ARMA Synthesis of Fading Channels-an Application to the Generation of Dynamic MIMO Channels

Hani Mehrpouyan and Steven D. Blostein
Department of Electrical and Computer Engineering
Queen's University, Kingston, Ontario, K7L3N6, Canada
Emails: 5hm@qlink.queensu.ca, steven.blostein@queensu.ca

***Abstract*— Adaptive transceivers play an important role in wireless communications and the design of MIMO systems. Therefore models that enable simulation of dynamic and time varying channels in a computationally scalable fashion are extremely valuable. Previously, the application of autoregressive moving average (ARMA) modeling to fading processes has been complicated by ill-conditioning and nonlinear parameter estimation. This paper presents a numerically stable and accurate method to synthesize ARMA rational approximations of correlated Rayleigh fading processes from more complex higher order representations. The resulting ARMA synthesis is then used in the generation of dynamic MIMO channel realizations with time varying Doppler at significantly lower computational complexities.**

I. Introduction

The properties of a mobile fading channel significantly influence the design of wireless devices. This has motivated extensive research into the statistical modeling of Rayleigh fading channels, which is also a core component of more complex scattering models. Clark's fading model [1], or a simplified version proposed by Jakes' [2], have been widely used for simulation. There also exist a variety of implementations of these models, ranging from the sum of sinusoids (SOS) [3]- [4], IDFT [5], AR [6], and ARMA schemes [7]. The limitations of SOS are outlined in [4] and were addressed, to some extent, in [3]. However the SOS model fundamentally requires the summation of numerous sinusoids to generate Rayleigh variates with the correct statistics. The IDFT method, can offer improved accuracy at a cost of requiring an inverse fast Fourier transform (IFFT) on a large block of samples ($N = 2^{15}$ or larger). Performing the IFFT operation or adding numerous sinusoids on such a large number of samples, however, results in large delay and overhead and may be computationally impractical for generating shorter sequences with different Doppler parameters, i.e. a temporally correlated MIMO channel with varying Doppler. In contrast, ARMA modeling potentially has similar accuracy to a large size IDFT, with significantly fewer computations. The higher order AR systems required for accurate modeling of the Jakes' frequency spectrum can, in principle, be approximated with considerably lower order ARMA filters. Previous ARMA-based methods proposed in [7], determine poles and zeros separately, and as a result, are still of very high order, typically ranging from 200-1000.

In the following, a low-order ARMA synthesis technique is developed that can generate high quality Rayleigh variates. In this paper the resulting ARMA system is used in the generation of temporally correlated and time varying fading channels. Such channels have numerous applications specifically in the design and performance analysis of adaptive channel estimation algorithms [8]. In [10] we propose the new ARMA synthesis technique and study the effect of finite precision effect on AR, SOS, IDFT, and ARMA techniques. This paper provides a brief overview of the work presented in [10] and applies the algorithm in the generation of temporally correlated and dynamic MIMO channels.

II. ARMA Model Generation

An ARMA(p, q) model of p poles and q zeros has the potential to generate digital filters with closely matching second-order statistics. The generation of such ARMA models allows for the overall order of the filter to be reduced since an AR(P) model of order P would require $P >> p + q$.

The relationship between the autocorrelation function $r_{xx}[m]$ and ARMA(p, q) parameters is given by [11]:

$$r_{xx}[m] \begin{cases} r^*_{xx}[-m] & m < 0 \\ -\sum_{k=1}^{p} a[k] r_{xx}[m-k] + \\ \sigma_w^2 \sum_{k=m}^{q} b[k] h[k-m] & 0 \le m \le q \\ -\sum_{k=1}^{p} a[k] r_{xx}[m-k] & m > q \end{cases} \quad (1)$$

where $r_{xx}[m], -\infty < m < \infty$ is the desired autocorrelation sequence of the fading process, $b[k], 0 \le k \le q$ and $a[k], 0 \le k \le p$ represent the coefficients of the numerator and denominator polynomials of the ARMA transfer function, respectively, $h[m], 0 \le m < \infty$ is the corresponding time-domain impulse response sequence, and σ_w^2 is the variance of the input driving sequence. Attempting to determine the ARMA parameters by solving Eq. (1) results in a non-linear set of equations, because the impulse response is also a function of the unknown ARMA parameters.

