UNKNOWN TARGET MOTION COMPENSATION FOR SPOTLIGHT-MODE SYNTHETIC APERTURE RADAR IMAGERY

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ABSTRACT

In spotlight-mode Synthetic Aperture Radar (SAR), a long integration time is required to achieve a high cross-range resolution. Unfortunately, target motion during this long integration time will lead to undesirable smeared SAR images. In this paper, the effect of subaperture integration time (SIT) on motion compensation for spotlight-mode SAR is investigated. Subaperture images are used to estimate the target velocity via object's centroid measurements. The estimated velocity is used to achieve motion compensation. The target is assumed to have a constant translational motion. The results demonstrate how the accuracy of motion compensation depends on the SIT and target velocity. With the analysis in this paper, we can optimize the choice of the SIT.

1. INTRODUCTION

SAR is an imaging system which is capable of producing high resolution images. The smear effects of target movement on SAR imaging are described in [1]. Various motion compensation methods have been proposed, and the subaperture-based technique is employed in [2] to compensate for the target rotational motion.

In this paper, a motion compensation technique is investigated and analyzed for spotlight-mode SAR. The target is assumed to have a constant translational motion. A mathematical model is first developed from a tomographic point of view to relate smeared and unsmeared images via a filtering process. The parameter of this filtering process is the target's velocity. No *a priori* knowledge of this velocity is assumed and this velocity is estimated using Kalman filtering on target's centroid measurements in a sequence of subaperture images. The variance of the target centroid measurement in a smeared subaperture image is determined by incorporating the spatial correlation among image pixels. The estimated velocity is used to achieve motion compensation.

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2. MODEL OF SMEARED SAR IMAGES

In this paper, we consider a *broadside* spotlight-mode SAR collection geometry where the radar flight path is level and the center pulse is launched perpendicular to the flight path. The waveform transmitted by the radar is a linear FM chirp pulse given by $Re\{s(t)\}$, where $s(t)$ is described by

$$s(t) = exp[j(\omega_0 t + \alpha t^2)], -\tau_c/2 \leq t \leq \tau_c/2 \quad (1)$$

where α is the chirp rate, ω_0 is the RF center frequency, and τ_c is the chirp duration.

If the target is stationary, the returned signal corresponding to the radar position at azimuthal angle θ is [3]

$$r_\theta(t) = A\ Re\{\int_{-u_1}^{u_1} p_\theta(u) s(t - \frac{2(R+u)}{C}) du\} \quad (2)$$

where A is a scale factor that accounts for propagation attenuation, R is the distance from the radar to the patch center, $p_\theta(u)$ is the projection function involving 2-D integration of the target reflectivity over a plane surface associated with angle θ at range u, and $R+u$ is the distance from the radar to the projection integration plane.

If the target is moving, by modifying (2), the returned signal at radar position θ can be expressed as

$$r_\theta(t) = ARe\{\int_{-u_1}^{u_1} p_\theta(u) s(t - \frac{2(R+u+\Delta u)}{C}) du\} \quad (3)$$

where Δu is the range variation due to the target motion.

Assume that the target only has a translational motion with range velocity v_r and cross-range velocity v_c. The aperture time is denoted as t', and let the center pulse be launched at $t' = 0$. If the radar velocity V_{radar} is much greater than the target velocity, and the target size is small compare to the distance between the radar and the patch, Δu can be approximated as [4]

$$\Delta u \approx v_r t' - \frac{V_{radar} v_c}{R_0} t'^2 \tag{4}$$

where R_0 is the distance between the radar and the patch center at $t' = 0$. Δu is a function of t', therefore it varies from pulse to pulse.

