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Synchronization in Cooperative Networks: Estimation of Multiple Carrier Frequency Offsets

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***Abstract*— The distributed nature of cooperative networks may result in multiple carrier frequency offsets (CFOs), which make the channel time-varying and overshadow the performance gains promised by collaborative communications. However, much of the analysis in the area of cooperative communications assumes perfect synchronization. This paper seeks to address frequency synchronization in cooperative communication systems, where CFO estimation using a training sequence is analyzed. The Cramer-Rao lower bound (CRLB) for various cooperative protocols is derived. In the next step, we propose two computationally efficient iterative estimators based on the MUltiple SIgnal Characterization (MUSIC) algorithm, that can estimate the CFOs from all the relays simultaneously. Unlike existing multiple CFO estimation algorithms, the proposed estimators are effective and accurate in the presence of both large and small CFO values and numerical and simulation results show that both methods outperform the existing algorithms and reach the CRLB at mid-to-high signal to noise ratio.**

I. Introduction

COOPERATIVE diversity, which is achieved when multiple terminals share their transmit and receive antennas, has been demonstrated to provide the advantages of *multi-input-multi-output (MIMO)* systems without requiring the nodes to be equipped with multiple antennas [1]. However, the majority of the analysis in the area of cooperative communications is focused on improving capacity and reliability when assuming perfect synchronization [2]. Moreover, even though many cooperative space coding techniques are proposed that provide full spatial diversity in the presence of multiple *carrier frequency offsets (CFOs)*, they still require accurate knowledge of the CFO at the receiver [3].

The presence of multiple CFOs in cooperative networks arises due to the distributed nature of the network and due to simultaneous transmissions from separate nodes with different oscillators, resulting in a rotation of the signal constellation and *signal to noise ratio (SNR)* loss. The amount of SNR loss and accuracy of channel estimation is highly dependent on CFO estimation precision at the receiver [4]. Thus, achieving frequency synchronization is key to future deployments of cooperative networks.

Multiple-input-single-output (MISO) systems are a critical component of cooperative communication networks. Previously proposed multiple CFO estimation methods for MISO systems include [5]–[7]. In [5], a maximum-likelihood estimator (MLE) for CFO estimation for MIMO systems is presented. However, the proposed MLE is computationally very complex. Moreover, as shown in [5] the MLE performs poorly when the CFOs are close to one another. To overcome this problem, the authors propose estimators that require the training sequences to be transmitted via time division multiplexing, which results in bandwidth inefficiency and significant delay. In [6], the authors propose a correlation based estimator (CBE) that requires orthogonal training sequences to be transmitted from different antennas. However, the CBE performs very poorly when normalized CFOs are larger than .05, since orthogonality between training sequences vanishes. Also, at low CFO values the CBE experiences an error floor. In [7] an iterative scheme that eliminates the error floor associated with initial CBE estimates is proposed. However, due to the application of CBE, the estimator in [7] performs poorly at large CFO values.

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While the assumption of small CFO values in [6], [7] might hold for point-to-point MIMO systems, it is not justifiable for cooperative systems with distributed relays. Thus, accurate CFO estimation in cooperative systems is still an open area of research. In [8] an estimator based on the maximum a posterior (MAP) scheme for single-relay 3-terminal *decode-and-forward (DF)* networks is presented. However, the approach in [8] is MLE-based and suffers from the same shortcomings as in [5]. To the best of author's knowledge, the topic of CFO estimation for *amplify-and-forward (AF)* cooperative networks has not been addressed to date.

The *Cramer-Rao lower bound (CRLB)* is the lower bound on the variance of an unbiased estimator [9], and is used as a quantitative performance measure for CFO estimators [5]. Moreover, the CRLB can be applied to determine the effect of network protocol and number of relays on CFO estimation accuracy in cooperative systems. The CRLB of CFO estimation for point-to-point systems is derived in [10] and [5]. In [8], the CRLB for 3-terminal DF cooperative networks is presented. However, the analysis is limited to single-relay DF networks.

This paper seeks to derive the CRLB for CFO estimation for DF and AF multi-relay cooperative systems. Moreover, the CRLBs are used to determine the effect of relay locations and SNR on frequency synchronization in cooperative networks. Next, two iterative multiple CFO estimation algorithms are proposed using the MUltiple SIgnal Characterization (MUSIC) method [11]. The proposed algorithms jointly estimate the CFOs and channel gains corresponding to the R relays at the destination and reach the CRLB with very few iterations. Unlike the algorithms presented in [5]–[7] the proposed estimators can accurately determine CFOs independent of their value.

This paper is organized as follows: Section II derives and analyzes the CRLBs for DF and AF cooperative networks. Section III provides an overview of the MUSIC algorithm and derives and outlines the proposed iterative estimators while Section IV presents numerical results and compares the proposed estimators' performance against the CRLB.

Notations: italic letters (ϕ) are scalars, bold lower case letters ($\boldsymbol{\phi}$) are vectors, bold upper case letters ($\boldsymbol{\Phi}$) are matrices, $\boldsymbol{\Phi}_{k,m}$ represents the kth row and mth column element of $\boldsymbol{\Phi}$, $\odot$ stands for Schur (element-wise) product, and $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\text{Tr}(\cdot)$ denote conjugate, transpose, conjugate transpose (hermitian), and trace, respectively.

II. Cramer-Rao Lower Bound

In this section the CRLB for CFO estimation for half-duplex multi-relay cooperative networks is presented. In Section II.A. it is shown that for DF relaying the CRLB for CFO estimation at the relays and destination can be transformed to the CRLB in the case of *single-input-single-output (SISO)* and *multi-input-single-output (MISO)* systems, respectively. However, for AF relaying, due to the more complex signal model at the destination the CRLB for joint CFO estimation at the destination is derived in Section II.B.

