

# Application of Phase Shift in Coherent Multi-Relay MIMO Communications

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**Abstract**—In this paper we propose a new low feedback and low complexity scheme that adjusts the phases at the relays, thus allowing for the coherent combining of signals from multiple relays in a cooperative multi-input-multi-output (MIMO) system using space division multiple access (SDMA). It is assumed that the channel state information (CSI) is not fed back to the source and the relays and the case of imperfect CSI at the destination is also taken into consideration. In determining the parameters, conventional convex optimization schemes cannot be applied due to the non-convex nature of the function. A low complexity method is used to optimize the system that utilizes bisection search. Numerical calculations and BER simulations demonstrate a significant performance gain over conventional AF.

**Index Terms**—Multi-Input-Multi-Output cooperative networks, application of phase shift, imperfect channel state information (CSI), capacity.

## I. INTRODUCTION

Cooperative communications has become a rapidly emerging and important area of research. Pioneering contribution can be found in [1] and [2]. Since future generations of cellular networks are migrating to higher carrier frequencies, MIMO cooperative communications is a potentially attractive method to combat the resulting severe signal attenuation.

In [3], a beamforming scheme has been proposed that aims to maximize the instantaneous receive SNR for single antenna systems at source, relay and destination. However, the scheme assumes that relays have access to CSI from the source and other relays. This is simply too complex for practical implementation. Furthermore, [3] only considers the case of perfect CSI and is targeted at single-antenna source, relays, and destination which as pointed out here and in [3] is easy to deal with and has a closed-form solution. On the other hand, the MIMO scenario is considerably more difficult to solve and has not been addressed before. Multi-antenna cooperative systems have been studied in [4] and [5], but only the scenario of single relay systems is investigated. In [6] various schemes for multi-antenna at source, relays, and destination based on amplify and forward (AF), decode and forward (DF), and a combination of the two (hybrid schemes) are investigated. However, the proposed optimum scheme requires that the relays have access to both the CSI from the source to the relays and from the relays to destination, to optimally maximize the capacity of the overall cooperative

system. Furthermore, the scenario of imperfect CSI and its effects on the overall system performance is not taken into consideration. In [7] new optimization schemes for multi-relay MIMO cooperative systems based on the triangularization of the MIMO relay channels has been proposed. It has been demonstrated that the proposed algorithm results in capacity gains in a cooperative relay system utilizing SDMA. However, similar to previous work, it is assumed that all the relays have access to perfect CSI from the source to the relays and the relays to the destination, which requires significant feedback and overhead. It is important to point out that the results presented in [7] assume that the relays are collocated, which is not very practical.

In this work we propose a new scheme that allows for the signals from multiple relays to combine coherently at the destination, without requiring any CSI to be fed back to the relays and the source. The scheme applies a phase adjustment at each relay to achieve coherent combining, which requires only the phases to be fed back to the relays from the destination, therefore reducing overhead significantly. The algorithm is applicable to both AF, DF, or any other cooperative protocol using SDMA. A closed-form solution for the signal antenna destination case has been provided, yet in the case of multiple antennas at the destination, the non-convex nature of the problem does not allow for an optimal closed-form solution. Nevertheless, a simple and iterative algorithm based on a bisection search algorithm allows for the phase shift required at each relay to be efficiently determined.

The following notation is used throughout this report: italic letters ( $x$ ) represent scalar quantities, bold lower case letters ( $\mathbf{x}$ ) represent vectors, bold upper case letters ( $\mathbf{X}$ ) represent matrices,  $(\cdot)^*$  represents the conjugate, and  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and conjugate transpose (hermitian), respectively.

