Linear Dispersion Over Time and Frequency

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***Abstract*— High rate linear dispersion codes (LDC) for space time channels can support arbitrary numbers of transmit and receive antennas. In contrast to the one-to-one transformations used in interleaving, these codes disperse data in linear combinations over space and time. To improve performance of orthogonal frequency division multiplexing (OFDM) for wireless fading channels, this paper investigates increasing frequency and time diversity using LDC. To overcome the requirement of constant channel gains over an entire LDC time interval, a new decoding algorithm for a special subclass of LDC is proposed. The newly proposed LDC-OFDM linearly disperses data over both time and frequency, i.e., over multiple subcarriers and OFDM blocks. Simulations show the bit error rate (BER) performance of rate-one LDC-OFDM with zero padding is superior to that of uncoded OFDM with zero padding. Further, compared to uncoded OFDM, LDC-OFDM may have improved performance without increasing the peak-to-average power ratio (PAPR).**

I. Introduction

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) has been accepted as a standard for high-data-rate communications. By serial-to-parallel convertion, OFDM transforms a single wideband multipath channel into multiple parallel narrowband flat fading channels, enabling simple equalization.

In practical OFDM system design, it is important to notice that uncoded OFDM cannot provide the same degree of diversity combining as uncoded single-carrier systems in severe frequency-selective fading environments, since the frequency responses of subcarriers differ from one another, and hence the optimal diversity combining weights chosen for one of the OFDM subcarriers is no longer optimal for the other OFDM subcarriers. In fading channels, very low signal-to-noise ratio (SNR) or channel nulls are experienced, at least over a fraction of the transmitted time, for some subchannels. This means that the information transmitted in these subbands will be lost. One technique to mitigate the above problem is the use of error-correcting codes across all subchannels at the price of reduced bandwidth efficiency, or coded OFDM (see e.g. [1]). Even if some sub-carriers experience a null in frequency response, they can be reconstructed from the coding information available in those bits that were successfully transmitted.

A critical issue for high-data-rate transmission is the coding rate for OFDM, which is related to bandwidth efficiency. In conventional OFDM schemes, the coding rate usually is less than one. Past research efforts have concentrated on schemes that produce good trade-offs between coding rate and error performance. This paper intends to investigate high-rate OFDM, and in particular, a proposed high-rate (rate up to one) coded OFDM system that improves BER performance. The proposed new LDC OFDM method exploits diversity across both multiple subcarrier channels and multiple OFDM blocks.

In recent years, multiple transmit and multiple receive (MIMO) antenna systems have attracted extensive interest and research. Very recently, Hassibi and Hochwald proposed a high-rate space time coding framework, linear dispersion codes (LDC) [2], which can support any configuration of transmit and receive antennas. LDC are designed to optimize the mutual information between the transmitted and received signals. It is shown in [2] that LDC may achieve a coding rate of up to one and outperform the well-known full-rate uncoded V-BLAST [3] scheme. In this paper, we propose to investigate whether LDC could similarly improve OFDM performance.

The paper is organized as follows. The proposed LDC decoding algorithm is discussed in Section II. In Section III the system structure model and signal detection of LDC-OFDM are described. Then simulation and analysis are presented in Section IV. The following notation is used in the following sections:$(\cdot)^\dagger$ denotes matrix pseudoinverse, $(\cdot)^T$ matrix transpose, $(\cdot)^H$ matrix transpose conjugate, and $C^{A\times B}$ denotes a complex matrix with dimensions $A \times B$.

II. New LDC decoding algorithm

Assume the data sequence has been modulated using complex-valued symbols chosen from an arbitrary, e.g. r-PSK or r-QAM, constellation. A linear dispersion code (LDC), S_{LD}, was first defined for multi-input, multi-output (MIMO) systems with M transmit antennas, N receive antennas, T channel uses and Q LDC constellation symbols as [2]

$$S_{LD} = \sum_{q=1}^{Q} (\alpha_q A_q + j\beta_q B_q) \quad (1)$$

where the LDC matrix is $S_{LD} \in C^{T\times M}$, $A_q \in C^{T\times M}, B_q \in C^{T\times M}, q = 1, ..., Q$ are called dispersion matrices, which transform data symbols into a space-time matrix. The constellation symbols are defined by

$$s_q = \alpha_q + j\beta_q, q = 1, ..., Q \quad (2)$$

The basic LDC system was originally formulated as follows

[2]:

$$X = \sqrt{\frac{\rho}{M}} \sum_{q=1}^{Q} (\alpha_q A_q + j\beta_q B_q) H + V \tag{3}$$

where space time MIMO channel matrix is $H \in C^{M \times N}$, received signal matrix $X \in C^{T \times N}$ and complex white Gaussian noise $V \in C^{T \times N}$.