As the radar moves, a set of returned signals are collected over an angular extent $\Delta\theta$. After deramp processing [3] of returned pulses, the phase-history data for the moving target $G_s(X,Y)$ is expressed as

$$G_s(X,Y) = H(X,Y) G_u(X,Y) \tag{5}$$

$$H(X,Y) = e^{j v_r \frac{T\lambda}{4\pi\Delta\theta} XY} e^{\frac{V_{radar} v_c}{R_0} (\frac{T\lambda}{4\pi\Delta\theta})^2 X^2 Y} \tag{6}$$

where T is the integration time, and λ is the wavelength of radar RF center frequency. The range spatial frequency Y is specified by $\frac{2}{C}(\omega_0 - \alpha\tau_c) \leq Y \leq \frac{2}{C}(\omega_0 + \alpha\tau_c)$, and the cross-range spatial frequency X is specified by $-\frac{2\pi}{\lambda}\Delta\theta \leq X \leq \frac{2\pi}{\lambda}\Delta\theta$. $G_u(X,Y)$ is the phase-history data if the target is stationary. Inverse DFT of sampled $G_s(X,Y)$ produces the smeared image of the moving target. Therefore, the complex-valued smeared image is the unsmeared complex-valued image convolved with the impulse response due to target motion. It can be shown that v_r causes a target shift in the cross-range in the SAR image, and the amount of shift is $v_r R_0 / V_{radar}$ [4].

3. TARGET CENTROID ESTIMATION

Statistical characterization of target centroid measurement is discussed in [5]; however, image pixel intensities are assumed to be independent random variables. In this section, we consider the variances of target centroid measurements where spatial correlations among image pixels exist. The pixel intensity at position (x,y) in a smeared SAR image is

$$I_s(x,y) = \mid g_u(x,y) \otimes h(x,y) \mid^2 \tag{7}$$

Due to speckle, $g_u(x,y)$'s are modelled as independent complex Gaussian random variables in the unsmeared image. After convolving with $h(x,y)$, $I_s(x,y)$'s become dependent random variables in the smeared image. The target centroid measurement is expressed as

$$(x_c, y_c) = \left(\frac{\sum_{x,y} x I_s(x,y)}{\sum_{x,y} I_s(x,y)} + \xi_x , \frac{\sum_{x,y} y I_s(x,y)}{\sum_{x,y} I_s(x,y)} + \xi_y \right) \tag{8}$$

where ξ_x and ξ_y take account of the effect of image quantization, and are assumed to be uniformly distributed within the cross-range and range resolution.

To calculate the variance of x_c (same for y_c), we consider the first term in x_c as the ratio M/S, $M = \sum_{x,y} x I_s(x,y)$ and $S = \sum_{x,y} I_s(x,y)$, and both M and S are random variables. The variance of x_c can be approximated as [6]:

$$var(x_c) = \frac{var(M)}{E^2(S)} - \frac{2E(M)}{E^3(S)} r + \frac{E^2(M)}{E^4(S)} var(S) + \frac{(\Delta x)^2}{12}, \tag{9}$$

where $r = E\{[M - E(M)][S - E(S)]\}$ is the correlation coefficient between M and S, and Δx is the range resolution.

To obtain $var(x_c)$, variances and expectations of M and S, as well as quantity r, have to be evaluated first. The derivations of quantities in (9) in terms of statistical characteristics of $g_u(x,y)$'s are very lengthy, and they are presented in [4].

4. TARGET VELOCITY ESTIMATION AND MOTION COMPENSATION

We estimate the target velocity by subaperture processing. The long integration time is divided into a set of consecutive short integration times. A set of target centroid measurements can be obtained from the low-resolution subaperture images. The ship velocity is estimated from these centroid measurements using a Kalman filter.

Let ΔT be the SIT. Choosing the true target centroid position and the target velocity as the state variables, the state equation of the Kalman filter is

$$\begin{pmatrix} x'_{c,n+1} \\ y'_{c,n+1} \\ v_{c,n+1} \\ v_{r,n+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'_{c,n} \\ y'_{c,n} \\ v_{c,n} \\ v_{r,n} \end{pmatrix} + \begin{pmatrix} (\Delta T)^2/2 & -\Delta T \cdot R_0/V_{radar} \\ 0 & (\Delta T)^2/2 \\ \Delta T & 0 \\ 0 & \Delta T \end{pmatrix} \begin{pmatrix} u_x \\ u_y \end{pmatrix} \tag{10}$$

where $(x'_{c,n}, y'_{c,n})^T$ is the true target centroid position in cross-range and range in the nth subaperture image. We use u_x and u_y to represent the random accelerations in cross-range and range, respectively. Term $-\frac{\Delta T}{V_{radar}} R_0$ takes account of the target shift in a SAR image due to range velocity.