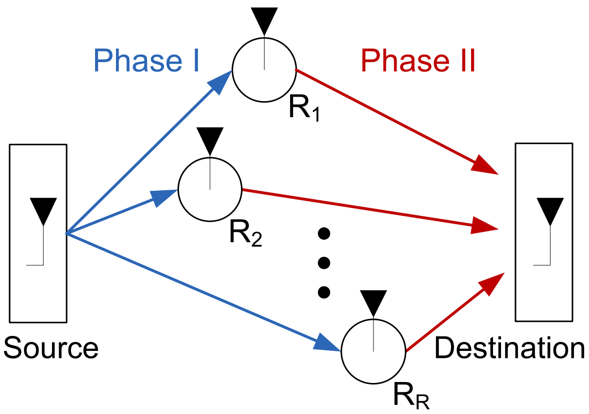

Fig. 1. The system model for the cooperative network.

Throughout this paper the following set of assumptions and system design parameters are considered:

1) In *phase I* the source broadcasts its *training sequence (TS)* to the relays and in *phase II* to efficiently estimate CFOs, the relays transmit R distinct TSs simultaneously to the destination, Fig. 1.
2) Frequency flat-fading channels are considered and the channels are assumed to not change over the length of the TS.
3) CFOs are modeled as unknown non-random parameters with no assumptions on their distributions.
4) Since accurate timing jitter can be extracted even in the presence of large CFOs, [5], [10], [12], we assume perfect timing synchronization.

Note that assumptions 2, 3, and 4 are in line with previous CFO estimation analyses performed for point-to-point systems in [5]–[7], [10] and are also intuitively justifiable, since the main sources of CFO are oscillator mismatch and Doppler shift. Both of these effects and the channel parameters do not significantly change throughout the short TS.

The baseband received signal model at the kth relay for $1 \leq k \leq R$ is given by

$$r_k(n) = \sqrt{p^{[s]}} h_k e^{j2\pi n \nu_k^{[sr]}} t^{[s]}(n) + v_k(n), \quad n = 1, \cdots, L \tag{1}$$

where:

- L denotes the length of the TS,
- $\mathbf{t}^{[\mathbf{s}]} \triangleq \{t^{[s]}(1), \cdots, t^{[s]}(L)\}$ is the known TS broadcast from the source to the relays,
- $\nu_k^{[sr]} = \nu_k^{[sr]} T$ is the normalized CFO from the source to the kth relay, where T is the symbol duration,
- h_k represents the channel gain from the source to the kth relay, $p^{[s]}$ is the transmitted power from the source,
- v_k is the *additive white Gaussian noise (AWGN)* at the kth relay with mean zero and variance $\sigma_{v_k}^2$ ($\mathcal{CN}(0, \sigma_{v_k}^2)$), and
- r_k represents the received signal at the kth relay.

A. Decode-and-Forward Cooperative Networks

Based on the above, the received signal at the destination, y for a DF network consisting of R relay nodes is given by

$$y(n) = \sum_{k=1}^{R} \sqrt{p_k^{[r]}} g_k e^{j2\pi n \nu_k^{[rd]}} t_k^{[r]}(n) + w(n), \; n = 1, \cdots, L \tag{2}$$

where:

- $\mathbf{t}_{\mathbf{k}}^{[\mathbf{r}]} \triangleq \{t_k^{[r]}(1), \cdots, t_k^{[r]}(L)\}$ is the kth relay's TS,
- g_k represents the channel gain from the kth relay to the destination, $p_k^{[r]}$ is the transmitted power from the kth relay,
- $\nu_k^{[rd]} = \triangle\nu_k^{[rd]} T$ is the normalized CFO from the kth relay to the destination, and
- $w(n)$ is the AWGN at the destination with $\mathcal{CN}(0, \sigma_w^2)$.

According to (1) and (2), for DF networks two sets of CFO qualities, $\boldsymbol{\nu}^{[\mathbf{sr}]} = \{\nu_1^{[sr]}, \cdots, \nu_R^{[sr]}\}$ and $\boldsymbol{\nu}^{[\mathbf{rd}]} = \{\nu_1^{[rd]}, \cdots, \nu_R^{[rd]}\}$ need to be estimated. Moreover, since the DF protocol requires the signals at the relays to be decoded, $\boldsymbol{\nu}^{[\mathbf{sr}]}$ needs to be estimated and equalized at the relays, where $\mathbf{t}^{[\mathbf{s}]}$ received in *phase I* is used for CFO estimation. The CRLB for the estimation of $\nu_k^{[sr]}$ at the kth relay is given by

$$\text{CRLB}(\nu_k^{[sr]}) = \frac{3}{4\pi^2 L(L-1)(2L-1)(p^{[s]}|h_k|^2/\sigma_{v_k}^2)}, \tag{3}$$

which is similar to the CRLB in the case of SISO point-to-point systems as derived in [10]. Note that in deriving (3), without loss of generality, it is assumed that $|t^{[s]}(n)|^2 = 1 \;\; \forall n$.

Based on the signal model in (2), the CRLB for the joint estimation of $\boldsymbol{\nu}^{[\mathbf{rd}]}$ in *phase II* is given by the diagonal elements of the $R \times R$ matrix

$$\mathbf{CRLB}(\boldsymbol{\nu}^{[\mathbf{rd}]}) = \mathbf{FIM}_{\mathbf{DF}}^{-1} = \frac{\sigma_w^2}{2} \left(\text{Re}\left[\mathbf{D}_{\mathbf{g}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\mathbf{rd}]}}^H \mathbf{D}_L^2 \mathbf{E}_{\boldsymbol{\nu}^{[\mathbf{rd}]}} \mathbf{D}_{\mathbf{g}}\right]\right)^{-1}, \tag{4}$$

where $\mathbf{FIM}_{\mathbf{DF}}$ is the *Fisher's information matrix*,

$$\mathbf{E}_{\boldsymbol{\nu}^{[\mathbf{rd}]}} \triangleq \begin{bmatrix} t_1^{[r]}(1) e^{j2\pi\nu_1^{[rd]}} & \cdots & t_R^{[r]}(1) e^{j2\pi\nu_R^{[rd]}} \\ \vdots & \vdots & \vdots \\ t_1^{[r]}(L) e^{jl2\pi\nu_1^{[rd]}} & \cdots & t_R^{[r]}(L) e^{jL2\pi\nu_R^{[rd]}} \end{bmatrix}$$

