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Single-cell vs. Multicell MIMO Downlink Signalling Strategies with Imperfect CSI

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***Abstract*—Cooperative base station (BS) signaling in multiple-input multiple-output (MIMO) multicell networks has been recently shown to achieve significant gains in system throughput and reliability. However, the promised gains of BS cooperation depend on the assumptions of perfect channel state information at the transmitters (CSIT) and ideal backhaul links. By focusing on the effects of imperfect CSIT on cooperative BS systems, this paper considers multi-antenna downlink communication with base stations (BSs) that perform joint transmitter null-space decomposition (JT-decomp) linear precoding strategy. An upper bound for the rate loss of JT-decomp precoding is provided, which demonstrate that the percentage rate loss of JT-decomp cooperative BS system decreases as signal-to-noise ratio (SNR) increases. This shows that JT-decomp precoding, in spite of the imperfect CSIT, preserves its spatial multiplexing gain at high SNR.**

I. Introduction

Spatial multiplexing cooperative base station (BS) systems, in which neighboring BSs are connected to form a virtual multiple-input multiple-output (MIMO) array, have been recently shown to significantly increase the spectral efficiency of the downlink cellular MIMO network [1], [2]. The main idea is to let BSs collaboratively and simultaneously transmit data streams to multiple mobile stations (MSs). Under full cooperation, the multicell MIMO downlink can be modeled as a classical MIMO broadcast channel with per-base power constraints. The optimal non-linear dirty-paper coding (DPC), with a pooled power constraint can then be used across the transmitters [3]. However, DPC is an information theoretic concept which is very difficult to implement in practical systems. Although not optimal, *joint transmitter null-space decomposition (JT-decomp)* is a more practical linear precoding technique that can achieve a significant fraction of the optimal DPC capacity by completely precanceling interference at the transmitters, and providing each MS with a block-diagonal inter-user interference-free channel [1], [2], [4], [5].

A full cooperative system can be impractical in realistic settings due to limitations in terms of globally available and perfect channel state information at the transmitter (CSIT), and backhaul capacity. Precoding techniques that exploit only local CSIT have been recently addressed in [6]. The impact of limited capacity backbone links and imperfect CSI on uplink network MIMO have been recently treated in [7]. Information theoretic analysis of cooperative MIMO networks with incomplete CSI is studied in [8]. For a hybrid signaling scheme that performs well under non-ideal scenarios, we finally refer to [9]. In this paper, we study the impact of imperfect channel knowledge on the performance of the JT-decomp cooperative BS system. For reference, we also investigate the effect of intercell co-channel interference on conventional cellular system as interference is the dominant impairment [10]. We consider a frequency-division duplexing (FDD) cellular network and assume that each MS has imperfect CSI and sends its imperfect CSI to the BSs through a noiseless, error-free, and delay-free feedback channel. The most important contributions are the following:

- A detailed channel estimation error model for cooperative BS cellular systems is first provided and the CSI is described.
- Upper bounds for the rate loss of JT-decomp and conventional cellular systems incurred, respectively, due to erroneous CSI and co-channel interference are also derived.

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II. System Model

We consider a cellular system that has at least two adjacent cells with reuse factor of one, each with one BS and one active co-channel MS. We denote by B and K the number of BSs and active co-channel MSs in the network, each equipped with N_t and N_r antennas, respectively. We define $\mathbf{H}_{bk}$ as the $N_r \times N_t$ channel matrix between BS_b and MS_k modeled as

$$\mathbf{H}_{bk} \triangleq \Phi_{bk}\mathbf{H}^w_{bk} \quad 1 \leq b \leq B,\, 1 \leq k \leq K \tag{1}$$

where Φ_{bk} accounts for path loss. The short term fading matrix $\mathbf{H}^w_{bk}$ is assumed to be white with independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ entries. The channel model is assumed to remain constant for a coherence block of T channel uses, but varies independently from block to block. The coherence block is split into training and data transmission phases of lengths (in symbols) τ and $T_d = T - \tau$, respectively. We assume FDD so that each MS estimates its channel from downlink training symbols and feeds the information back to its serving BSs through a noiseless, error-free, and delay-free feedback channel.

Let $\mathbf{x}_b$ be the $N_t \times 1$ precoded data signal transmit vector for BS_b. The baseband equivalent of the transmitted signal vector is

$$\mathbf{y}_k = \sum_{b=1}^{B} \mathbf{H}_{bk}\mathbf{x}_b + \mathbf{n}_k, \quad 1 \le k \le K \tag{2}$$

where $\mathbf{y}_k$ is the $N_r \times 1$ received vector at MS$_k$, and $\mathbf{n}_k$ is the AWGN vector with covariance matrix $N_0\,\mathbf{I}_{N_r}$.