In LDC design, minimizing average pairwise error probability (PEP) is shown to be numerically difficult for high rate systems [2]. Rather, LDC design was achieved by formulating a power-constrained optimization problem based on mutual information [2]. The following remarks are in order:

1) The above LDC system model (3) requires ($M \times N$) MIMO block fading channels that are valid only when the channel is constant for at least T channel uses.
2) From Eq. (23) and (24) in [2], we observe that (3) leads to LDC decoding that requires block fading channel knowledge.

This paper considers applying LDC to multicarrier systems, which a special type of MIMO channel with the same number of inputs and outputs. For LDC in this channel, this paper defines the data symbol coding rate of LDC as

$$R_{LDC}^{sym} = \frac{Q}{MT} \tag{4}$$

When $Q = MT$, the coding rate of LDC is one, while the data symbol coding rate of LDC is less than one when $Q < MT$.

In this paper, the MIMO formulation (3) is transformed and modified so as to apply LDC across both multiple subcarriers and multiple OFDM blocks to achieve both frequency and time diversity. In OFDM systems, one full OFDM block transmission, considered as one channel use in this paper, is comprised of the number of time slots in one discrete Fourier transformed OFDM symbol plus a guard interval. We do not assume that the channel coefficients are constant across multiple OFDM blocks, since each OFDM block already occupies many time slots. Rather, it is assumed that the channel coefficients are constant over only one OFDM block.

In the following, we consider a special subclass of dispersion matrices with the constraint

$$A_q = B_q, q = 1, ..., Q \tag{5}$$

In the following, we are able to remove the direct dependency of the LDC decoding procedure on the channel matrix. That is to say, we consider a special subclass of LDC codes that permit the channel coefficients to be changed over each OFDM block instead of over T channel uses. This enables an LDC decoding layer to be independent of the specific equalizers used and enables LDC to become more widely applicable to enhancing different standards.

In this case, we define the $T \times M$ LD coded matrix S_{LD} to be

$$S_{LD} = \sum_{q=1}^{Q} s_q A_q \tag{6}$$

Define vec operation of $m \times n$ matrix K as

$$vec(K) = \begin{bmatrix} K_{.1}^T & K_{.2}^T & ... & K_{.n}^T \end{bmatrix}^T \tag{7}$$

where $K_{.i}$ is the i^{th} column of K. Reordering S_{LD} and each matrix A_q into a $TM \times 1$ column vector respectively by $vec(S_{LD})$ and $vec(A_q)$, we transform (6) into

$$vec(S_{LD}) = \begin{bmatrix} vec(A_1) & ... & vec(A_Q) \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_Q \end{bmatrix} \tag{8}$$

To decode the data symbol vector, we could invert (8) by calculating the Moore-Penrose pseudo-inverse of LDC encoding matrix $G = \begin{bmatrix} vec(A_1) & ... & vec(A_Q) \end{bmatrix}$. If G has full column rank, we obtain the least squares solution

$$G^{\dagger} = (G^H G)^{-1} G^H \tag{9}$$

Obviously, if G is a square matrix whose inverse exists, then a unique solution of $s_q, q = 1, ..., Q$ could be found.

Symbol-by-symbol detection follows LDC signal estimation. The procedure basically includes two steps, which we call two-step-estimation:

1) *Signal estimation per channel use:*
   Signals in each channel use are estimated. No immediate signal detection is performed. (In each step, channel knowledge for each channel use is required in each estimate; in different channel uses, channel matrices could be different.)
2) *Data symbol estimation and detection per LDC block:*
   After signal estimation for T channel uses (corresponding to one LDC block) is completed, source data symbols are estimated from estimated LDC-encoded symbols. (In this step, channel knowledge is not required). Bit detection is then performed.