$\mathbf{D}_{\mathbf{h}} \triangleq \text{diag}(\sqrt{p_1^{[r]}} g_1, \cdots, \sqrt{p_R^{[r]}} g_R)$, and $\mathbf{D}_L \triangleq \text{diag}(2\pi, 4\pi, \cdots, 2L\pi)$. The CRLB in (4) is similar to the CRLB for joint CFO estimation in the case of MISO systems in [5]. Although the derivations for (3) and (4) are omitted due to lack of space, they are similar to results presented in [10] and [5], respectively. Based on (4) the following remark is in order:

- When TSs transmitted from all the relays are the same or highly correlated and the CFOs from the relays are close to each other ($\nu_1^{[rd]} \simeq \nu_2^{[rd]} \simeq \cdots \simeq \nu_R^{[rd]}$), the matrix $\mathbf{FIM_{DF}}$ becomes singular. Thus, there does not exist an unbiased estimator that can jointly estimate $\boldsymbol{\nu}^{\mathbf{[rd]}}$ [13]. Moreover, the CRLB (the inverse of $\mathbf{FIM_{DF}}$) approaches infinity, which means that the estimation error for the CFOs becomes unbounded. Therefore, the TS transmitted from each relay needs to be distinct.

B. Amplify-and-Forward Cooperative Networks

For AF relaying the signal model at the destination is given by

$$y(n) = \underbrace{\sum_{k=1}^{R} \zeta_k \sqrt{p_k^{[r]} p^{[s]}} g_k h_k e^{j2\pi n \nu_k^{[\text{sum}]}} t_k^{[r]}(n) t^{[s]}(n)}_{\textbf{desired signal}} + \underbrace{\sum_{k=1}^{R} \zeta_k \sqrt{p_k^{[r]}} g_k e^{j2\pi n \nu_k^{[rd]}} t_k^{[r]}(n) v_k(n) + w(n)}_{\textbf{overall noise}}, \quad (5)$$

where:

- $\zeta_k \triangleq 1/\sqrt{p^{[s]}|h_k|^2 + \sigma_{v_k}^2}$ satisfies the kth relay's power constraint, $\nu_k^{[\text{sum}]} \triangleq \nu_k^{[rd]} + \nu_k^{[sr]}$, and
- $\mathbf{t_k^{[r]}}$ is used to modulate the received TS, $\mathbf{t^{[s]}}$ to ensure the kth relay has a distinct TS.

Eq. (5) follows from the fact that the received signal, $r_k(n)$, is amplified and forwarded without being decoded.

According to (5), $2R$ quantities containing CFOs are present in the signal model:

1) $\boldsymbol{\nu}^{\mathbf{[sum]}} \triangleq \{\nu_1^{[\text{sum}]}, \nu_2^{[\text{sum}]}, \cdots, \nu_R^{[\text{sum}]}\}$, which result in the rotation of the signal constellation and
2) $\boldsymbol{\nu}^{\mathbf{[rd]}} \triangleq \{\nu_1^{[rd]}, \cdots, \nu_R^{[rd]}\}$, which affect the AWGN.

The presence of CFOs, $\boldsymbol{\nu}^{\mathbf{[rd]}}$ does not affect signal detection, since $\boldsymbol{\nu}^{\mathbf{[rd]}}$ only result in a phase shift of the noise. Therefore, the terms $\{\nu_1^{[\text{sum}]}, \nu_2^{[\text{sum}]}, \cdots, \nu_R^{[\text{sum}]}\}$ are the only CFO-related quantities that affect system performance and need to be estimated.

Derivation of the CRLB for the joint estimation of $\boldsymbol{\nu}^{[sum]}$: For notational convenience we introduce the following variables:

- $c_k(n) \triangleq t_k^{[r]}(n) t^{[s]}(n)$ and $\tilde{v}_k \triangleq t_k^{[r]}(n) v_k(n)$,
- $\alpha_k \triangleq \zeta_k \sqrt{p_k^{[r]} p^{[s]}} g_k h_k$ and $\beta_k \triangleq \zeta_k \sqrt{p_k^{[r]}} g_k$,
- $\mathbf{D}_{\boldsymbol{\alpha}} \triangleq \text{diag}(\alpha_1, \cdots, \alpha_R)$ is an $R \times R$ matrix,
- $\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \triangleq [\mathbf{e_1}, \mathbf{e_2}, \cdots, \mathbf{e_R}]$ is an $L \times R$ matrix, and
- $\mathbf{e_k} \triangleq \left[c_k(1) e^{j2\pi \nu_k^{[\text{sum}]}}, \cdots, c_k(L) e^{j2\pi L \nu_k^{[\text{sum}]}}\right]^T$.

Based on the above definitions, (5) can be rewritten as

$$y(n) = \sum_{k=1}^{R} \alpha_k e^{j2\pi n \nu_k^{[\text{sum}]}} c_k(n) + \beta_k e^{j2\pi n \nu_k^{[rd]}} \tilde{v}_k(n) + w(n). \quad (6)$$

According to (6), $\mathbf{y} \triangleq \{y(1), y(2), \cdots, y(L)\}^T$, the vector of the received signals at the destination, is distributed as

$$\mathbf{y} \sim \mathcal{CN}(\boldsymbol{\mu}_{\mathbf{y}}, \boldsymbol{\Sigma}_{\mathbf{y}}), \quad (7)$$

where

$$\begin{cases} \boldsymbol{\mu}_{\mathbf{y}} &= \sum_{k=1}^{R} \alpha_k \mathbf{e_k} \\ \boldsymbol{\Sigma}_{\mathbf{y}} &= \left(\sum_{k=1}^{R} |\beta_k|^2 \sigma_v^2 + \sigma_w^2\right) \mathbf{I} \end{cases}, \quad (8)$$

and $\mathbf{I}$ is the $L \times L$ identity matrix. In arriving at (8), without loss of generality, it is assumed that unit-amplitude TS are transmitted ($|t_k^{[r]}(n)| = 1 \ \forall_{k,n}$) and the variances of the noise terms at the relays are all the same ($\sigma_{v_1}^2 = \sigma_{v_2}^2 = \cdots = \sigma_{v_k}^2 = \sigma_v^2$).