A. Conventional Single-cell Signalling

Here communication is restricted to only active in-cell base-mobile pairs, and each BS has knowledge of CSI of its own cell. Thus (2) can be expressed as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{x}_k + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K} \mathbf{H}_{\bar{k}k}\mathbf{x}_{\bar{k}} + \mathbf{n}_k \tag{3}$$

with $\bar{k}$ denotes the value not being k, $\mathbf{H}_{kk}$ and $\{\mathbf{H}_{\bar{k}k}\}$ are, respectively, the in-cell and inter-cell interfering channel matrices, and $\mathbf{x}_k$ is given by

$$\mathbf{x}_k = \mathbf{W}_{kk}\mathbf{s}_k \tag{4}$$

where $\mathbf{s}_k \in \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{N_r}\right)$ denotes the data intended for MS$_k$, $\mathbf{W}_{kk}$ is a $N_t \times N_r$ linear precoding matrix expressed as

$$\mathbf{W}_{kk} = \mathbf{\Omega}_{kk}\sqrt{\frac{P_k^{\mathrm{Tx}}}{N_r}}\mathbf{I}_{N_r} \tag{5}$$

where $\mathbf{\Omega}_{kk}$ is a linear transmit filter which consists of the first N_r right singular vectors of $\mathbf{H}_{kk}$, and P_k^{Tx} is the power allocated to MS$_k$. Substituting (4) in (3) yields

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{W}_{kk}\mathbf{s}_k + \mathbf{n}_{k,ce} \tag{6}$$

where vector $\mathbf{n}_{k,ce} \triangleq \mathbf{n}_k + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K} \mathbf{H}_{\bar{k}k}\mathbf{W}_{\bar{k}\bar{k}}\mathbf{s}_{\bar{k}}$ is the *effective combined* interference plus noise vector, which can be shown to have covariance matrix $\mathbf{R}_{\mathbf{n}_{k,ce}} = N_0\mathbf{I}_{N_r} + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K} \frac{P_{\bar{k}}^{\mathrm{Tx}}}{N_r}\left[\left(\mathbf{H}_{\bar{k}k}\mathbf{\Omega}_{\bar{k}\bar{k}}\right)\left(\mathbf{H}_{\bar{k}k}\mathbf{\Omega}_{\bar{k}\bar{k}}\right)^{\dagger}\right]$ with $\mathbf{H}_{\bar{k}k}\mathbf{\Omega}_{\bar{k}\bar{k}}$ the $N_r \times N_r$ effective channel matrix.

B. BS Cooperation Signalling

Here each BS has complete knowledge of all data symbols intended for all MSs and CSI of all other BSs. Transmissions from BSs occur simultaneously for all MSs in the network, and the signal transmitted by BS$_b$ is

$$\mathbf{x}_b = \sum_{k=1}^{K} \mathbf{W}_{bk}\mathbf{s}_k \tag{7}$$

where $\mathbf{W}_{bk}$ is a $N_t \times N_r$ transmit linear precoding matrix of MS$_k$ from BS$_b$ given by

$$\mathbf{W}_{bk} \triangleq \mathbf{G}_{bk}\,\mathbf{\Lambda}_{bk} = \mu_k\,\mathbf{G}_{bk} \tag{8}$$

where $\mathbf{\Lambda}_{bk} = \mu_k\mathbf{I}_{N_r}$ with μ_k the power allocated to each substream of MS$_k$, and $\mathbf{G}_{bk}$ is a linear transmit filter matrix. BS$_b$ is also subject to an average transmit power constraint of P_b^{Tx}, i.e.,

$$\begin{aligned}\mathbb{E}\{\|\mathbf{x}_b\|_{\mathrm{F}}^2\} &= \sum_{k=1}^{K}\mathbb{E}\{tr(\mu_k^2\mathbf{G}_{bk}\mathbf{s}_k\mathbf{s}_k^{\dagger}\mathbf{G}_{bk}^{\dagger})\}\\ &= \sum_{k=1}^{K}\mu_k^2\|\mathbf{G}_{bk}\|_{\mathrm{F}}^2 = P_b^{\mathrm{Tx}}.\end{aligned} \tag{9}$$

Given P_b^{Tx}, the powers $\{\mu_k\}_{k=1}^{K}$ can be obtained by solving (9). In matrix form, $\boldsymbol{\mathcal{G}}\boldsymbol{\mu} = \mathbf{P}^{\mathrm{Tx}}$, where these quantities are, respectively, defined as

$$\begin{bmatrix}\|\mathbf{G}_{11}\|_{\mathrm{F}}^2 & \cdots & \|\mathbf{G}_{1K}\|_{\mathrm{F}}^2\\ \vdots & \ddots & \vdots\\ \|\mathbf{G}_{B1}\|_{\mathrm{F}}^2 & \cdots & \|\mathbf{G}_{BK}\|_{\mathrm{F}}^2\end{bmatrix}\begin{bmatrix}\mu_1^2\\ \vdots\\ \mu_K^2\end{bmatrix} = \begin{bmatrix}P_1^{\mathrm{Tx}}\\ \vdots\\ P_B^{\mathrm{Tx}}\end{bmatrix}. \tag{10}$$