Suboptimal methods that simultaneously estimate all the $a[k]$ and $b[k]$ parameters are presented in [12]. However due to the fact that the autocorrelation sequence under consideration is a narrowband process and is not rational [2], none of the above schemes reliably result in a stable ARMA filter.

The proposed solution first employs a high-order AR approximation to synthesize a rational model.

For an order-P AR process, Eq. (1) simplifies to

$$r_{xx}[m] \begin{cases} r_{xx}^*[-m] & m < 0 \\ -\sum_{k=1}^{P} ar[k] r_{xx}[m-k] + \sigma_w^2 & 0 \leq m \leq q \\ -\sum_{k=1}^{P} ar[k] r_{xx}[m-k] & m > q. \end{cases} \tag{2}$$

This gives rise to the following Yule-Walker equations, which when solved yield $ar[k], 1 \leq k \leq P$, the parameters of the AR(P) filter:

$$\begin{bmatrix} r_{xx}[0] & r_{xx}[-1] & r_{xx}[-2] & \dots & r_{xx}[-P+1] \\ r_{xx}[1] & r_{xx}[0] & r_{xx}[-1] & \dots & r_{xx}[-P+2] \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ r_{xx}[P] & r_{xx}[P-1] & r_{xx}[P-2] & \dots & r_{xx}[0] \end{bmatrix}$$

$$\begin{bmatrix} ar[1] \\ ar[2] \\ . \\ . \\ . \\ ar[P] \end{bmatrix} = - \begin{bmatrix} r_{xx}[0] \\ r_{xx}[1] \\ . \\ . \\ . \\ r_{xx}[P] \end{bmatrix} \tag{3}$$

This system of equations (3) can be efficiently solved using the Levinson-Durbin algorithm. Details on application to Rayleigh fading channels can be found in [6].

The resulting ARMA system is then determined by formulating a system identification problem [9]. The input to the ARMA system consists of the sequence $x(n)$ that is generated by the AR(P) system driven by white noise $w[k] \sim WGN(0, \sigma_w^2)$. The input-output equation is given by

$$a_0 x(n) = -\sum_{k=1}^{p} a_k x(n-k) + \sum_{k=0}^{q} b_k w(n-k). \tag{4}$$

Setting $a_0 = b_0 = 1$, without loss of generality, Equation (4) can be expressed as

$$x[n] = \mathbf{z}^T(n-1)\mathbf{c}_{ARMA} + w(n) \tag{5}$$

where

$$\mathbf{z}[n] = [-x(n) \dots - x(n-p+1) \;\; w(n) \dots w(n-q+1)]^T \tag{6}$$

and the vector of filter coefficients,

$$\mathbf{c}_{ARMA} = [a[1] \dots a[p] \;\; b[1] \dots b[q]]^T . \tag{7}$$

Assuming that the excitation $w(n)$ is known we may predict $x(n)$ from past values, using the following linear predictor:

$$\hat{x}(n) = \mathbf{z}^T(n-1)\hat{\mathbf{c}}_{ARMA} \tag{8}$$

$$\hat{\mathbf{c}}_{ARMA} = \left[\hat{a}[1] \dots \hat{a}[p] \;\; \hat{b}[1] \dots \hat{b}[q]\right]. \tag{9}$$

The prediction error

$$e(n) = x(n) - \hat{x}(n) = x(n) - \mathbf{z}^T(n-1)\hat{\mathbf{c}}_{ARMA} \tag{10}$$

equals $w(n)$ if $\mathbf{c}_{ARMA} = \hat{\mathbf{c}}_{ARMA}$. Minimization of the total squared error

$$\xi(c) = \sum_{n=N_i}^{N_f} e^2(n) \tag{11}$$

leads to the system of linear equations

$$\hat{\mathbf{R}}_z \hat{\mathbf{c}}_{ARMA} = \hat{\mathbf{r}}_z \tag{12}$$

where the correlation of the output AR process

$$\hat{\mathbf{R}}_z = \sum_{n=N_i}^{N_f} \mathbf{z}(n-1)\mathbf{z}^T(n-1) \tag{13}$$