In contrast to the originally proposed LD codes [2], the above LDC signal detection procedure eliminates the requirement that channel coefficients be constant over multiple channel uses. To facilitate two-step-estimation, we choose a special class of LDC codes, which make LDC coded symbols uncorrelated. General design of this special class of LDC codes is still an open issue.

An example of that special class of LDC codes is shown as follows. The group of dispersion matrices of this code satisfies the constraint of (5) and $M = T$ [2], which is

$$\begin{aligned} A_{M(k-1)+l} &= B_{M(k-1)+l} \\ &= \frac{1}{\sqrt{M}} D^{k-1} \Pi^{l-1}, k = 1, ..., M; l = 1, ..., M \end{aligned} \tag{10}$$

where $D = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{j\frac{2\pi}{M}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\frac{2\pi(M-1)}{M}} \end{bmatrix}$,

$$\prod = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & \ddots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

Using the above matrices, the data symbol coding rate of LDC is one. In this case, since $\begin{bmatrix} vec(A_1) & \ldots & vec(A_Q) \end{bmatrix}$ is a full-rank square matrix, its inverse exists and may be pre-calculated. LDC encoding and decoding only requires one multiplication between a pre-calculated matrix and a signal vector. The per-data-symbol complexity of encoding and decoding is thus independent of the total number of constellation symbols, which are encoded in the coding matrix, as well as proportional to the data symbol coding rate of LDC.

III. LDC CODED OFDM SYSTEM

A. Wideband OFDM model

During transmission, for each block of N IFFT transformed complex symbols, a block of P symbols are corrupted in a frequency selective channel with order L taps. Each of the paths experiences independent Rayleigh fading.

As discussed in Section II, a key assumption is that the channel experiences slow fading so that channel coefficients are constant over one OFDM block, considered as one channel use, while channel coefficients could change in subsequent OFDM blocks. The second step of the proposed LDC decoding algorithm in Section II is not directly dependent on channel knowledge. Choosing $P \geq N+L$, the inter-block interference due to the previous transmitted block is eliminated by a guard interval. We consider zero-padded OFDM,

$$x_{OFDM}^{(i)} = H_0^{(i)} F_N^H s_{OFDM}^{(i)} + v_{OFDM}^{(i)}, i = 1, ..., T, \quad (11)$$

with the i-th received OFDM block $x_{OFDM}^{(i)} \in C^{P \times 1}$, the frequency selective channel matrix $H_0^{(i)} \in C^{P \times N}$ corresponding to the i-th OFDM block, normalized IFFT matrix $F_N^H \in C^{N \times N}$, the i-th complex symbol vector $s_{OFDM}^{(i)} \in C^{N \times 1}$. The Toeplitz channel matrix $H_0^{(i)}$ is always guaranteed to be invertible, regardless of the channel zero locations [4]. Zero-mean white additive complex Gaussian noise vector with variance ${\sigma_n}^2$ is represented by $v_{OFDM}^{(i)}$. We also assume $H_0^{(i)}$, $s_{OFDM}^{(i)}$, and $v_{OFDM}^{(i)}$ are statistically independent.

B. LDC-OFDM system

The proposed LDC decoding algorithm in Section II is applied to the wideband OFDM channel described above. In OFDM systems, since the number of subcarriers is typically much higher than the number of antennas in space time MIMO systems, LDC has more freedom to choose larger dispersion matrices. In addition, the ability to guarantee low correlation across subcarriers in OFDM also serves as an advantage for LDC-OFDM.

One LDC-OFDM block, illustrated in Figure 2, consists of T adjacent OFDM blocks. An LDC-OFDM system includes D LDC blocks, each with LDC matrices occupying M_k subcarriers and T OFDM blocks $\in C^{T \times M_k}, k = 1, ..., D$, with $\sum_k M_k = N$. One LDC-OFDM block could be organized into the $T \times N$ matrix:

$$S_{LDC-OFDM-block} = \begin{bmatrix} \left[s_{OFDM}^{(1)}\right]^T \\ \vdots \\ \left[s_{OFDM}^{(T)}\right]^T \end{bmatrix} \quad (12)$$

where $S_{LDC-OFDM-block} \in C^{T \times N}$ and $S_{OFDM}^{(i)}$, which has been used in (11), is the transmitted complex symbol vector before the inverse Fourier tranformation in the transmitter for the i^{th} OFDM transmitted block. $\left[S_{OFDM}^{(i)}\right]^T$ consists of all the D row vectors $S_{LD(i,.)}^{(k)}, k = 1, ..., D$, where $S_{LD(i,.)}^{(k)} \in C^{1 \times M_k}$ is the i-th row of the k-th LDC matrix codeword $S_{LD}^{(k)}$ in a single LDC-OFDM block. $S_{LD(i,.)}^{(k)}$ occupies M_k subcarriers, and it is not necessary that the M_k subcarriers are adjacent.