A possible solution, if it exists, is

$$\boldsymbol{\mu} = \left[\mu_1^2\cdots\mu_K^2\right]^T = \boldsymbol{\mathcal{G}}^{-1}\mathbf{P}^{\mathrm{Tx}}. \tag{11}$$

Note that if $\boldsymbol{\mathcal{G}}^{-1}$ does not exist and/or $\boldsymbol{\mu}$ in (11) contains negative entries, then the solution can be refined as [1]

$$\boldsymbol{\mu} = \left[\mu^2\cdots\mu^2\right]^T \quad \text{with} \quad \mu^2 = \min_{1\le b\le B}\left(\frac{P_b^{\mathrm{Tx}}}{\sum_{k=1}^{K}\|\mathbf{G}_{bk}\|_{\mathrm{F}}^2}\right). \tag{12}$$

Next, by substituting (7) in (2), after some manipulation we get

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{W}_k\mathbf{s}_k + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\mathbf{H}_k\mathbf{W}_{\bar{k}}\mathbf{s}_{\bar{k}} + \mathbf{n}_k \tag{13}$$

where the $N_r \times BN_t$ matrix $\mathbf{H}_k \triangleq [\mathbf{H}_{1k}\mathbf{H}_{2k}\cdots\mathbf{H}_{Bk}]$, and $\mathbf{W}_k \triangleq [\mathbf{W}_{1k}^{\dagger}\mathbf{W}_{2k}^{\dagger}\cdots\mathbf{W}_{Bk}^{\dagger}]^{\dagger} = \mu_k[\mathbf{G}_{1k}^{\dagger}\mathbf{G}_{2k}^{\dagger}\cdots\mathbf{G}_{Bk}^{\dagger}]^{\dagger} = \mu_k\mathbf{G}_k$ is of size $BN_t \times N_r$. For JT-decomp precoding, we have

$$\mathbf{W}_k \triangleq \mu_k\mathbf{G}_k = \mu_k\check{\mathbf{V}}_k\mathbf{V}_k' \tag{14}$$

where $\check{\mathbf{V}}_k$ consists of the right singular vectors corresponding to the null space of $\mathbf{H}_{\mathrm{tot}}^{(-k)} \triangleq [\mathbf{H}_1^{\dagger}\cdots\mathbf{H}_{k-1}^{\dagger}\mathbf{H}_{k+1}^{\dagger}\cdots\mathbf{H}_K^{\dagger}]^{\dagger}$, and $\mathbf{V}_k'$ contains the first N_r right singular vectors of the virtual channel $\mathbf{H}_k' \triangleq \mathbf{H}_k\check{\mathbf{V}}_k$.

III. Channel Sounding Model

We now present model for the channel estimation error for cooperative BS cellular systems. It is assumed that (i) channel sounding is performed on the white channel matrix $\mathbf{H}_{bk}^{w}$ [11], and (ii) the midamble pilot symbols of sounding signals are used to estimate the channel gains of the downlink channels as employed in the IEEE 802.16.e standard.

A significant amount of interference is expected during channel estimation by the fact that (i) BSs have to transmit simultaneously, and (ii) non-orthogonality of the channel sounding signals among the BSs. An estimation strategy that maintains time/frequency orthogonality among the BSs, with a small increase in overhead is proposed in [12]. We build

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on this method to present a model for the channel estimation error for cooperative BS systems. The idea is to interleave the midambles of the BSs so that they are not transmitted simultaneously from each BS. This way each MS can estimate its channel without interference from other BSs. Thus the coherence interval is $T = \tau + (B-1)\tau + T_d$ with $\tau \geq N_t$ and $(B-1)\tau$ the overhead. We emphasize that the overhead is taken into account in the analysis by the factor $\left(1 - \frac{B\tau}{T}\right)$.

The $N_t \times \tau$ ($\tau \geq N_t$ [11]) downlink training symbol matrix $\mathbf{S}_b$ to estimate the radio link between BS$_b$ and all MSs can be formed as $\mathbf{S}_b \triangleq [\mathbf{s}_b^1 \cdots \mathbf{s}_b^\tau]$, where $\{\mathbf{s}_b^i\}_{i=1}^{\tau}$ are the $N_t \times 1$ training vectors transmitted by BS$_b$ to all MSs. The training symbols are normalized such that $tr(\mathbf{S}_b\mathbf{S}_b^\dagger) = \tau P_b^{\text{Tx}}$ is the total training power transmitted from BS$_b$, and the training matrix is of the form $\mathbf{S}_b = \sqrt{\frac{\tau P_b^{\text{Tx}}}{N_t}}\mathbf{U}_b$, where $\mathbf{U}_b$ is $N_t \times \tau$ unitary (i.e., $\mathbf{U}_b\mathbf{U}_b^\dagger = \mathbf{I}_{N_t}$). The corresponding $N_r \times \tau$ matrices of received and noise vectors at MS$_k$ are, respectively, $\mathbf{Y}_k^{(b)} \triangleq [\mathbf{y}_k^{(b),1} \cdots \mathbf{y}_k^{(b),\tau}]$ and $\mathbf{N}_k \triangleq [\mathbf{n}_k^1 \cdots \mathbf{n}_k^\tau]$. The channel output transmitted from BS$_b$ observed by MS$_k$ is