## II. CHANNEL AND SYSTEM MODEL

A wireless network consisting of a designated source-destination pair and  $K$  relays is considered (see Fig. 1). The source and destination, equipped with  $M$  and  $N$  antennas, respectively, are denoted as  $S$  and  $D$ . The  $k$ th relay

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employs a single transmit/receive antenna and is denoted by  $R_k (k = 1, 2, \dots, K)$ . The choice of a single antenna for the relays is motivated to keep the feedback to a minimum and to reduce the complexity at the relays. Furthermore, the relays are assumed to be fixed and a part of the infrastructure and the channel from source to relays is fed forward to the destination (which is done in both AF and DF for coherent detection). We assume that there is no direct link between  $S$  and  $D$  due to large distance. Data is transmitted from source through relays to the destination over two time slots in half-duplex mode. Vertical Bell Laboratories Layered Space-Time Architecture (VBLAST) is deployed and Frequency-flat fading channels are assumed during each transmission block.

Denote the signal transmitted from the source as  $\mathbf{s}$ . The signal model at the  $k$ th relay is given by

$$r_k = \sqrt{\frac{P}{M}} \mathbf{h}_k^T \mathbf{s} + n_k \quad (1)$$

where  $r_k$  is the received signal and  $P$  is the power available at the source. The vector  $\mathbf{s}$  is the  $M \times 1$  transmit signal vector with covariance matrix  $\mathbf{I}_M$ ,  $n_k$  is the zero mean unit variance complex additive white Gaussian noise at the  $k$ th relay, and  $\mathbf{h}_k^T$  is the  $1 \times M$  channel vector from the source to the  $k$ th relay.  $\mathbf{h}_k$  can be further expressed as  $\mathbf{h}_k = \sqrt{\alpha_k} \tilde{\mathbf{h}}_k$ , where the entries of  $\tilde{\mathbf{h}}_k$  are independent and identically distributed (i.i.d) complex Gaussian random variables with unit variance, and  $\alpha_k$  represents the path loss effect, and can be expressed as  $\alpha_k = \left(\frac{d}{d_0}\right)^n$ , where  $d$  is the distance between the transmitter and receiver,  $d_0$  is the reference distance and  $n$  is the path loss exponent. In all the protocols to be discussed, we assume that the relays are fixed and part of the infrastructure.

### III. RELAYING SCHEME

#### A. Phase shift

The phase adjustment required at each relay can be efficiently calculated at the destination and fed back to the relays since the destination has knowledge of both  $\mathbf{h}_k$  and  $\mathbf{g}_k$ , the channel from the  $k$ th relay to the destination, reducing overhead and complexity. It is also important to note that the analysis provided here is for AF (named amplify and phase shift and forward (ASPF)) and it can be easily extended to include the case of DF (named decode phase shift and forward (DPSF)), since DF is a special case of the scenario presented here. (The noise at the relays  $n_k$ , is eliminated since it is assumed that only the relays which successfully decode the signal, through a cyclic redundancy check (CRC), forward their signals to the destination).

Considering the phase adjustment the transmitted pulse at each relay is

$$t_k = \sqrt{\eta_k} \frac{\sum_{i=1}^M \sqrt{\frac{P}{M}} h_{k,i} s_i + n_k}{\sqrt{\sum_{i=1}^M \frac{P}{M} |h_{k,i}|^2 + 1}} e^{j\theta_k}, \quad (2)$$

where  $\eta_k$  is the power available at the  $k$ th relay and  $\theta_k$  is the phase shift at the  $k$ th relay. In (2),  $h_{k,i}$  is the channel from the  $i$ th antenna to the  $k$ th relay.

#### B. Determining the Capacity

The received signal model at the destination is

$$\mathbf{y} = \sum_{i=1}^K \mathbf{g}_i t_i + \mathbf{n}_d \quad (3)$$

where the vector  $\mathbf{g}_i$  is written as  $\mathbf{g}_i = \sqrt{\beta_i} \tilde{\mathbf{g}}_i$ , where each entry of  $\tilde{\mathbf{g}}_i$  is an i.i.d complex Gaussian random variable. The vector  $\mathbf{n}_d$  is the  $N \times 1$  circular complex additive white Gaussian noise with covariance matrix  $\mathbf{I}$ . Substituting (2) into (3),

$$\mathbf{y} = \sum_{i=1}^K \mathbf{g}_i \mathbf{h}_i^T \sqrt{\frac{\frac{P}{M} \eta_i}{\frac{P}{M} |\mathbf{h}_i|^2 + 1}} e^{j\theta_i} \mathbf{s} + \sum_{i=1}^K \mathbf{g}_i n_i \sqrt{\frac{\eta_i}{\frac{P}{M} |\mathbf{h}_i|^2 + 1}} e^{j\theta_i} + \mathbf{n}_d \quad (4)$$