C. LDC-OFDM receiver

The receiver for LDC-OFDM is illustrated in Figure 3. The receiver first estimates the signals in T OFDM blocks. Second, in the receiver, the estimated $S_{LDC-OFDM-block}$ is reorganized into D LDC blocks. The D LDC demodulators operate in parallel, followed by data bit detection. Denote the LDC encoding matrix of the k-th LDC matrix codeword $S_{LD}^{(k)} \in C^{T \times M_k}$ as $G^{(k)}$, which encodes source data symbol vector with zero mean, unit variance, $s^{(k)} = \begin{bmatrix} s_1^{(k)} & s_1^{(k)} & \cdots & s_{Q_k}^{(k)} \end{bmatrix}^T$ into $vec(S_{LD}^{(k)})$, where Q_k is the number of source data symbols in $s^{(k)}$. For simplicity, we consider the case that $G^{(k)} = G, k = 1, ..., D$ are unitary matrices and $Q_k = Q, k = 1, ..., D$, then covariance matrices of $S_{OFDM}^{(i)}, i = 1, ..., T$ are identity matrices.

1) First estimation step - OFDM Demodulation: In the proposed LDC decoding algorithm, LDC decoding is independent of OFDM signal estimation. Thus the proposed LDC-OFDM system could be made backwards-compatible with conventional OFDM systems. Reiterating, a significant advantage arising from LDC-OFDM decoding is that it is not required that channel coefficients remain constant over multiple OFDM blocks. This proposed system could thus be used in dynamic environments.

In the next section, minimum-norm zero-forcing (ZF) and minimum-mean-squared-error MMSE equalizers are chosen to investigate error performance. Assuming that OFDM symbols are normalized with unit variance, the respective equalizers are given by [4]

$$G_{ZF}^{(i)} = F_N (H_0^{(i)})^{\dagger} \quad (13)$$

$$G_{MMSE}^{(i)} = F_N \left(H_0^{(i)}\right)^H \left(\sigma_n^2 I_P + H_0^{(i)} \left(H_0^{(i)}\right)^H\right)^{-1} \quad (14)$$

and

$$\widehat{s_{OFDM}^{(i)ZF}} = G_{ZF}^{(i)} x_{OFDM}^{(i)} \quad (15)$$

$$\widehat{s_{OFDM}^{(i)MMSE}} = G_{MMSE}^{(i)} x_{OFDM}^{(i)}. \quad (16)$$

where $i = 1, ..., T$

2) Second estimation step - LDC-OFDM Block Demodulation: Reorganizing the estimation results of first estimation step into estimated D LDC matrix codewords, $\widehat{S_{LD}^{(k)}}, k = 1, ..., D$, the estimation data symbol vectors corresponding to D LDC blocks are

$$\widehat{s^{(k)}} = \left[G^{(k)}\right]^{\dagger} vec(\widehat{S_{LD}^{(k)}}), k = 1, ..., D \quad (17)$$

IV. Simulation and Analysis

A. Simulation setup

Perfect channel estimation (amplitude and phase) is assumed at the receiver but not at the transmitter. The number of subcarriers of OFDM, N, is 64. In the simulation, the LDC in (10) are adopted as dispersion matrices in all LDC codewords. The D LDC demodulators each decode $T \times M_k$ LDC matrices, where M_k is the number of subcarriers and T is the number of OFDM blocks. In particular, we set

$$M_1 = ... = M_D = M = T = \frac{N}{D}. \quad (18)$$

To assess performance as a function of LDC matrix size, N is fixed while D is varied. Data symbols use 4-PSK modulation. The frequency selective channel has an $(L+1)$ paths exhibiting an exponential power delay profile, where $L = 12$ is chosen. All simulation curves were obtained from 10,000 Monte Carlo iterations per OFDM block.