$$\mathbf{Y}_k^{(b)} = \Phi_{bk}\mathbf{H}_{bk}^w\mathbf{S}_b + \mathbf{N}_k \qquad 1 \leq b \leq B. \tag{15}$$

Premultiplying both sides of (15) by Φ_{bk}^{-1} and postmultiplying the result by $\mathbf{S}_b^+$ yields the noisy estimate

$$\tilde{\mathbf{H}}_{bk}^w = \Phi_{bk}^{-1}\mathbf{Y}_k^{(b)}\mathbf{S}_b^+ = \mathbf{H}_{bk}^w + \mathbf{N}_k' \tag{16}$$

where $\mathbf{N}_k' \triangleq \frac{\sqrt{N_t}}{\Phi_{bk}\sqrt{\tau P_b^{\text{Tx}}}}\mathbf{N}_k\mathbf{U}_b^+$ has i.i.d. $\mathcal{CN}\left(0, \varsigma_{bk}^2\right)$ entries with $\varsigma_{bk}^2 = \frac{N_t N_0}{\tau\Phi_{bk}^2 P_b^{\text{Tx}}}$.

The minimum MSE (MMSE) channel estimation of $\mathbf{H}_{bk}^w$ is now performed based on (16), which yields [13]

$$\hat{\mathbf{H}}_{bk}^w = \mathbb{E}\{\mathbf{H}_{bk}^w|\tilde{\mathbf{H}}_{bk}^w\} = \frac{1}{1+\varsigma_{bk}^2}\tilde{\mathbf{H}}_{bk}^w. \tag{17}$$

Furthermore, $\mathbf{H}_{bk}^w$ is expressed as the sum of $\hat{\mathbf{H}}_{bk}^w$ and the estimation error matrix $\hat{\mathbf{H}}_{bk}^{w,e}$ as

$$\mathbf{H}_{bk}^w = \hat{\mathbf{H}}_{bk}^w + \hat{\mathbf{H}}_{bk}^{w,e} \tag{18}$$

where the entries of $\hat{\mathbf{H}}_{bk}^{w,e}$ and $\hat{\mathbf{H}}_{bk}^w$ are, respectively, i.i.d. $\mathcal{CN}(0, \xi_{bk,e}^2)$ and i.i.d. $\mathcal{CN}(0, 1-\xi_{bk,e}^2)$ with $\xi_{bk,e}^2 = \frac{\varsigma_{bk}^2}{1+\varsigma_{bk}^2}$, where $\varsigma_{bk}^2 = \frac{N_t N_0}{\tau\Phi_{bk}^2 P_b^{\text{Tx}}}$. The CSI model for cooperative BS cellular systems is

$$\mathbf{H}_{bk} = \hat{\mathbf{H}}_{bk} + \hat{\mathbf{H}}_{bk}^e \tag{19}$$

where $\mathbf{H}_{bk}$ is the true channel matrix, $\hat{\mathbf{H}}_{bk}$ and $\hat{\mathbf{H}}_{bk}^e$ are, respectively, the estimated and estimation error channel matrices with i.i.d. $\mathcal{CN}(0, \Phi_{bk}^2 - \Phi_{bk}^2\xi_{bk,e}^2)$ and $\mathcal{CN}(0, \Phi_{bk}^2\xi_{bk,e}^2)$ entries. In the sequel, we assume that all BSs have the same power constraint P^{Tx}, therefore $\hat{\mathbf{H}}_{bk}$ and $\hat{\mathbf{H}}_{bk}^e$ consist, respectively, of i.i.d. $\mathcal{CN}\left(0, \frac{\tau\Phi_{bk}^4 P^{\text{Tx}}}{\tau P^{\text{Tx}}\Phi_{bk}^2 + N_t N_0}\right)$ and $\mathcal{CN}\left(0, \frac{N_t N_0 \Phi_{bk}^2}{\tau P^{\text{Tx}}\Phi_{bk}^2 + N_t N_0}\right)$ entries.