Through simple manipulation, (4) is reformulated as:

$$\mathbf{y} = \sum_{i=1}^K \mathbf{U}_i \mathbf{s} + \mathbf{n} \quad (5)$$

where

$$\mathbf{U}_i = \mathbf{g}_i \mathbf{h}_i^T \sqrt{\frac{\frac{P}{M} \eta_i}{\frac{P}{M} |\mathbf{h}_i|^2 + 1}} e^{j\theta_i}. \quad (6)$$

and

$$\mathbf{n} = \sum_{i=1}^K \mathbf{g}_i n_i \sqrt{\frac{\eta_i}{\frac{P}{M} |\mathbf{h}_i|^2 + 1}} e^{j\theta_i} + \mathbf{n}_d \quad (7)$$

The noise covariance matrix is

$$\Sigma_{\mathbf{n}} = E[\mathbf{n}\mathbf{n}^H] = \sum_{k=1}^K \mathbf{g}_k \mathbf{g}_k^H \eta_k \frac{1}{\frac{P}{M} |\mathbf{h}_k|^2 + 1} + \mathbf{I}_N \quad (8)$$

The capacity of the cooperative system may be calculated via [8]

$$C(\theta_1, \theta_2, \dots, \theta_K) = \max_{\theta_1, \theta_2, \dots, \theta_K} \frac{1}{2} \log_2 \det \left( \mathbf{I} + \Sigma_{\mathbf{n}}^{-1} \left( \sum_{l=1}^K \mathbf{U}_l \right) \left( \sum_{m=1}^K \mathbf{U}_m \right)^H \right). \quad (9)$$

Assuming the set of phases  $(\theta_1, \dots, \theta_{k-1}, \theta_{k+1}, \dots, \theta_K)$ , is fixed and separating the terms with  $\theta_k$  and without  $\theta_k$ , we arrive at Eq. (10) which can be further simplified to

$$C(\theta_k) = \frac{1}{2} \log_2 \det(\Sigma_1) + \frac{1}{2} \max_{\theta_k} \log_2 \det \left( \mathbf{I} + \mathbf{A} e^{j\theta_k} + \mathbf{A}^H e^{-j\theta_k} \right), \quad (12)$$

where  $\Sigma_1$  is expressed in Eq. (11) and

$$\mathbf{A} = \Sigma_1^{-1} \Sigma_n^{-1} \frac{\mathbf{U}_k}{e^{j\theta_k}} \left( \sum_{m=1, m \neq k}^K \mathbf{U}_m \right)^H. \quad (13)$$

Furthermore it can be shown that,

$$\frac{\partial C(\theta_k)}{\partial \theta_k} = \frac{j}{2 \ln 2} \times \text{Tr} \left[ \left( \mathbf{I} + \mathbf{A} e^{j\theta_k} + \mathbf{A}^H e^{-j\theta_k} \right)^{-1} \left( \mathbf{A} e^{j\theta_k} - \mathbf{A}^H e^{-j\theta_k} \right) \right]. \quad (14)$$

$$C(\theta_k) = \max_{\theta_k} \frac{1}{2} \log_2 \det \left[ \boldsymbol{\Sigma}_1 + \left( \sum_{l=1, l \neq k}^K \mathbf{U}_l \right) \mathbf{U}_k^H \boldsymbol{\Sigma}_n^{-1} + \boldsymbol{\Sigma}_n^{-1} \mathbf{U}_k \left( \sum_{m=1, m \neq k}^K \mathbf{U}_m \right)^H \right] \quad (10)$$

$$\boldsymbol{\Sigma}_1 = \mathbf{I} + \boldsymbol{\Sigma}_n^{-1} \left( \mathbf{U}_k \mathbf{U}_k^H + \left( \sum_{m=1, m \neq k}^K \mathbf{U}_m \right) \left( \sum_{l=1, l \neq k}^K \mathbf{U}_l \right)^H \right) \quad (11)$$