To determine the CRLB, the $R \times R$ FIM needs to be determined. In the case of parameter estimation in a complex Gaussian observation sequence, the entries of FIM are given by [9]

$$\mathbf{FIM}(\boldsymbol{\lambda})_{k,m} = 2\text{Re}\left[\frac{\partial \boldsymbol{\mu}_{\mathbf{y}}^H}{\partial \lambda_k} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\mu}_{\mathbf{y}}}{\partial \lambda_m}\right] + \text{Tr}\left[\boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\boldsymbol{\Sigma}_{\mathbf{y}}}{\lambda_k} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\boldsymbol{\Sigma}_{\mathbf{y}}}{\lambda_m}\right], \quad (9)$$

where $\boldsymbol{\lambda} = \left\{\nu_1^{[\text{sum}]}, \nu_2^{[\text{sum}]}, \cdots, \nu_R^{[\text{sum}]}\right\}$,

$$\frac{\partial \boldsymbol{\mu}_{\mathbf{y}}}{\partial \nu_k^{[\text{sum}]}} = \alpha_k \frac{\partial \mathbf{e_k}}{\partial \nu_k^{[\text{sum}]}}, \quad \text{and} \quad (10)$$

$$\frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \nu_k^{[\text{sum}]}} = 0. \quad (11)$$

The term $\partial \mathbf{e_k}/\partial \nu_k^{[\text{sum}]}$ in (10) is a column vector with its nth elements given by

$$\frac{\partial \mathbf{e_k}}{\partial \nu_k^{[\text{sum}]}}(n) = j2\pi n c_k(n) e^{j2\pi n \nu_k^{[\text{sum}]}}, \qquad n = 1, \cdots, L \quad (12)$$

Using (9) and (12) the entries of the FIM are determined as

$$\mathbf{FIM}(\boldsymbol{\nu}^{\mathbf{[sum]}})_{k,m} = 2\text{Re}\left\{\alpha_k^* \alpha_m \mathbf{e_k^H} \mathbf{D_L} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \mathbf{D_L} \mathbf{e_m}\right\} \quad (13)$$

Let $\mathbf{FIM_{AF}} = \mathbf{FIM}(\boldsymbol{\nu}^{\mathbf{[sum]}})$. Then the CRLB for the estimation of $\boldsymbol{\nu}^{\mathbf{[sum]}}$ is given by the diagonal elements of the inverse of $\mathbf{FIM_{AF}}$, which are calculated as

$$\mathbf{CRLB}(\boldsymbol{\nu}^{\mathbf{[sum]}}) = \left(2\text{Re}\left\{\mathbf{D}_{\boldsymbol{\alpha}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D_L} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \mathbf{D_L} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}\right\}\right)^{-1}. \quad (14)$$

The following remark is in order:

- Based on (14), similar to the case of the DF protocol, to accurately estimate the specific CFO for each relay node (nonsingular $\mathbf{FIM_{AF}}$), the transmitted TSs need to be distinct ($\mathbf{c_1} \neq \mathbf{c_2} \neq \cdots \neq \mathbf{c_R}$). Thus, as shown in (5), we propose that the kth relay modulates a specific and known TS, $\mathbf{t_k^{[r]}}$ onto the received signal using analog phase shifting or digital signal processing before forwarding to the destination (see, e.g., [2] and references therein).

III. Proposed CFO Estimators

In this section, a brief overview of the MUSIC-based CFO estimation algorithm is provided [11]. Next the proposed multiple CFO estimators, iterative-MUSIC (I-MUSIC) and iterative correlation-based-MUSIC (I-C-MUSIC) are outlined. For readability purposes the case of DF cooperative networks is discussed first.

A. MUSIC Algorithm for CFO Estimation

The MUSIC algorithm is a spectral estimation method based on the eigen-decomposition of the covariance matrix of a received signal. To apply the MUSIC CFO estimator the TS, $\mathbf{t_k^{[r]}}$ of length L symbols is assumed to be transmitted in M blocks of length N symbols ($M = L/N$).

Under the assumption of narrow-band transmitted signals and constant channels gains over the length of each block, the signal model in (2) can be rewritten in vector form as

$$\mathbf{y}(m) = \boldsymbol{\Gamma}(\boldsymbol{\nu})\mathbf{s}(m) + \mathbf{w}(m), \qquad m = 1 \cdots M \tag{15}$$

where:

- $\boldsymbol{\Gamma}(\boldsymbol{\nu}) \triangleq [\boldsymbol{\gamma}(\nu_1), \cdots, \boldsymbol{\gamma}(\nu_R)]$ is an $N \times R$ matrix with $\boldsymbol{\gamma}(\nu_k) \triangleq [e^{j2\pi\nu_k}, e^{j4\pi\nu_k}, \cdots, e^{jN\pi\nu_k}]^T$,
- $\nu_k = \nu_k^{[rd]}$ for $k = 1 \cdots R$,
- $\mathbf{y}(m) \triangleq [y(m+1), \cdots, y(m+N)]^T$,
- $\mathbf{w}(m) \triangleq [w(m+1), \cdots, w(m+N)]^T$, and
- $\mathbf{s}(m) \triangleq [s_1(m), \cdots, s_R(m)]^T$, with its k$th$ element given by $s_k(m) \triangleq \sqrt{p_k^{[r]}} g_k t_k^{[r]}(m)$.