IV. Rate Loss Analysis

Theorem 1 *For the conventional cellular system, the rate loss ΔR_k for MS$_k$ incurred due to intercell co-channel interference is upper bounded by*

$$\Delta R_k \leq N_r\left(1 - \frac{\tau}{T}\right)\log_2\left(N_0 + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\Phi_{\bar{k}k}^2 P_{\bar{k}}^{\text{Tx}}\right). \tag{20}$$

Proof: The rate of an ideal[1] MS$_k$ in an interference-free single cell is given by [1]

$$R_k^{\text{U.B.}} = \left(1 - \frac{\tau}{T}\right)\mathbb{E}\Big\{\log_2 det\left(\mathbf{R}_{\mathbf{n}_k} + \mathbf{H}_{kk}\mathbf{W}_{kk}\mathbf{W}_{kk}^\dagger\mathbf{H}_{kk}^\dagger\right) - \log_2 det\left(\mathbf{R}_{\mathbf{n}_k}\right)\Big\}. \tag{21}$$

Based on (6), the net ergodic achievable rate at MS$_k$ is

$$R_k^{\text{L.B.}} = \left(1 - \frac{\tau}{T}\right)\mathbb{E}\Big\{\log_2 det\left(\mathbf{R}_{\mathbf{n}_{k,ce}} + \mathbf{H}_{kk}\mathbf{W}_{kk}\mathbf{W}_{kk}^\dagger\mathbf{H}_{kk}^\dagger\right) - \log_2 det\left(\mathbf{R}_{\mathbf{n}_{k,ce}}\right)\Big\}. \tag{22}$$

From (21) and (22), the upper bound on the rate loss for MS$_k$ is

$$\begin{aligned}\Delta R_k &= R_k^{\text{U.B.}} - R_k^{\text{L.B.}}\\ &\overset{(a)}{\leq} \left(1 - \frac{\tau}{T}\right)\mathbb{E}\left\{\log_2 det\left(\mathbf{R}_{\mathbf{n}_{k,ce}}\right)\right\}\\ &\overset{(b)}{\leq} N_r\left(1 - \frac{\tau}{T}\right)\log_2\left(N_0 + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\Phi_{\bar{k}k}^2 P_{\bar{k}}^{\text{Tx}}\right)\end{aligned} \tag{23}$$

where (a) follows by dropping the non-negative term $\log_2 det\left(\mathbf{R}_{\mathbf{n}_k}\right)$, (b) follows from Jensen's inequality and the fact that $\mathbb{E}\left\{\left(\mathbf{H}_{\bar{k}k}\mathbf{\Omega}_{\bar{k}\bar{k}}\right)\left(\mathbf{H}_{\bar{k}k}\mathbf{\Omega}_{\bar{k}\bar{k}}\right)^\dagger\right\} = N_r\Phi_{\bar{k}k}^2\mathbf{I}_{N_r}$ as $\mathbf{H}_{\bar{k}k}\mathbf{\Omega}_{\bar{k}\bar{k}}$ is a $N_r \times N_r$ complex Gaussian matrix whose entries are i.i.d. $\mathcal{CN}\left(0, \Phi_{\bar{k}k}^2\right)$ since $\mathbf{\Omega}_{\bar{k}\bar{k}}$ is a unitary matrix (i.e., $\mathbf{\Omega}_{\bar{k}\bar{k}}^\dagger\mathbf{\Omega}_{\bar{k}\bar{k}} = \mathbf{I}_{N_r}$) independent of $\mathbf{H}_{\bar{k}k}$. ■

Theorem 2 *Under the simplifying approximation of the aggregated estimation error channel matrix $\hat{\mathbf{H}}_k^e$ as $\hat{\mathbf{H}}_k^e \approx \Phi_{\tilde{b}k}^2\xi_{\tilde{b}k,e}^2\hat{\mathbf{F}}_k^e$ with $\tilde{b} = \underset{1\leq b\leq B}{\arg\max}\ \Phi_{bk}^2$ and $\hat{\mathbf{F}}_k^e$ the normalized aggregated estimation error channel matrix with i.i.d. $\mathcal{CN}(0,1)$ entries, the rate loss ΔR_k for MS$_k$ in range of the coordinated BSs that employ JT-decomp precoding incurred under erroneous CSI with respect to ideal CSI is upper bounded by*

$$\Delta R_k \leq N_r\left(1 - \frac{B\tau}{T}\right)\log_2\left(N_0 + N_r\hat{\mu}_k^2\Phi_{\tilde{b}k}^2\xi_{\tilde{b}k,e}^2 + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}N_r\hat{\mu}_{\bar{k}}^2\Phi_{\tilde{b}k}^2\xi_{\tilde{b}k,e}^2\right). \tag{24}$$

[1] (21) is also the net rate for cell-center MSs [1].