### C. Determining the Phase Shift

1) *Single antenna at the receiver:* We first consider the case of one receive antenna at the destination and provide a closed-form solution for the optimal  $\theta_k$  when fixing  $\forall \theta_i$ ,  $i \neq k$ ,  $(\theta_1, \dots, \theta_{k-1}, \theta_{k+1}, \dots, \theta_K)$ . The following analysis also pertains to the feasibility of maximizing locally with respect to  $\theta_k$  when all other phases are fixed. Taking the first derivative of  $C$  with respect to  $\theta_k$ , and equating it to zero results in two roots,  $\theta_{k,1}$  and  $\theta_{k,2}$ . It can be further shown that  $\frac{\partial^2 C}{\partial \theta_k^2} |_{\theta_k = \theta_{k,1}}$  and  $\frac{\partial^2 C}{\partial \theta_k^2} |_{\theta_k = \theta_{k,2}}$  are of opposite signs. Therefore, either  $\theta_{k,1}$  or  $\theta_{k,2}$  represent a maximum.  $\theta_{k,1}$  and  $\theta_{k,2}$  are determined explicitly via

$$\begin{aligned} \theta_{k,1} &= \frac{1}{2} \arctan \left( \frac{\Im\{a\}}{\Re\{a\}} \right), \\ \theta_{k,2} &= \pi + \frac{1}{2} \arctan \left( \frac{\Im\{a\}}{\Re\{a\}} \right), \end{aligned} \quad (15)$$

where  $a$  is the scalar version of  $\mathbf{A}$  in (13).

2) *Multiple antennas at the receiver:* As stated previously, due to the non-convex nature of the problem in the case of multiple antennas at the receiver, there does not exist a closed-form solution for the optimum phase set. Therefore, a bisection search algorithm is used. From (9) it is observed that the system capacity is a function of the phases  $\theta_1, \theta_2, \dots, \theta_K$ . Since the log function is monotonically increasing, maximizing  $C(\theta_1, \theta_2, \dots, \theta_K)$  is equivalent to  $\max_{\theta_1, \theta_2, \dots, \theta_K} \det(\boldsymbol{\Psi})$ , where

$$\boldsymbol{\Psi} = \mathbf{I} + \boldsymbol{\Sigma}_n^{-1} \left( \sum_{l=1}^K \mathbf{U}_l \right) \left( \sum_{m=1}^K \mathbf{U}_m \right)^H \quad (16)$$

From (6) we have

$$\begin{aligned} \mathbf{U}_i(\theta_1, \theta_2, \dots, \theta_K) &= \\ \mathbf{U}_i(\theta_1 + 2m_1\pi, \theta_2 + 2m_2\pi, \dots, \theta_K + 2m_K\pi) & \end{aligned} \quad (17)$$

where  $m_1, m_2, \dots, m_K$  can be arbitrary integers. From (9) and (17) we have

$$\begin{aligned} C(\theta_1, \theta_2, \dots, \theta_K) &= \\ C(\theta_1 + 2m_1\pi, \theta_2 + 2m_2\pi, \dots, \theta_K + 2m_K\pi) & \end{aligned} \quad (18)$$

In other words,  $C(\theta_1, \theta_2, \dots, \theta_K)$  is a periodic function of  $\theta_1, \theta_2, \dots, \theta_K$  with a period of  $2\pi$  for the phase of each relay.

