the transmitter. The number of OFDM subcarriers, N_C, is 32. In both DLD-STFC and MIMO-LDC-OFDM simulations, all LDC codewords in the FT-LDC stage are encoded using Eq. (31) of [3]. All LDC codewords in the ST-LDC stage of DLD-STFC are encoded by the newly proposed U-LDC [9]. LDC codewords in LD-STFC are encoded by Eq. (31) of [3] or newly proposed U-LDC [9].

The symbol coding rates of all systems are unity, so compared with uncoded MIMO-OFDM systems, no bandwidth is lost. The sizes of all LDC codewords in the FT-LDC stage of DLD-STFC and MIMO-LDC-OFDM are identically $T \times N_F$, as are the sizes of LDC codewords in the ST-LDC stage of DLD-STFC, $T \times N_T$, as are the sizes of LDC codewords in LD-STFC, $T \times N_{LD}$, where $N_{LD} = N_{LD_m} N_T$, and N_{LD_m} is the size of the subcarrier partition on each transmit antenna for an LDC codeword.

An evenly spaced LDC subcarrier mapping (ES-LDC-SM) for the FT-LDC of DLD-STFC and MIMO-LDC-OFDM, as well as LD-STFC, is used in simulations.

The frequency selective channel has $(L+1)$ paths exhibiting an exponential power delay profile, and a channel order of $L = 3$ is chosen. Data symbols use QPSK modulation in all simulations. No spatial correlation is assumed in simulations. The signal-to-noise-ratio (SNR) reported in all figures is the average symbol SNR per receive antenna. The matrix channel is assumed to be constant over different integer numbers of OFDM blocks, and i.i.d. between blocks. We term this interval as the channel change rate (CCR).

B. Performance comparison between DLD-STFC and uncoded MIMO-OFDM

Figure 1 shows the performance comparison of Bit Error Rate (BER) vs. SNR between LDC-STFC and uncoded MIMO-OFDM for various combinations of N_F and T in a two transmit and two receive (2×2) antenna system. Clearly, in frequency-selective Rayleigh fading channels, BER performance of DLD-STFC is notably better than that of uncoded MIMO-OFDM. The larger the dispersion matrices used, the greater the performance improvement is achieved, at a cost of increased decoding delay. Note that the simulations use U-LDC based DLD-STFC. Though we do not claim that U-LDC has full diversity, we conjecture that U-LDC achieves close to full diversity for PSK constellations. This is also expected from results in Section VI.

C. Effect of channel dynamics in DLD-STFC

Figure 2 depicts performance of LD-STFC under various different rates of channel parameter change in a 2×2 antenna system.

As discussed in Section V, STFC diversity order is maximized only if the channel provides block-wise temporal independence. As shown, the performance of DLD-STFC is significantly influenced by channel dynamics, i.e., time correlation. At high SNRs, the faster the channel changes, the better the performance. This indicates that DLD-STFC effectively exploits available temporal diversity across multiple OFDM blocks. In the future, testing on a more accurate model of channel dynamics is needed to obtain a more accurate assessment.

D. Performance comparison between DLD-STFC and MIMO-LDC-OFDM

Figure 3 compares DLD-STFC to MIMO-LDC-OFDM with same sized FT-LDC codewords in a four transmit and four receive antenna system. While at low SNRs, the performance difference between DLD-STFC and MIMO-LDC-OFDM is small, at high SNRs, DLD-STFC noticeably outperforms MIMO-LDC-OFDM. The performance gain arises from the increased spatial diversity due to the ST-LDC coding stage of DLD-STFC.

E. Performance comparison between DLD-STFC and LD-STFC

We compare space and frequency diversity of DLD-STFC with ES-LDC-SM and LDC-STFC with ES-LDC-SM in a two transmit and two receive antenna system, and remove the effects of time diversity in the channels through setting CCR to be a multiple of T.

Figure 4 compares DLD-STFC to LDC-STFC with different sized $N_T \times T \times N_{freq}$ STF blocks. In Figure 4, DLD-STFC with STF block size $2 \times 8 \times 8$ has performance similar to that of LD-STFC with STF block size $2 \times 16 \times 8$, while DLD-STFC with STF block size $2 \times 8 \times 8$ performs better than LD-STFC with STF block size $2 \times 8 \times 8$. The reason is that the diversity order of $T \times M$ U-LDC is no larger than $\min\{T, M\}$ for each matrix dimension. Thus LD-STFC with STF block size $2 \times 16 \times 8$ has the potential to achieve the same space and frequency diversity order as LD-STFC with STF block size $2 \times 8 \times 8$. For similar sized STF blocks, DLD-STFC utilizes smaller sized LDC codewords, thus reducing complexity.