and the cross correlation

$$\hat{\mathbf{r}}_z = \sum_{n=N_i}^{N_f} \mathbf{z}(n-1)x(n). \tag{14}$$

Therefore a total of $p+q$ equations need to be solved to determine the parameters of the ARMA model. To ensure that the resulting ARMA filter is minimum phase the poles and zeros outside the unit-circle are reflected. Table I demonstrates one such ARMA filter for $p = 12$ and $q = 12$. Figure 3 also compares the normalized level-crossing rate, defined as the rate at which the envelope crosses a specified level in the negative direction [2], is computed for each variate and compared with the theoretical value given in [2]. The ARMA (12, 12) filter provides better matching to the theoretical rates at the lower envelope levels compared to the IDFT ($N = 2^{15}$) scheme.

1930-529X/07/$25.00 © 2007 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.

III. **Simulation Results**

Using the method outlined above, an AR(50) ($P = 50$) and AR (100) ($P = 100$) model were approximated by ARMA(12,12) and ARMA(17,20) models respectively. The normalized maximum Doppler frequency, $f_m = .05Hz$ and $N = 2^{20}$ inputs and outputs were used to determine the parameters of the ARMA model. Figures 1 and 2 represent the autocorrelation sequence of the Rayleigh variates generated using the example ARMA(17,20) and ARMA(12,12) models respectively. Comparing the second-order statistics of the variates generated using the ARMA(12,12) and ARMA(17,20) filter to that of the AR(50), it is clear that the ARMA filters closely match Jakes' autocorrelation sequence. For $f_m = .05\ Hz$ and $N = 2^{20}$, we obtained accurate results with $p = 17$ and $q = 20$.

Next, using the quality measures described in [5], the quality of the variates $y(n)$ generated using the ARMA(12,12) or ARMA(17,20)filter are compared to that of the SOS [3], IDFT [5], and AR [6] models. The two quality measures are defined as follows. The first, termed *the mean power margin*, is defined by [5]

$$g_{mean} = \frac{1}{\sigma_y^2 L} trace\{C_y C_{\hat{y}}^{-1} C_y\} \quad (15)$$

and the second, the *maximum power margin*, is defined by [5]

$$g_{max} = \frac{1}{\sigma_y^2} \max\{diag\{C_y C_{\hat{y}}^{-1} C_y\}\} \quad (16)$$

where σ_{y^2} is the variance of the reference distribution. In (15) and (16), the $L \times L$ matrix $C_{\hat{y}}$ is defined to be the covariance matrix of any length-L subset of adjacent variates produced by the random variate generator. Due to the stationarity of the generator output, the covariance matrix of all such subsets will be identical. The $L \times L$ covariance matrix of a reference vector of L ideally distributed variates is similarly defined to be C_y. The matrix C_y represents the desired covariance matrix, and is known exactly (in this case the zeroth order bessel function). $L = 200$ in this paper to keep the results consistent with the simulation results presented in [3], [5], and [6].

Table II compares the quality of the generated variates for the ARMA, IDFT and SOS methods. Perfect variate generation corresponds to 0 dB for both measures. An autocorrelation sequence length of 200 was considered for evaluation of [5], Eq. (22) and [5], Eq. (23). The results presented in Table II demonstrate that the variate generating capability of the ARMA (17, 20) is comparable or better than the that of AR(50) filter and the IDFT method ($N = 2^{15}$), in terms of quality.