### D. Channel Estimation

In the case of imperfect channel estimation, the model proposed in [9] is applied to the multi-relay scheme proposed here. When orthogonal training sequences are transmitted from each source transmit antenna or relay, i.e.  $\mathbf{b}_i^H \mathbf{b}_j = \delta_{i,j}$  ( $\delta_{i,j} = 1$  if  $i = j$  and  $\delta_{i,j} = 0$  if  $i \neq j$ ), the received signal at the  $i$ th received antenna is given by

$$\mathbf{a}_i = \sum_{j=1}^M \sqrt{\frac{P}{M}} l_{i,j} \mathbf{b}_j + \mathbf{n}_i \quad (19)$$

where  $l_{i,j}$  is the channel gain from the  $j$ th transmit antenna to the  $i$ th receive antenna, and  $\mathbf{n}_i$  is the zero mean complex Gaussian noise vector at the  $i$ th receive antenna with  $E[\mathbf{n}_i \mathbf{n}_i^H] = \sigma_e^2 \mathbf{I}$ . We obtain a noisy version of  $l_{i,j}$ ,  $\hat{l}_{i,j}$  by computing

$$\hat{l}_{i,j} = \sqrt{\frac{M}{P}} \mathbf{b}_j^H \mathbf{a}_i = l_{i,j} + \sqrt{\frac{M}{P}} \mathbf{b}_j^H \mathbf{n}_i = l_{i,j} + x_{i,j} \quad (20)$$

where  $x_{i,j}$  is zero-mean complex Gaussian noise with  $P[x_{i,j} x_{i,j}^*] = \sigma_e^2 = M\sigma^2/P$ . Note that  $x_{i,j}$  values are independent of  $l_{i,j}$  and i.i.d.  $\forall i, j$ . Based on the above, Eqs. (6) and (8) can be estimated as

$$\hat{\mathbf{U}}_i = (\mathbf{g}_i + \mathbf{c}_i)(\mathbf{h}_i + \mathbf{d}_i)^T \sqrt{\frac{\frac{P}{M} \eta_i}{\frac{P}{M} |\mathbf{h}_i|^2 + 1}} e^{j\theta_i}, \quad (21)$$

$$\begin{aligned} \hat{\boldsymbol{\Sigma}}_{\mathbf{n}} &= E[\hat{\mathbf{n}} \hat{\mathbf{n}}^H] \\ &= \sum_{k=1}^K E[(\mathbf{g}_k \mathbf{g}_k^H + \sigma_e^2 \mathbf{I}_N) \eta_k \\ &\quad \times \frac{\sigma_e^2}{\sum_{i=1}^M \frac{P}{M} (|h_{k,i}|^2 + \sigma_e^2) + 1} + \mathbf{I}_N], \end{aligned} \quad (22)$$

where  $\mathbf{c}_i$  and  $\mathbf{d}_i$  are estimation error vectors for  $\mathbf{g}_i$  and  $\mathbf{h}_i$ , respectively and  $\sigma_e^2$  is the scaled estimation error variance. Consider the quasi-static channel where the channel does not change in  $T$  time slots. The time average of (20) over  $T$  time slots

$$\text{avg}(\hat{h}_{i,j,T}) = \frac{\hat{h}_{i,j,1} + \hat{h}_{i,j,2} + \dots + \hat{h}_{i,j,T}}{T} = h_{i,j} + \bar{x}_{i,j,T} \quad (23)$$

where  $\hat{h}_{i,j,t}$  represents the estimated channel coefficient at time  $t$ . The variance of the averaged estimation noise  $\bar{x}_{i,j,T}$  over time  $T$  decreases as  $T$  increases. This is clearly represented in the simulation results presented in the next section.

### E. Phase Optimization via the Bisection Search Algorithm

Step 1, choose an initial set of randomly generated phases. Denote these phases as  $\theta_1^{(1)}, \theta_2^{(1)}, \dots, \theta_K^{(1)}$ . Step 2, repeat until system capacity reaches a stopping criterion (capacity difference from previous iteration falls below a threshold). Cycle through each of  $K$  relay phases, fixing all but one. Determine the phase that maximizes  $\det(\Psi)$  (Eq. (16)).