Proof: With perfect estimation, JT-decomp precoding completely precancels interference, which yields

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \mathbf{n}_k. \tag{25}$$

Thus the net ergodic achievable rate at MS_k is

$$R_k^{\mathtt{P.E.}} = \left(1-\frac{B\tau}{T}\right)\mathbb{E}\Big\{\log_2 det\left(\mathbf{R}_{\mathbf{n}_k} + \mathbf{H}_k\mathbf{W}_k\mathbf{W}_k^\dagger\mathbf{H}_k^\dagger\right) - \log_2 det\left(\mathbf{R}_{\mathbf{n}_k}\right)\Big\}. \tag{26}$$

With imperfect estimation, (13) can be modified as

$$\mathbf{y}_k = \hat{\mathbf{H}}_k\hat{\mathbf{W}}_k\mathbf{s}_k + \hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_k\mathbf{s}_k + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_{\bar{k}}\mathbf{s}_{\bar{k}} + \mathbf{n}_k \tag{27}$$

by the fact that with erroneous CSI the JT-decomp precoding matrices cannot perfectly remove interference, where they only achieve $\{\hat{\mathbf{H}}_k\hat{\mathbf{W}}_{\bar{k}}\}_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}=\{\mathbf{0}\}$ with $\hat{\mathbf{W}}_k = \hat{\mu}_k\hat{\tilde{\mathbf{V}}}_k\hat{\mathbf{V}}'_k$ denoting the estimated precoding matrix calculated based on the erroneous CSI, $\hat{\mathbf{H}}_k = [\hat{\mathbf{H}}_{1k}\cdots\hat{\mathbf{H}}_{Bk}]$, and $\hat{\mathbf{H}}_k^e = [\hat{\mathbf{H}}_{1k}^e\cdots\hat{\mathbf{H}}_{Bk}^e]$. The net ergodic achievable rate at MS_k can be written as

$$\begin{aligned} R_k^{\mathtt{I.E.}} = {} & \left(1-\frac{B\tau}{T}\right)\mathbb{E}\Bigg\{\log_2 det\Bigg(\mathbf{R}_{\mathbf{n}_k} + \hat{\mathbf{H}}_k\hat{\mathbf{W}}_k\hat{\mathbf{W}}_k^\dagger\hat{\mathbf{H}}_k^\dagger \\ & + \hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_k\hat{\mathbf{W}}_k^\dagger\hat{\mathbf{H}}_k^{e\dagger} + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_{\bar{k}}\hat{\mathbf{W}}_{\bar{k}}^\dagger\hat{\mathbf{H}}_k^{e\dagger}\Bigg) \\ & - \log_2 det\Bigg(\mathbf{R}_{\mathbf{n}_k} + \hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_k\hat{\mathbf{W}}_k^\dagger\hat{\mathbf{H}}_k^{e\dagger} \\ & + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_{\bar{k}}\hat{\mathbf{W}}_{\bar{k}}^\dagger\hat{\mathbf{H}}_k^{e\dagger}\Bigg)\Bigg\}. \end{aligned} \tag{28}$$

Based on (26) and (28), the rate loss is

$$\begin{aligned} \Delta R_k = {} & R_k^{\mathtt{P.E.}} - R_k^{\mathtt{I.E.}} \\ \overset{(a)}{\leq} {} & \left(1-\frac{B\tau}{T}\right)\mathbb{E}\Bigg\{\log_2 det\Bigg(\mathbf{R}_{\mathbf{n}_k} + \hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_k\hat{\mathbf{W}}_k^\dagger\hat{\mathbf{H}}_k^{e\dagger} \\ & + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_{\bar{k}}\hat{\mathbf{W}}_{\bar{k}}^\dagger\hat{\mathbf{H}}_k^{e\dagger}\Bigg)\Bigg\} \\ \overset{(b)}{\leq} {} & \left(1-\frac{B\tau}{T}\right)\log_2 det\Bigg(\mathbf{R}_{\mathbf{n}_k} + \mathbb{E}\left\{\hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_k\hat{\mathbf{W}}_k^\dagger\hat{\mathbf{H}}_k^{e\dagger}\right\} \\ & + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}\mathbb{E}\left\{\hat{\mathbf{H}}_k^e\hat{\mathbf{W}}_{\bar{k}}\hat{\mathbf{W}}_{\bar{k}}^\dagger\hat{\mathbf{H}}_k^{e\dagger}\right\}\Bigg) \\ \overset{(c)}{\leq} {} & N_r\left(1-\frac{B\tau}{T}\right)\log_2\Bigg(N_0 + N_r\hat{\mu}_k^2\Phi_{\tilde{b}k}^2\xi_{\tilde{b}k,e}^2 \\ & + \sum_{\substack{\bar{k}=1\\ \bar{k}\neq k}}^{K}N_r\hat{\mu}_{\bar{k}}^2\Phi_{\tilde{b}k}^2\xi_{\tilde{b}k,e}^2\Bigg) \end{aligned} \tag{29}$$