VIII. Conclusion

This paper proposes two new types of block-based high-rate (up to full rate) space-time-frequency (STF) codes: (1) double linear dispersion space-time-frequency-coding (DLD-STFC), and (2) linear dispersion space-time-frequency-coding (LD-STFC). High-rate linear dispersion codes significantly improve the performance under frequency selective fading channels

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

not only in single input single output (SISO) channel OFDM transmission first shown in [6] but also in MIMO-OFDM transmission as proposed in this paper. For instance, for 2×2 MIMO systems in Figure 1, $N_F = 8$ and $T = 8$, DLD-STFC obtain, respectively, 9.8 dB gains over uncoded MIMO-OFDM at a BER of 10^{-3}. At high SNRs, the performance of DLD-STFC is notably better than that of MIMO-LDC-OFDM, achieving 2.4 dB gain at a BER of 10^{-4} in a 4×4 MIMO system as shown in Figure 3. Reiterate that, due to full rate codes used, no bandwidth is lost in Figures 1 and 3! Both DLD-STFC and LD-STFC systems simultaneously exploit the diversity of space, time, and frequency available in wideband space time multicarrier communications channels. This paper provides insights into diversity properties of STF block systems, which show the potential diversity order the proposed systems can achieve . From error union bound analysis, more restrictive LDC code design criteria are developed. This paper shows that the three-stage DLD-STFC technique has relatively attractive performance/ complexity tradeoffs.

This work was made possible through support of the Natural Sciences and Engineering Research Council of Canada, Communications and Information Technology Ontario, Bell Mobility and Samsung.

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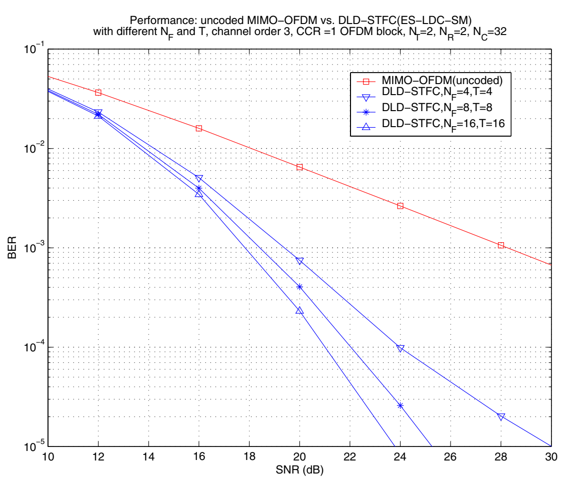

Fig. 1. BER Performance of MIMO-OFDM vs DLD-STFC with different sizes of dispersion matrices

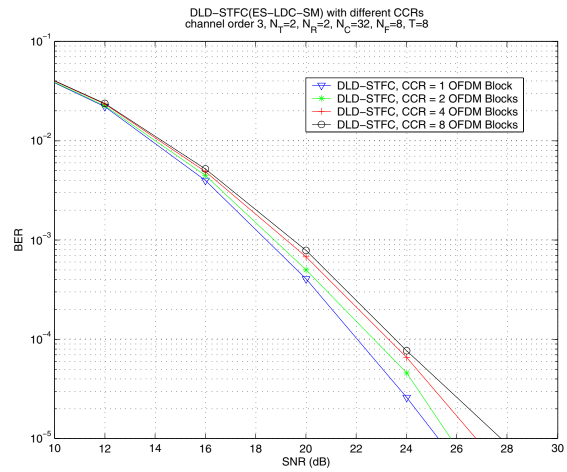

Fig. 2. BER Performance of DLD-STFC under different CCRs (channel change rates)

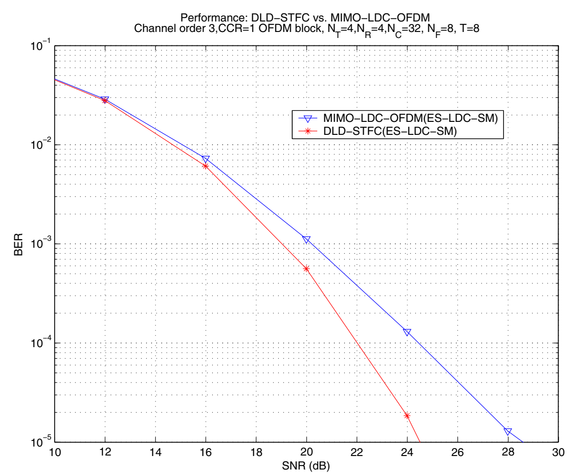

Fig. 3. BER Performance of MIMO-LDC-OFDM vs DLD-STFC with the same size of N_F

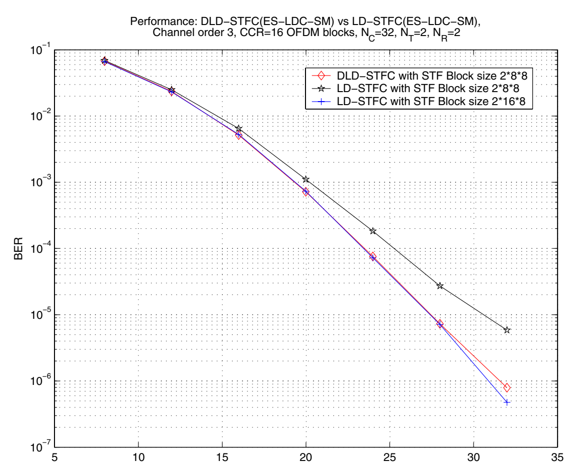

Fig. 4. BER Performance of LD-STFC vs DLD-STFC with different sizes of STF blocks