Denote the set of phases as  $\theta_1^{(o)}, \theta_2^{(o)}, \dots, \theta_K^{(o)}$ , where the superscript indicates that a local maximum is reached. The iterative algorithm converges to a local maximum since the overall capacity is upper-bounded by the total power constraint at the source and the bisection search algorithm monotonically increases capacity at each iteration.

## IV. SIMULATION RESULTS

Fig. 2 shows the capacity of the APSF scheme for the two-hop cooperative system and compares it with AF. The capacity gain from APSF compared to that of AF is considerable. This performance gain is achieved with minimal added feedback and computational complexity because the proposed algorithm only requires the set of phases to be fed back to the relays and the iterative bisection search algorithm converges quickly. However, it should be pointed out that from an implementation point of view, APSF requires added hardware at the relays compared to AF, which increases hardware costs and complexity. The required hardware maybe implemented using adjustable analog phase shifters (see [10]) or through the use of digital signal processors. Fig. 2 also represents the performance of APSF under imperfect CSI and it is clear that APSF results in significant performance gains even under imperfect channel estimation.

The case of DPSF is not simulated in this section due to the complexity associated with DF, which requires the signals first to be decoded at the relays before they are forwarded to the destination. However, the performance of APSF, AF, and DF are compared in terms of Average Bit Error Rate (ABER) in Fig. 4 (QPSK modulation combined with VBLAST is applied). Fig. 3 illustrates the setup used under DF for the BER results represented in Fig. 4. As illustrated in Fig. 3 for successful decoding of the signal from the source each relay needs to be equipped with a minimum of  $M$  receive antennas. Thus, a MIMO DF cooperative system requires significantly more complex hardware at the relays compared to AF and APSF. Fig. 4 also demonstrates that unlike APSF, the performance of AF and DF does not improve as the number of relays is increased. Fig. 5 represents the corresponding capacities of AF and APSF with different numbers of relays which agree with the BER results presented in Fig. 4.

Fig. 6 represents a comparison between APSF and AF when relays are placed in three different spatial configurations in the cell: Position 1, where relays are close to the S (1 km from S), Position 2 where the relays are approximately equal distance from S and D (2 km from S), and Position 3,

where relays are closer to D (3 km from S).  $d_0$  is set to 1 km and the path loss exponent  $n$  is chosen to be 2.7 which corresponds to an urban area cellular network. A four-relay network is considered. From the results presented in Fig. 6, it is clear that the APSF performance gains previously demonstrated still hold independently of these different spatial relay distributions throughout the cell. Since the results presented in Fig. 6 are scenario dependent, more general quantitative conclusions regarding the locations of the relays and BER performance cannot be made. Overall system performance analysis as a function of relay location is a topic that merits further research.

## V. CONCLUSION

A low complexity optimization scheme for the multi-relay cooperative MIMO systems has been presented. The phase adjustment introduced at the relays allows for coherent combining of the signals at the destination, resulting in considerable performance gain.

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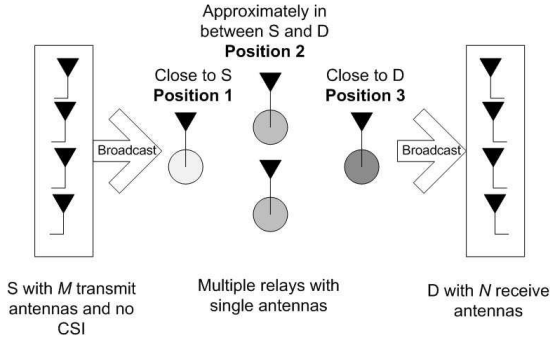


Fig. 1. Two-hop MIMO system based on APSF.