where step (a) follows by dropping the non-negative term $\log_2 det\left(\mathbf{R}_{\mathbf{n}_k}\right)$, and in step (b) we use Jensen's inequality. It is possible to express $\hat{\mathbf{H}}_k^e$ as $\hat{\mathbf{H}}_k^e = \hat{\mathbf{F}}_k^e\,\mathrm{blockdiag}\left\{\Phi_{1k}\xi_{1k,e}\mathbf{I}_{N_t},\cdots,\Phi_{Bk}\xi_{Bk,e}\mathbf{I}_{N_t}\right\}$ where $\hat{\mathbf{F}}_k^e$ is the normalized channel matrix with unit-variance entries [10]. Now, in order to proceed further with the analysis, we approximate $\hat{\mathbf{H}}_k^e$ as $\hat{\mathbf{H}}_k^e \approx \Phi_{\tilde{b}k}\xi_{\tilde{b}k,e}\hat{\mathbf{F}}_k^e$, where $\tilde{b} = \arg\max_{1\leq b\leq B}\Phi_{bk}^2$, by the fact that MS_k is within range of the coordinated BSs ($\Phi_{1k}^2\xi_{1k,e}^2 \approx \ldots \approx \Phi_{Bk}^2\xi_{Bk,e}^2$ (see Sec. III)). Therefore, $\hat{\mathbf{H}}_k^e\hat{\tilde{\mathbf{V}}}_k\hat{\mathbf{V}}'_k \approx \Phi_{\tilde{b}k}\xi_{\tilde{b}k,e}\hat{\mathbf{F}}_{k,\mathrm{eff}}^e$ where $\hat{\mathbf{F}}_{k,\mathrm{eff}}^e \triangleq \hat{\mathbf{F}}_k^e\hat{\tilde{\mathbf{V}}}_k\hat{\mathbf{V}}'_k$ is the $N_r \times N_r$ Gaussian matrix with i.i.d. $\mathcal{CN}(0,1)$ entries as $\hat{\tilde{\mathbf{V}}}_k\hat{\mathbf{V}}'_k$ is unitary. Under this simplifying approximation, step (c) follows from $\mathbb{E}\big\{\big(\hat{\mathbf{H}}_k^e\hat{\tilde{\mathbf{V}}}_k\hat{\mathbf{V}}'_k\big)\big(\hat{\mathbf{H}}_k^e\hat{\tilde{\mathbf{V}}}_k\hat{\mathbf{V}}'_k\big)^\dagger\big\} = N_r\Phi_{\tilde{b}k}^2\xi_{\tilde{b}k,e}^2\mathbf{I}_{N_r}$ as $\hat{\mathbf{H}}_k^e\hat{\tilde{\mathbf{V}}}_k\hat{\mathbf{V}}'_k$ is a complex Gaussian matrix with $\mathcal{CN}\big(0,\Phi_{\tilde{b}k}^2\xi_{\tilde{b}k,e}^2\big)$ entries. It is noteworthy to mention that *Theorem 2* is also applicable to all MSs in the cell. ■

The following insights can be drawn from *Theorems 1* and *2*. The rate loss in *Theorem 1* increases with intercell co-channel interference. For cell-edge MSs and at high SNR, intercell co-channel interference is the dominant impairment and obviously the rate loss increases and does not saturate. From *Theorem 2*, it is obvious that imperfect CSI incurs a rate loss due to residual inter-user interference. However, as the SNR increases these residual terms' contribution to rate loss diminishes by the fact that at high SNR $\xi_{bk,e}^2 \rightarrow 0$ (see Sec. III). The multiplexing gain of the JT-decomp system is thus preserved, and its percentage rate loss decreases as SNR increases

V. Numerical Results

In the following, a system with B BSs and K MSs, each equipped with N_t and N_r antennas, respectively, is referred to as an $[(B,N_t),(K,N_r)]$ system. Henceforth, we consider: (i) a $[(3,2),(3,2)]$ downlink urban micro-cellular network, (ii) the inter-BS distance is 500m, (iii) MSs are randomly located with a uniform distribution in a limited cell area so that any MS is at least 202 m from its nearest BS as depicted in Fig. 1, (iv) the path loss exponent is 3.7, (v) Rayleigh fading model, (vi) all BSs have equal average transmit power constraint P, (vii) the average transmit SNR is defined as $\mathrm{SNR} \triangleq \frac{P}{N_0}$, and (viii) the coherence interval $T = 100$, and the training interval $\tau = 4$.