Based on the signal model in (15) the temporal covariance matrix of $\mathbf{y}(m)$ can be straightforwardly determined as

$$\mathbf{Q}_{\mathbf{y}} = \boldsymbol{\Gamma}(\boldsymbol{\nu})\mathbf{S}\boldsymbol{\Gamma}^H(\boldsymbol{\nu}) + \sigma_w \mathbf{I}, \tag{16}$$

where $\mathbf{S} = E[\mathbf{s}(m)\mathbf{s}^H(m)]$ and $\mathbf{I}$ is the $N \times N$ identity matrix. Let $\psi_1 \geq \psi_2, \cdots, \geq \psi_N$ denote the eigenvalues of $\mathbf{Q}_{\mathbf{y}}$. If the CFO values are different, rank$(\boldsymbol{\Gamma}(\boldsymbol{\nu})\mathbf{S}\boldsymbol{\Gamma}^H(\boldsymbol{\nu})) = R$ and it follows that $\psi_k > \sigma_w$ for $k = 1, \cdots, R$ and $\psi_k = \sigma_w$ for $k = R+1, \cdots, N$.

Denote the the unit-eigenvectors corresponding to $\psi_1, \cdots, \psi_R$ and $\psi_{R+1}, \cdots, \psi_N$ as $\boldsymbol{\Psi}^{[\mathbf{R}]} = [\boldsymbol{\psi}_1, \cdots, \boldsymbol{\psi}_R]$ and $\boldsymbol{\Psi}^{[\mathbf{N}]} = [\boldsymbol{\psi}_{R+1}, \cdots, \boldsymbol{\psi}_N]$, respectively. Next, observe that

$$\mathbf{Q}_{\mathbf{y}}\boldsymbol{\Psi}^{[\mathbf{N}]} = \boldsymbol{\Gamma}(\boldsymbol{\nu})\mathbf{S}\boldsymbol{\Gamma}^H(\boldsymbol{\nu})\boldsymbol{\Psi}^{[\mathbf{N}]} + \sigma_w\boldsymbol{\Psi}^{[\mathbf{N}]} = \sigma_w\boldsymbol{\Psi}^{[\mathbf{N}]}. \tag{17}$$

Therefore, the MUSIC estimate of $\boldsymbol{\nu}$ is given by [11]

$$\hat{\boldsymbol{\nu}} = \left(\arg\max_{\nu} \boldsymbol{\gamma}^H(\nu)\boldsymbol{\Psi}^{[\mathbf{N}]}\left(\boldsymbol{\Psi}^{[\mathbf{N}]}\right)^H \boldsymbol{\gamma}^H(\nu)\right)^{-1}. \tag{18}$$

In practice the covariance matrix $\mathbf{Q}_{\mathbf{y}}$ is unknown but can be estimated from the received data according to

$$\hat{\mathbf{Q}}_{\mathbf{y}} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{y}(m)\mathbf{y}^H(m). \tag{19}$$

Although accurate, the outlined MUSIC-based estimator performs poorly when the CFO values are close to one another [11] and does not assign the estimated CFOs to corresponding relays.

B. I-MUSIC and I-C-MUSIC for DF Networks

Unlike the above MUSIC CFO estimator, we propose that the relay nodes transmit distinct TSs, where each TS is a combination of coherent and non-coherent training symbols such that

$$\{t_k^{[r]}(1) = \cdots = t_k^{[r]}((m-1)N\}_{k=1}^{R} \quad \text{, and} \tag{20}$$
$$\{t_k^{[r]}((m-1)N) \neq \cdots \neq t_k^{[r]}(mN)\}_{k=1}^{R}.$$

*1) **Initialization of I-MUSIC**:* Let q denote the number of distinct CFOs present in $\mathbf{y}$. Methods for estimating q are well researched in the literature and will not be examined here. The following two possible scenarios are considered:

Scenario 1) q=R: Let $\boldsymbol{\nu}$ and $\hat{\boldsymbol{\nu}}$ denote $\boldsymbol{\nu}^{[\mathbf{rd}]}$ and $\hat{\boldsymbol{\nu}}^{[\mathbf{rd}]}$ throughout this subsection, respectively. Since $\mathbf{y}$ is distributed as $\mathcal{CN}(\boldsymbol{\mu}_{\mathbf{y}}, \sigma_w^2\mathbf{I})$, the negative *log likelihood function (LLF)* of the CFOs and the channels gains is given by

$$\delta(\boldsymbol{\nu}, \mathbf{g}) = \|\mathbf{y} - \mathbf{E}_{\boldsymbol{\nu}}\mathbf{g}\|^2. \tag{21}$$

For a given $\boldsymbol{\nu}$, the minimizer of (21) and the ML estimates of the channel gains, $\hat{\mathbf{g}}$ are given by

$$\mathbf{p}^{[\mathbf{r}]} \odot \hat{\mathbf{g}} = (\mathbf{E}_{\boldsymbol{\nu}}^H\mathbf{E}_{\boldsymbol{\nu}})^{-1}\mathbf{E}_{\boldsymbol{\nu}}^H\mathbf{y}, \tag{22}$$

where $\mathbf{p}^{[\mathbf{r}]} \triangleq \{\sqrt{p_1^{[r]}}, \cdots, \sqrt{p_R^{[r]}}\}^T$ and $\hat{\mathbf{g}} \triangleq \{\hat{g}_1, \cdots, \hat{g}_R\}$. To estimate the CFOs, $\mathbf{y}^{[\mathbf{c}]} \triangleq [y(1), \cdots, y((m-1)N)]^T$ is used as the input to the MUSIC algorithm, where in (18) the search is performed for q maxima instead of R. Next, using $\hat{\boldsymbol{\nu}}$, $\mathbf{y}^{[\mathbf{c}]}$, and (22) the channel gain corresponding to each CFO is determined.