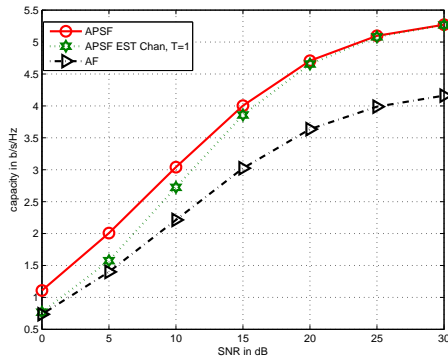


Fig. 2. capacity comparison of conventional AF, APSF, and APSF protocols with imperfect channel estimation over 1 block of training data for 4 relays system. Power at relays is set to 10dB,  $M=N=2$ .

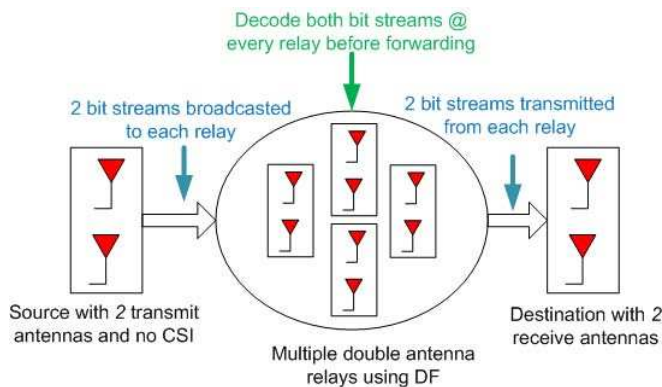


Fig. 3. The setup for the BER analysis of the  $M = 2$  and  $N = 2$  DF system using VBLAST.

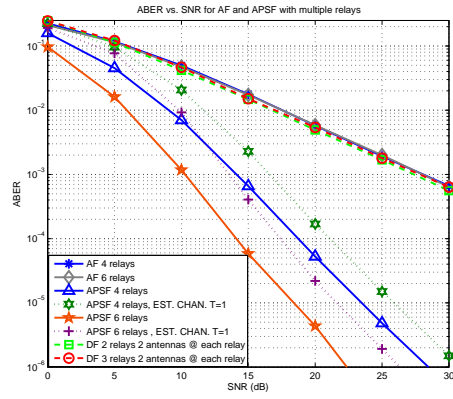


Fig. 4. Comparison of the BER performance between the conventional AF, APSF (with 4 and 6 single antenna relays, QPSK modulation and  $M=N=2$ ), and DF (with 2 and 3 double antenna relays, QPSK modulation and  $M=N=2$ ) under the assumption of perfect and imperfect channel estimation. The plots in the case of AF 4, and 6 relays overlap since there is almost no BER gain due to addition of more relays in the case of AF.

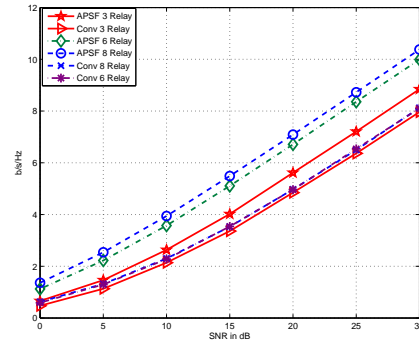


Fig. 5. Comparison of the capacity between the conventional AF, and APSF, with 3, 4, 6, and 8 relays with  $M=N=2$ . (The AF capacity for 3, 4, and 6 relays overlap) The numerical results for the capacity corroborate the BER simulation results presented in Fig. 4.

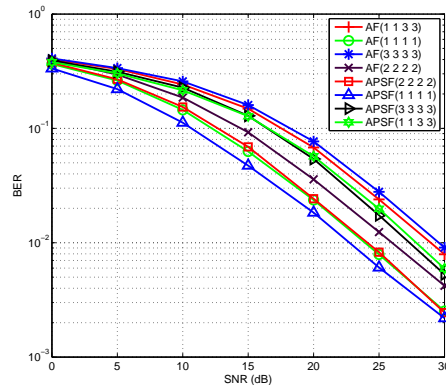


Fig. 6. Comparison of the BER of AF and APSF for 4 relays distributed at different locations within the network (1 refers to a relay close to the S, 2 refers to a relay in the middle, and 3 refers to a relay close to D. Refer to Figure 1).