IV. **Correlated MIMO Channel Generation**

Simulation models of the fading channel play an important role in the progression of research in the wireless communications and MIMO system design. Temporally correlated MIMO channels with varying Dopplers are of particular interest in the design and development of adaptive channel estimators. Since the speed of a mobile station is varying during the transmission, the Doppler spread (or the maximum Doppler frequency) can also be varying [8]. Therefore it is important to be able to generate such MIMO channels accurately and efficiently to facilitate analysis and simulation of MIMO systems. The approach in [13] is used for the generation of temporally correlated MIMO fading channels. Consequently the IDFT, SOS, and ARMA schemes are compared based on the overall computational complexity and delay associated with each method. It is also important to note that the technique presented in this paper can be easily expanded to include spatially correlated channels as well, by simply applying the approach presented in [14] to the temporarily correlated channel realizations (see figure 4.

$$\begin{bmatrix} h_{1,1}[t] & h_{1,2}[t] & h_{1,3}[t] & ... & h_{1,n_t}[t] \\ h_{2,1}[t] & h_{2,2}[t] & h_{2,3}[t] & ... & h_{2,n_t}[t] \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ h_{n_r,1}[t] & h_{n_r,2}[t] & h_{n_r,3}[t] & ... & h_{n_r,n_t}[t] \end{bmatrix} \quad (17)$$

Equation (17) represents an n_r (number of received antennas) by n_t (number of transmit antennas) channel matrix, where, $h_{x,y}$ represents the channel gain between receive antenna x and transmit antenna y. According to the model described in [13] the elements $h_{x,y}[t]$ to $h_{x,y}[t+N]$ are correlated based on Jakes' fading model. Therefore, a total number of $n_r.n_t$ sequences of variates of length N need to be generated to populate the required channel matrices. The delay and number of Multiplies Per Unit time (MPU) associated with the generation of temporally correlated and time varying MIMO channels using the IDFT and ARMA approach is outlined in Table III. It is important to note that the generation of quality random variates using the IDFT method requires $N > 2^{15}$. Thus, for small values of N the IDFT approach is not scalable unlike the ARMA model. The problem is exacerbated under the time varying scenario due to the continuously varying maximum Doppler frequency, which requires requires the FFT operation to be performed on $n_r.n_t$ sequences of length N. Table IV represents a common simulation scenario and compares the delay associated with the ARMA and IDFT schemes. The SOS scheme was not considered since the complexity associated with performing sinusoidal functions is dependent on the algorithm used. However, it is important to note that a minimum of 24 sinusoids is required to ensure that high quality variates are generated.

V. Conclusion

By separating the issues of ill-conditioning and ARMA/AR equivalences, ARMA filters for the generation

1930-529X/07/$25.00 © 2007 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.

of Rayleigh random variates were considered. The inherent nonlinearity in determining the ARMA filter coefficients was addressed by first using an AR approximation filter. The AR filter was then modeled by a significantly lower order ARMA filter through a linear system identification process, developing a reduced order ARMA model for generating Rayleigh variates. It was also demonstrated that using the IDFT or SOS schemes for generating temporally correlated MIMO channels results in significantly more delay compared to the ARMA synthesis scheme presented here.

ACKNOWLEDGMENT

This research was supported by the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