To assign the pairs of $\hat{\boldsymbol{\nu}}$ and $\hat{\mathbf{g}}$ to specific nodes, the **distinct** TSs, $\mathbf{y}^{[\mathbf{n\text{-}c}]} \triangleq [y((m-1)N+1), \cdots, y(mN)]^T$, and the LLF in (21) are used, to carry out the minimization

$$\hat{\boldsymbol{\nu}}^{[\mathbf{A}]}, \hat{\mathbf{g}}^{[\mathbf{A}]} = \arg\min_{\hat{\boldsymbol{\nu}}, \hat{\mathbf{g}}} \delta(\boldsymbol{\nu}, \mathbf{g}) = \|\mathbf{y}^{[\mathbf{n\text{-}c}]} - \breve{\mathbf{E}}_{\boldsymbol{\nu}}\mathbf{g}\|^2, \tag{23}$$

where $\breve{\mathbf{E}}_{\hat{\boldsymbol{\nu}}} \triangleq [\breve{\mathbf{e}}_{\mathbf{1}}, \cdots, \breve{\mathbf{e}}_{\mathbf{R}}]$ with $\breve{\mathbf{e}}_{\mathbf{k}} \triangleq \left[t_k^{[r]}((m-1)N+1)e^{j2\pi((m-1)N+1)\nu_k}, \cdots, t_k^{[r]}(mN)e^{j2\pi mN\nu_k}\right]^T$ and $\hat{\boldsymbol{\nu}}^{[\mathbf{A}]}$ and $\hat{\mathbf{g}}^{[\mathbf{A}]}$ represent the set of estimated CFO and channel gains corresponding to each relay node, respectively.

Scenario 2) q<R: Table I outlines the steps for determining the CFOs for this case. Note that $(\cdot)^{[\mathbf{i}]}$ represents the ith iteration.

TABLE I

INITIALIZATION STEPS FOR I-MUSIC AND I-C-MUSIC

Step 1)	**Initialization** Using (18) to determine the set of q distinct CFOs, $\hat{\boldsymbol{\nu}}^{[\mathbf{q}]}$.
Step 2)	**Iteration** For $i = 1, 2, \cdots, \binom{R-1}{q-1}$ • Construct $(\hat{\boldsymbol{\nu}})^{[\mathbf{i}]} = \hat{\boldsymbol{\nu}}^{[\mathbf{q}]} \bigcup \left(\hat{\boldsymbol{\nu}}^{[\mathbf{R\text{-}q}]}\right)^{[\mathbf{i}]}$, where $\hat{\boldsymbol{\nu}}^{[\mathbf{R\text{-}q}]}$ is a combination of frequencies selected from $\hat{\boldsymbol{\nu}}^{[\mathbf{q}]}$. • Using (22) determine $(\hat{\mathbf{g}})^{[\mathbf{i}]}$ corresponding to $(\hat{\boldsymbol{\nu}})^{[\mathbf{i}]}$. • Determine $\left(\hat{\boldsymbol{\nu}}^{[\mathbf{A}]}\right)^{[\mathbf{i}]}$ and $\left(\hat{\mathbf{g}}^{[\mathbf{A}]}\right)^{[\mathbf{i}]}$ using (23). Select $\left(\hat{\boldsymbol{\nu}}^{[\mathbf{A}]}\right)^{[\mathbf{i}]}$ and $\left(\hat{\mathbf{g}}^{[\mathbf{A}]}\right)^{[\mathbf{i}]}$ that result in the smallest LLF value, $\delta(\left(\hat{\boldsymbol{\nu}}^{[\mathbf{A}]}\right)^{[\mathbf{i}]}, \left(\hat{\mathbf{g}}^{[\mathbf{A}]}\right)^{[\mathbf{i}]})$ for $i = 1, \cdots \binom{R-1}{q-1}$ as $\hat{\boldsymbol{\nu}}^{[\mathbf{A}]}$ and $\hat{\mathbf{g}}^{[\mathbf{A}]}$, the set of estimated CFO and channel gains corresponding to each relay node, respectively.

Despite the above advantages, the initialization step of I-MUSIC suffers from an error floor at high SNRs, since the resolution of the MUSIC algorithm dictated by the length of TS block, N, influences estimation accuracy and not SNR. Therefore, an iterative step is used to remove the error floor and reach the CRLB.

*2) **Iterative Step for I-MUSIC**:* Since the TSs are known, the effect of data modulation corresponding to the ith node can be eliminated. Using (2) and assuming unit-amplitude phase shift keying training symbols we have

$$\begin{aligned}\tilde{y}_i(n) &= y(n)\left(t_i^{[r]}(n)\right)^* \\ &= \underbrace{\sqrt{p_i^{[r]}} g_i e^{j2\pi n\nu_i^{[rd]}}}_{\textbf{desired term}} \\ &\quad + \underbrace{\sum_{k=1, k\neq i}^{R} \sqrt{p_k^{[r]}} g_k e^{j2\pi n\nu_k^{[rd]}} t_{k,i}^{[d]}(n)}_{\textbf{interference}} + \underbrace{\tilde{w}_i(n)}_{\textbf{noise}},\end{aligned} \tag{24}$$

where $t^{[d]}_{k,i}(n) \triangleq t^{[r]}_k(n)\left(t^{[r]}_i(n)\right)^*$ and $\tilde{w}_i(n) \triangleq w(n)\left(t^{[r]}_i(n)\right)^*$. Note that $\tilde{w}_i(n)$ has the same statistical properties as $w(n)$.