Choosing the target centroid measurement from the subaperture image as the measurement variables, the

measurement equation of the Kalman filter is

$$\begin{pmatrix} x_{c,n} \\ y_{c,n} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x'_{c,n} \\ y'_{c,n} \\ v_{c,n} \\ v_{r,n} \end{pmatrix} + \begin{pmatrix} \epsilon_x \\ \epsilon_y \end{pmatrix} \tag{11}$$

where $(x_{c,n}, y_{c,n})^T$ are the centroid measurements in cross-range and range in the nth subaperture image, and (ϵ_x, ϵ_y) represent the target centroid measurement noises due to both quantization and smearing. The variance of the centroid measurement was described in Section 3. To take account of the imperfect knowledge of measurement noise when using a Kalman filter to estimate the velocity, only the variance due to image resolution (last term in (9)) is used as the centroid measurement noise variance.

With the estimated target velocity from the Kalman filter, phase error is compensated in the phase-history domain for the smeared high-resolution image. The resultant high-resolution image is formed from the compensated phase-history data. From the Kalman filter we can obtain the steady state variances of target velocity estimates, therefore we will be able to assess how well we can restore the smeared high-resolution image.

5. EXPERIMENTAL RESULTS

In our experiments, the target is a ship at sea experiencing a constant translational motion. We choose the radar center frequency to be 9.5GHz, $R_0 = 100$km and $V_{radar} = 200$m/sec. The range resolution is 0.5m, and the total integration time of 15.8sec is used to achieve 0.5m cross-range resolution. For illustrative purposes, two SITs are considered, 0.3sec and 1.5sec. The cross-range resolutions in subaperture images are 26.3m for the 0.3sec case and 5.26m for the 1.5sec case.

The variances of ship centroid and velocity estimates based on the image sequence are shown in Table 1. The centroid estimate variance is larger for a higher ship velocity, and smaller for a longer SIT due to the finer image resolution. Despite the mismatch between the centroid measurement variance in the Kalman filter and the true measurement variance, we can still estimate the ship velocity accurately with a small variance and bias [4]. A tradeoff exists in the choice of SIT. A longer SIT results better centroid estimates; however, fewer subaperture images may result a poorer Kalman filter velocity estimate. Our experiment and analysis confirm that the 1.5sec subaperture case can achieve more accurate velocity estimate than the 0.3sec case.

Motion compensations for smeared images are shown in Figs.1 to 4 where the ship is moving at 4m/sec and 45 degrees heading. In Fig.1, there is no smearing due to target motion, while in Fig.2 smearing occurs due to uncompensated ship motion. Fig.3 and 4 show subaperture-based motion compensated images for integration times of 0.3 and 1.5sec, respectively. In Fig.3, the estimated ship velocity is assumed to be $\sqrt{0.0020}$m/sec and $\sqrt{0.0546}$m/sec off the true range and cross-range velocity, respectively. In Fig.4, the estimated ship velocity is assumed to be $\sqrt{0.0012}$m/sec and $\sqrt{0.0149}$m/sec off the true velocity. As shown, the boundary for the 1.5sec subaperture compensated ship image more closely resembles the unsmeared ship image because of the lower range and cross-range velocity estimate variances compared to the 0.3sec case.

6. CONCLUSIONS

A subaperture-based motion compensation technique for spotlight-mode SAR is investigated and analyzed in this paper. The motion compensation is achieved by target velocity estimation, where the velocity is estimated by Kalman filtering on target's centroid measurements in a sequence of subaperture images. A tradeoff exists in the choice of the SIT. Our analysis quantifies the accuracy of target velocity estimate versus SIT. Therefore, we can choose the most suitable SIT.