[1] R. H. Clarke, "A statistical theory of mobile-radio reception," *Bell Syst. Tech. J.*, pp. 957-1000, Jul.-Aug. 1968..
[2] W.C. Jakes, "Microwave Mobile Communications," *New York: John Wiley and Sons*, 1974.
[3] C. Xiao; Y. R. Yahong, N. C. Beaulieu, "Novel Sum-of-Sinusoids Simulation Models for Rayleigh and Rician Fading Channels," *IEEE Trans. on W. Communcations*, pp.3667-3679, vol. 5, no. 12, Dec. 2006.
[4] M. F. Pop & N. C. Beaulieu, "Limitations of sum-of-sinusoids fading channel simulators," *IEEE Trans. on Communcations*, IEEE Trans. Commun., vol. 49, pp. 699-708, Apr.2001.
[5] D. J. Young & N. C. Beaulieu, "The Generation of Correlated Rayleigh Random Variates by Inverse Discrete Fourier Transform," *IEEE Trans. on Communications*, pp.1114-1127, vol. 48, no. 7, July 2000.
[6] K. E. Baddour & N. C. Beaulieu, "Autoregressive Modeling for Fading Channel Simulation," *IEEE Trans. on Wireless Communications*, pp.1650-1662, vol. 4, no. 4, July 2005.
[7] W. Turin, R. Jana, C. Martin, & J. Winters, "Modeling wireless channel fading," *IEEE Vehicular Technology Conference*, pp.1740-1744, vol. 3, no. 4, Oct 2001.
[8] Jungwoo Lee, "Multiple hypothesis channel estimation for the MIMO-OFDM system," *IEEE VTS 60th on Vehicular Tech. Con.*, pp.1599-1603, vol. 3, Sept. 2004.
[9] D. G. Manolakis, V. K. Ingle, & S. M. Kogon, "Statitics and Adaptive Signal Processing," *McGraw-Hill Series*, 2000.
[10] H. Mehrpouyan & S.D. Blostein, "ARMA Synthesis of Fading Channels," *Submitted to IEEE Trans. on Wireless Communications*, 2006. (revised June 2007)
[11] A. El-Jaroudi, J. Makhoul, "Digital Spectral Analysis With Applications," *IEEE Acoustics, Speech, and Signal Proc.*, pp.2162-2165, vol. 4, May 1989.
[12] Kay, S. M., "Modern Spectral Estimation," *Prentice-Hall*, 1987.
[13] M. Patzold , U. Killat, & F. Laue, "A deterministic digital simulation model for Suzuki processes with application to a shadowed Rayleigh land mobile radio channel," *IEEE Trans. on Inf. Theory*, pp.292-297, vol.39, no.1, Jan. 1993.
[14] L. Schumacher, K. Pedersen, & P. Mogensen, From antenna spacings to theoretical capacities - guidelines for simulating MIMO systems, *13th IEEE International Symposium on Personal, Indoor and Mobile Radio Comm.*, pp. 587-592, vol. 2, Sept. 2002.

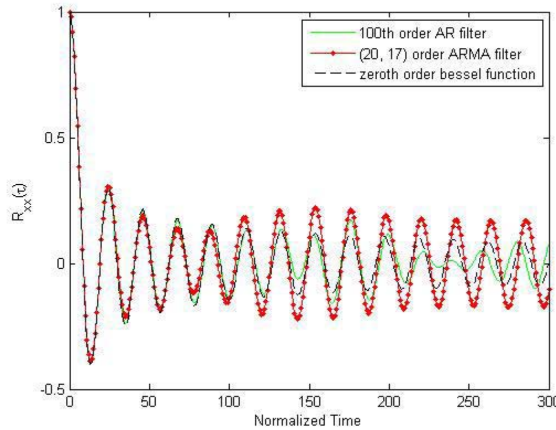

Fig. 1. Autocorrelation for ARMA(17,20) and AR(100) filters

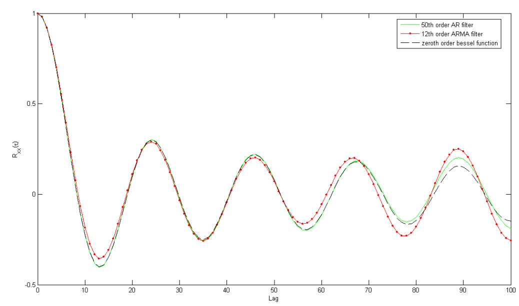

Fig. 2. Autocorrelation for ARMA(12,12) and AR(50) filters

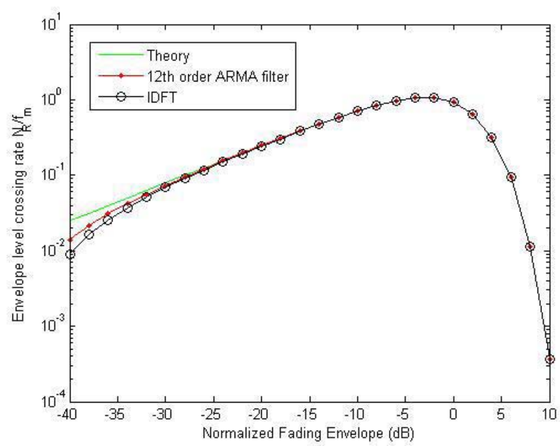

Fig. 3. Empirical level-crossing rates for the ARMA(12,12) and IDFT 2^{15}

1930-529X/07/$25.00 © 2007 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.