The initial estimates of CFOs and channel gains, $\left(\hat{\boldsymbol{\nu}}^{[\mathbf{rd}]}\right)^{[\mathbf{1}]}$ and $(\hat{\mathbf{g}})^{[\mathbf{1}]}$, respectively, are used to reduce the interference term in (24) according to

$$f_i(n) = \tilde{y}_i(n) - \sum_{k=1,k\neq i}^{R} \sqrt{p^{[r]}_k}\hat{g}_k e^{j2\pi n\hat{\nu}^{[rd]}_k} t^{[d]}_{k,i}(n), \tag{25}$$

where $\mathbf{f_i} = \{f_i(1), \cdots, f_i(L)\}$ is used in the next iteration to estimate the CFO corresponding to the ith node. As stated previously the MLE does not perform well when the CFO values are close to one another, therefore at every iteration the channel gain for the ith node can be estimated using

$$\sqrt{p^{[r]}_i}\hat{g}_i = \frac{1}{L}\sum_{n=1}^{L}\frac{f_i(n)}{e^{j2\pi n\hat{\nu}^{[rd]}_i}}, \tag{26}$$

which is based on the expectation conditional maximization (ECM) algorithm outlined in [7]. The iteration stops when the absolute difference between the LLF of two iterations is smaller than a threshold value χ

$$\left| \|\mathbf{y} - \mathbf{E}_{\left(\hat{\boldsymbol{\nu}}^{[\mathbf{rd}]}\right)^{[\mathbf{o+1}]}}\left(\hat{\mathbf{g}}\right)^{[\mathbf{o+1}]}\|^2 - \|\mathbf{y} - \mathbf{E}_{\left(\hat{\boldsymbol{\nu}}^{[\mathbf{rd}]}\right)^{[\mathbf{o}]}}\left(\hat{\mathbf{g}}\right)^{[\mathbf{o}]}\|^2 \right| \leq \chi, \tag{27}$$

where $\left(\hat{\boldsymbol{\nu}}^{[\mathbf{rd}]}\right)^{[\mathbf{o}]}$ and $(\hat{\mathbf{g}})^{[\mathbf{o}]}$ represent the frequency offset and channel gain estimates corresponding to the oth iteration. Table II summarizes the I-MUSIC algorithm.

TABLE II
I-MUSIC CFO ESTIMATOR

Step 1)	**Initialization** Use the initialization step for I-MUSIC.

Step 2)	**Iteration**
	$o = 1$
	While (27) holds *do*
	• For $i = 1, 2, \cdots, R$
	– Compute $(\mathbf{f_i})^{[o]}$ using (25).
	– Use $(\mathbf{f_i})^{[o]}$ as input to (18) to determine $\left(\hat{\nu}^{[rd]}_i\right)^{[o+1]}$
	– Compute the channel gain corresponding to $\left(\hat{\nu}^{[rd]}_i\right)^{[o+1]}$, $(\hat{g}_i)^{[o+1]}$, using (26).
	• $o = o + 1$
	end While

*3) **I-C-MUSIC**:* Since at each iteration the CFO for each node is estimated separately and due to the use of phase shift keying training symbols, the computational complexity of I-MUSIC can be reduced through single-CFO estimation techniques already proposed in the literature. One approach is the correlation-based estimator [14], which estimates the ith node's CFO using

$$2\pi\hat{\nu}^{[rd]}_i = \sum_{n=1}^{L-1}\varpi(n)\text{angle}\{f^*_i(n)f_i(n+1)\}, \tag{28}$$

where $\varpi(n)$ is a window designed to reduce the estimator's variance (see [14] for details). The same steps outlined in Table II can be used to implement I-C-MUSIC, with the exception that (28) is used instead of (18).

C. I-MUSIC and I-C-MUSIC for AF Networks

In the case of AF networks as shown in Section II, $\boldsymbol{\nu}_{\mathbf{sum}}$ are the only CFO-related quantities that need to be estimated at the destination. Therefore, by combining the noise terms in (5) the signal model at the destination is represented as

$$y(n) = \underbrace{\sum_{k=1}^{R}\zeta_k\sqrt{p^{[r]}_k p^{[s]}}\varrho_k e^{j2\pi n\nu^{[\text{sum}]}_k} t^{[r]}_k(n)t^{[s]}(n)}_{\textbf{desired signal}} + \underbrace{z_{\text{sum}}(n)}_{\textbf{overall noise}} \tag{29}$$

where

$$\varrho_k = g_k h_k, \text{ and}$$
$$z_{\text{sum}}(n) = \sum_{k=1}^{R}\zeta_k\sqrt{p^{[r]}_k}g_k e^{j2\pi n\nu^{[rd]}_k} t^{[r]}_k(n)v_k(n) + w(n).$$

We note that the signal model in (29) is similar to that of DF networks in (2). Therefore, I-MUSIC and I-C-MUSIC can be applied to efficiently estimate $\boldsymbol{\nu}_{\mathbf{sum}}$ in the case of AF networks.

IV. NUMERICAL RESULTS AND DISCUSSIONS

Throughout this section the propagation loss is modeled as $\beta = (d/d_0)^{-m}$, [4], where d is the distance between the transmitter and receiver, d_0 is the reference distance, and m is the path loss exponent. The following results are based on $d_0 = 1$km and $m = 2.7$, which corresponds to urban area cellular networks. A cooperative network consisting of 2 relays is considered. Without loss of generality only the CFO estimation performance for the first relay is presented. Specific channels are used in all simulations as in [5] and [7]. More precisely, $\mathbf{h}$ and $\mathbf{g}$ are drawn from independent and identically distributed zero-mean complex Gaussian processes with unit variance and these channels are used in all runs. For our particular channels $\mathbf{h} = [.2790 - .9603i, .8837 + .4681i]^T$ and $\mathbf{g} = [.7820 + .6233i, .9474 - .3203i]^T$. The length of TS, $L = 24$ and $N = 8$. The threshold in (27), $\chi = .0001$. The relays distance from the source and destination, $d^{[sr]} = 1$km, $d^{[rd]} = 1$km unless specified. Finally, $\sigma^2_v = \sigma^2_w$.