The question remains on how to allocate the M_k subcarriers of each LDC code among the N subcarriers. In frequency selective channels, neighboring subcarriers are more likely to fade simultaneously than subcarriers spaced at larger intervals. To understand the effects of subcarrier spacing on BER performance, simulations were performed both without subcarrier spacing and with $(M_k - 1)$ subcarrier intervals within each LD code matrix.

B. Comparison of LDC-OFDM and OFDM

Figure 4 shows the Bit Error Rate (BER) performance vs. receiver input average symbol SNR of the zero-padded LDC-ZP-OFDM system with a subcarrier spacing interval $(M_k - 1)$ in LDC. Various combinations of M with ZF or MMSE equalizers are used, and compared to ZP-OFDM.

It can be seen that LDC-ZP-OFDM is very effective in frequency selective Rayleigh fading channels. The LDC-ZP-OFDM systems with MMSE equalizers significantly outperform that of LDC-ZP-OFDM systems with ZF equalizers. Under both equalizers, BER performance of LDC-ZP-OFDM is notably better than that of ZP-OFDM for SNRs higher than 16.5 dB. Within the whole range of SNRs shown in Figure 4, MMSE and LDC-ZP-OFDM outperform both ZF and MMSE ZP-OFDM receivers. The larger the dispersion matrices used, the greater the performance improvement achieved, at a cost of increased decoding delay. Despite LDC's increased delay in decoding, we note that a symbol coding rate of one is used, resulting in no bandwidth expansion.

C. Comparison of LDC-OFDM with different subcarrier spacings

In Figure 5, $M = 8$ is used for the above channel. The curves show a BER performance comparison of LDC-ZP-OFDM between subcarrier spacings at the extremes of zero and $(M - 1)$ for LDC. No obvious performance differences are found.

D. Peak-to-Average Power Ratio comparison

It is well known that low Peak-to-Average Power Ratio (PAPR) is critical to OFDM systems. Simulation results in Figure 6 show the PAPR of LDC-OFDM systems and OFDM systems is similar. Thus, LDC-OFDM systems may improve BER without increasing PAPR.

V. Conclusion

Inspired by a technique proposed for space-time processing, we have applied linear dispersion codes to improve OFDM performance in multipath fading channels. The attractive LD codes can be advantageously combined with OFDM transmission to enable simple decoding. Large LDC matrices can be designed. A novel LDC decoding algorithm is proposed for a special subclass of LDC matrices with the constraint (5), which eliminates the direct dependency between LDC decoding and channel knowledge. At a cost of increased decoding delay, the proposed LDC decoder can support channels that change across OFDM blocks. Exploiting both frequency and time diversity available in frequency selective wideband OFDM channels, the performance of the proposed LDC-OFDM has high transmission bandwidth efficiency and improved BER. For instance, as shown in Figure 4, with MMSE equalization, LDC-ZP-OFDM using dispersion matrices (10) with 8 subcarriers per LDC block, 3.5 dB and 7.6 dB gains over ZP-OFDM are observed at BERs of 10^{-2} and 10^{-3}, respectively.

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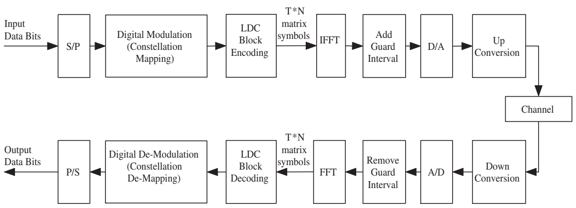