7. REFERENCES

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Ship heading 45 degrees	Centroid Estimate Variance (m^2)		Velocity Estimate Variance (m^2/sec^2)	
	Range	Cross-range	Range	Cross-range
$v = 4$ m/sec, 0.3 integration time	1.155	62.454	0.0020	0.0546
$v = 4$ m/sec, 1.5 integration time	0.507	3.965	0.0012	0.0149
$v = 8$ m/sec, 0.3 integration time	1.200	65.663	0.0031	0.0920
$v = 8$ m/sec, 1.5 integration time	0.632	5.056	0.0017	0.0195

Table 1. Variances of ship centroid measurements and velocity estimates

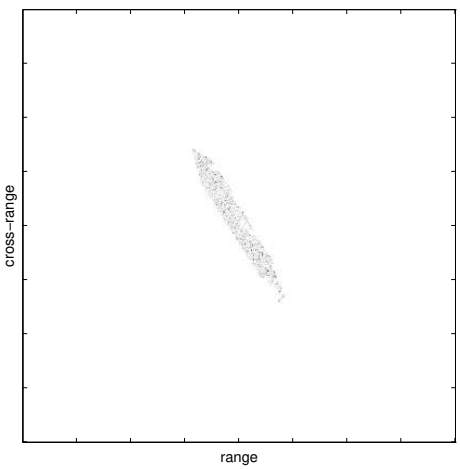

Fig. 1: Unsmeared image
(imaging area : 200 m $\times$ 200 m)

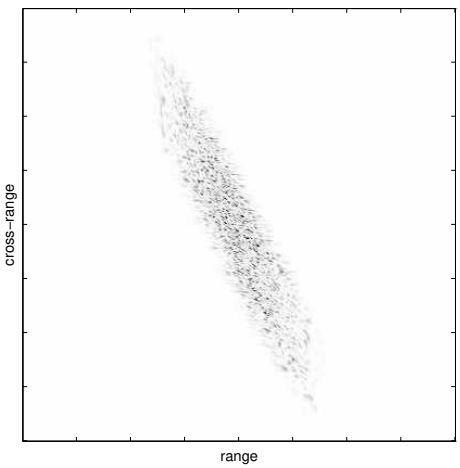

Fig. 2: Smeared image for ship speed
at 4 m/sec and 45 degrees heading

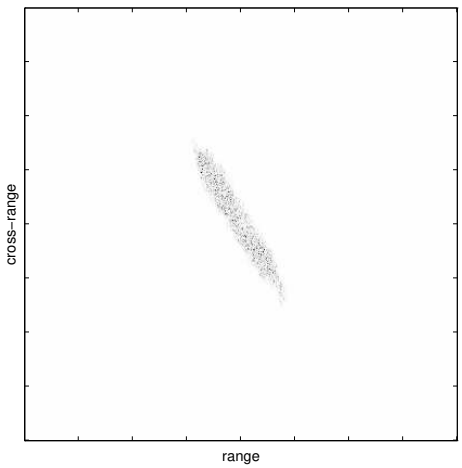

Fig. 3: De-smeared image with 0.3 sec
subaperture integration time

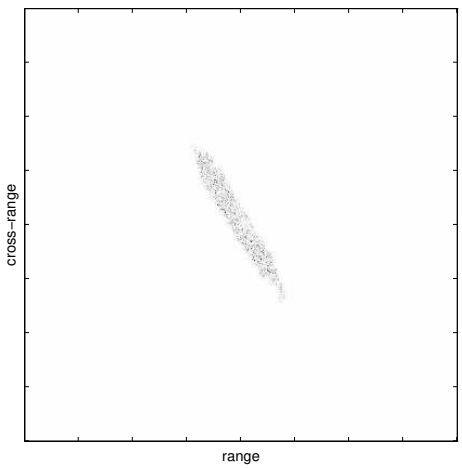

Fig. 4: De-smeared image with 1.5 sec
subaperture integration time