Fig. 2 compares the performance of I-MUSIC and I-C-MUSIC for the estimation of $\boldsymbol{\nu}^{[\mathbf{rd}]}$ in DF networks against the CRLB in Eq. (4), the MLE in [5], and the expectation conditional maximization (ECM) algorithm in [7]. The normalized CFOs are $\boldsymbol{\nu}^{[\mathbf{rd}]} = \{.22, .2\}$, which are specifically chosen to be close to one another. Simulation results show that I-MUSIC is close to the CRLB but does not reach the CRLB. This is due to the inherent shortcoming of the MUSIC algorithm as also discussed in [11]. However, I-C-MUSIC reaches the CRLB and is also computationally less complex, but it shows poorer performance at low SNR. Fig. 2 also shows that both algorithms outperform the MLE and ECM methods at mid-to-high SNR. The MLE, on the other hand, requires that while one relay transmits the other one stays silent. This approach effectively reduces the length of the transmitted TS by an order of $R = 2$ and results in higher MSE. As expected the ECM estimator fails, since the initial CFO estimates are so poor whenever the CFOs are larger than .05 (the results in [7] are based on normalized CFO values of .01 and .015).

Fig. 3 compares the performance of I-MUSIC and I-C-MUSIC for the estimation of $\nu^{[\mathrm{sum}]}$ in AF networks against the CRLB in (14). The normalized CFOs are $\nu^{[\mathrm{sum}]} = \nu^{[\mathrm{sr}]} + \nu^{[\mathrm{rd}]} = \{.11, .1\} + \{.11, .1\}$. Note that similar to the case of DF I-C-MUSIC reaches the CRLB while I-MUSIC is very close to the CRLB and demonstrates better performance at low SNR values.

Fig. 4 shows the effect of network topology on frequency synchronization in distributed cooperative systems by comparing the CRLB in (4) and (14) for DF and AF protocols, respectively. Three different relay locations are taken into consideration ($d^{[sr]} + d^{[rd]} = 2$km for all nodes): $d^{[rd]} = .5$km, $d^{[sr]} = d^{[rd]}$, and $d^{[rd]} = 1.5$km. Fig. 4 A. and Fig. 4 B. show the CRLB for CFO estimation for DF and AF networks, respectively. According to Fig. 4, the best overall CFO estimation in DF networks is achieved when the nodes are closer to the destination. However, in the case of AF moving the nodes closer to the destination from the mid point does not result in improved estimation performance due to the noise at the relay nodes, which is also amplified and forwarded to the destination. The closer the node is to the destination (while at the same being further away from the source), the greater is the effect of the amplified noise at the destination. Therefore, moving the relay closer does not result in any performance gain in terms of CFO estimation. These properties can be combined with relay selection methods to achieve better frequency synchronization in distributed cooperative networks.

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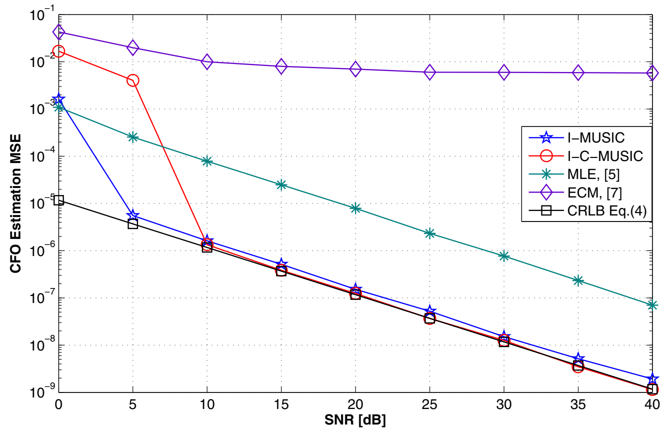

Fig. 2. The MSE of I-MUSIC and I-C-MUSIC for the estimation of $\nu_1^{[rd]}$ for **DF networks** VS. the algorithms in [5] and [7] and the CRLB in Eq. (4) ($L = 24$).

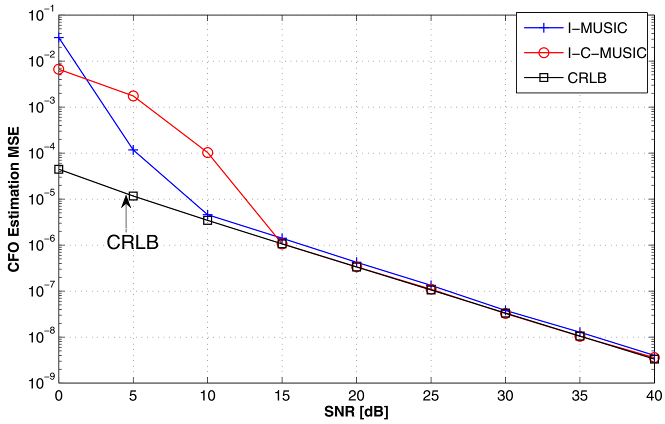

Fig. 3. The MSE of I-MUSIC and I-C-MUSIC for the estimation of $\nu_1^{[\mathrm{sum}]}$ for **AF networks** VS. the CRLB in Eq. (14) ($L = 24$).

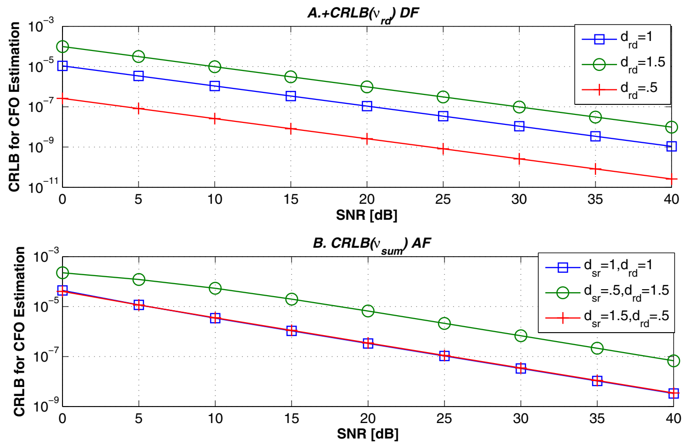

Fig. 4. CRLBs for CFO estimation with the relays at different locations with $L = 24$. **A.** The CRLBs for CFO estimation for DF cooperative networks. **B.** The CRLB for CFO estimation for AF cooperative networks.