The effects of intercell co-channel interference on the rate at MS_1 of the conventional cellular system is depicted in Fig. 2. The simulated rate loss and the upper bound in *Theorem 1* are also shown. There are several key observations: (i) the simulated rate curve $R_1^{\mathtt{L.B.}}$ saturates at high SNR, (ii) the simulated and the calculated rate loss curves from *Theorem 1* do not saturate at high SNR. This shows that the percentage rate loss of the conventional system increases as SNR increases mainly due to the power of intercell co-channel interference. For instance, the *relative percentage* rate losses at SNRs of 8 dB and 18 dB are about 38% and 68%, respectively. All the above verify the observations given in the analysis of Sec. IV.

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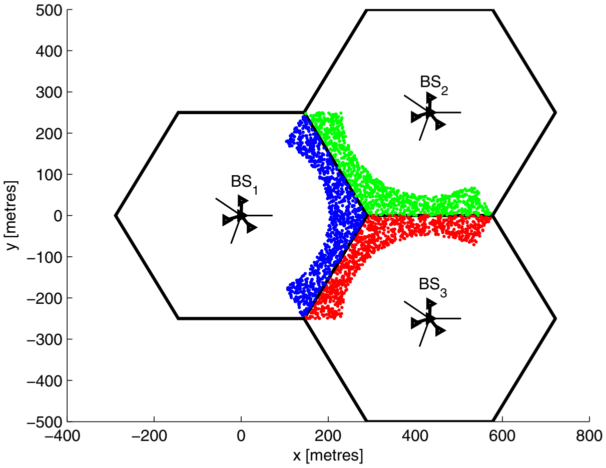

Fig. 1. Snapshot of users locations in the network over some 1000 trials

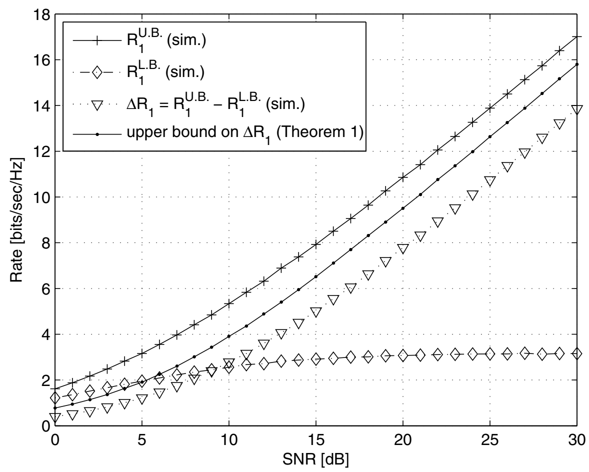

Fig. 2. Rate loss ΔR_1 at MS_1 of a $[(3,2),(3,2)]$ conventional cellular system. Coherence block $T = 100$ and training length $\tau = 4$.

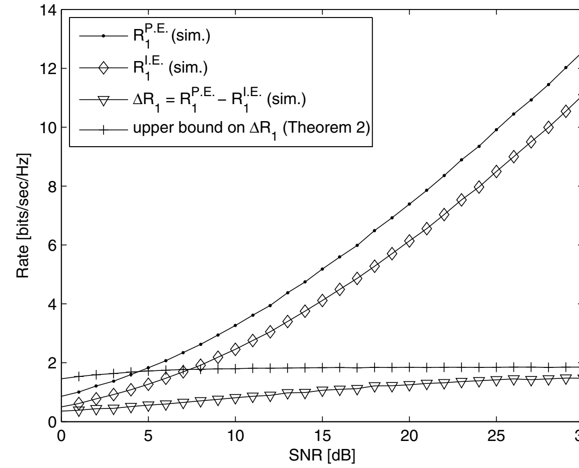

Fig. 3. Rate loss ΔR_1 at MS_1 of a $[(3,2),(3,2)]$ JT-decomp cooperative BS system. Coherence block $T = 100$ and training length $\tau = 4$.

The influence of channel estimation errors on the rate at MS_1 of the JT-decomp system is detailed in Fig. 3. The calculated upper bound on rate loss in *Theorem 2* is also shown. Inspection of Fig. 3 reveals: (i) unlike the conventional system, as SNR increases the perfect and imperfect estimation rate curves of JT-decomp system, as predicted by *Theorem 2*, do not saturate; (ii) the simulated and the calculated rate loss curves from *Theorem 2* saturate at high SNR. The relative percentage rate losses at SNRs of 5 dB, 18 dB, and 30 dB are about 30%, 19%, and 12% respectively. In agreement with *Theorem 2*, the percentage rate loss decreases as SNR increases. In summary, JT-decomp delivers higher throughput gain in all cases.

VI. Conclusion

We have investigated the impact of erroneous CSI on the performance of the JT-decomp cooperative BS cellular system. The most important results and insights are the following: (i) we present a channel estimation error model for BS cooperation signalling scheme; (ii) we have shown that, unlike conventional systems, the percentage rate loss of the JT-decomp system decreases as SNR increases, which demonstrates that the latter preserves its spatial multiplexing gain; and (iii) we provide upper bounds closed-form analytical expressions for the rate loss per user.

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