Fig. 1. Proposed LDC-OFDM system model

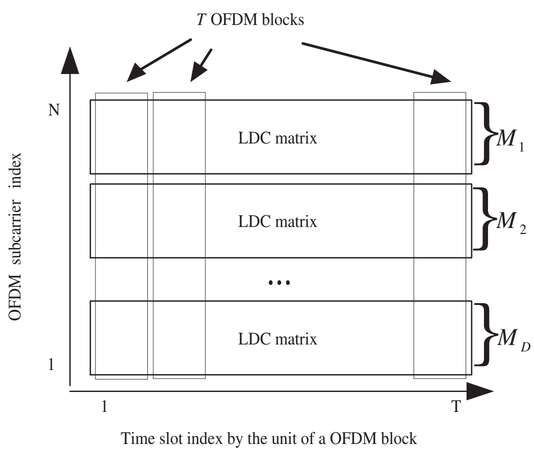

Fig. 2. LDC-OFDM blocks in the time-frequency plane

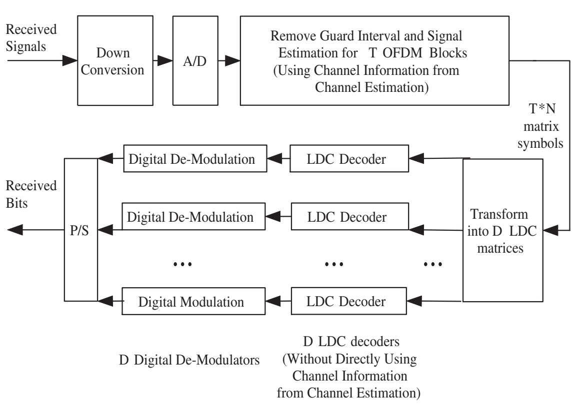

Fig. 3. Proposed LDC-OFDM receiver structure

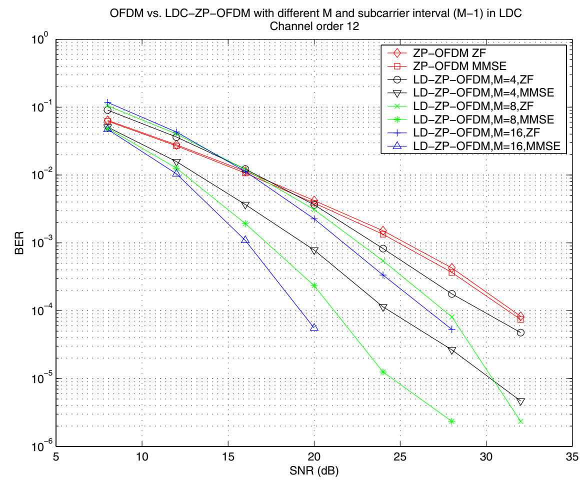

Fig. 4. BER Performance of OFDM vs LDC-ZP-OFDM with subcarrier interval

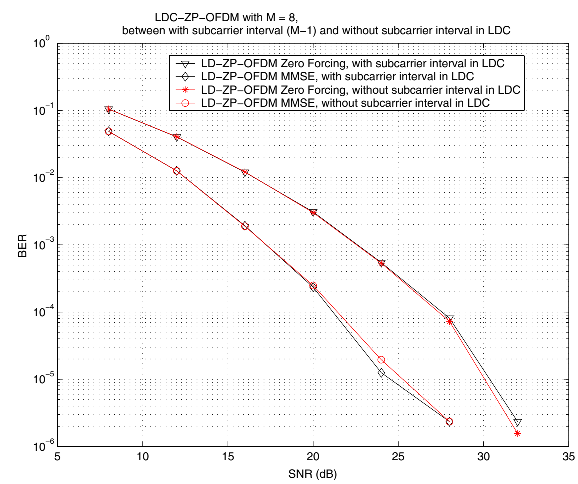

Fig. 5. For LDC-ZF OFDM, BER comparison between with subcarrier interval (M-1) and without subcarrier interval in LDC

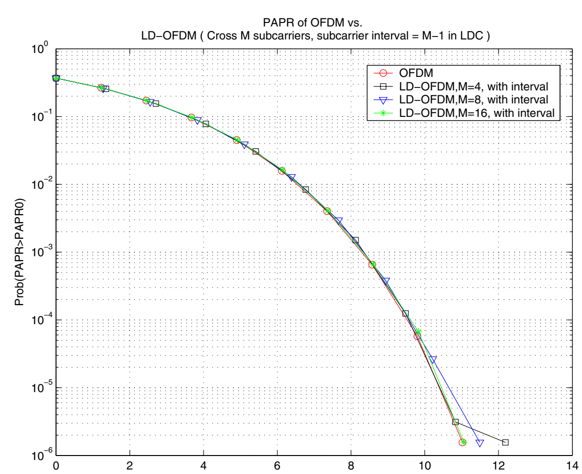

Fig. 6. PAPR performance of OFDM vs. LDC-OFDM