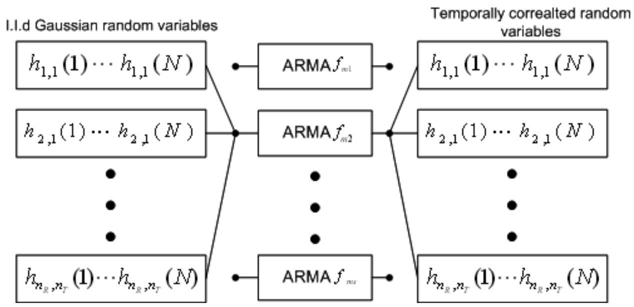

Fig. 4. Generation of temporally correlated MIMO channels with predetermined ARMA filters based on different f_m

TABLE I

SAMPLE 12TH ORDER ARMA FILTER COEFFICIENTS

MA
1
-1.68275350730153 + 9.84295554125048e-005i
0.644493633643956 - 0.00552636919352567i
0.451738794999222 + 0.00824658942989798i
0.123342380119134 - 0.00374437678854544i
-0.128604399156942 - 0.00100379923323234i
-0.203762810993031 + 0.0026365168246727i
-0.0111181314546748 - 0.00439704009999398i
0.300142078737244 + 0.00522041809851936i
0.230639493213332 - 0.00279908934476658i
-0.386701675011671 + 0.00098743714053473i
0.248224215812672 + 0.000270400025694159i
-0.0100465776239465 - 0.000552606858192417i
AR
1
-3.95261454893067 + 0.001379855990846i
4.91746254320627 - 0.00858034579707567i
-0.790857010320365 + 0.0205212671735957i
-1.67331524265772 - 0.0222065358526091i
-0.541840469843435 + 0.00705789606509577i
0.560034497561668 + 0.00822280902983019i
0.93259503416618 - 0.0146248513219662i
0.393724529149383 + 0.0180394625325917i
-0.7266067904956 - 0.0154556888501931i
-1.07673702321772 + 0.00609149805686471i
1.33830293471128 + 8.48013829379823e-007i
-0.380077391146616 - 0.000445954880891053i

TABLE II

A COMPARISON OF THE ARMA, AR, IDFT, AND SOS METHODS OF GENERATING BANDLIMITED RAYLEIGH VARIATES FOR COVARIANCE SEQUENCE LENGTH 200

	g_{mean}	g_{max}
ARMA Filtering(12,12)	0.56 dB	0.68 dB
ARMA Filtering(17,20)	0.31 dB	0.34 dB
AR Filtering(20)	2.6 dB	2.9 dB
AR Filtering(50)	0.26 dB	0.4 dB
IDFT Method ($N = 2^{15}$)	0.34 dB	0.53 dB
IDFT Method ($N = 2^{20}$)	0.0012 dB	0.0013 dB
SOS (24 Sinusoids)	0.012 dB	0.015 dB

TABLE III

COMPARISON OF THE ARMA & IDFT METHODS OF GENERATING TEMPORALLY CORRELATED AND TIME-VARYING MIMO CHANNELS BASED ON DELAY AND MPUS. φ REPRESENTS THE NUMBER OF DOPPLER REALIZATIONS

Temporally Correlated		
	Delay	MPUs
IDFT	$n_r.n_t.N$ $(N > 2^{15})$	N
ARMA Filtering(12,12)	$n_r.n_t.N$	12
Temporally Correlated with Time Varying Doppler		
IDFT	$n_r.n_t.N.\varphi$ $(N > 2^{15})$	N
ARMA Filtering(12,12)	$n_r.n_t.N.\varphi$	12

TABLE IV

A NUMERICAL EXAMPLE DEMONSTRATING THE SIMPLICITY OF THE PROPOSED ARMA SCHEME COMPARED TO THE IDFT APPROACH $(N = 1000, N_{IDFT} = 2^{15}, n_r = 8, n_t = 4, \varphi = 30, CPU = 3GHz)$

.

IDFT (Temporally Correlated)	ARMA (Temporally Correlated)	IDFT (Temp. Corr. & Time Varying)	ARMA (Temp. Corr. & Time Varying)
5.24 (sec)	0.128 (sec)	157 (sec)	.38 (sec)