Soft-Decision Successive Interference Cancellation CDMA Receiver with Amplitude Averaging and Robust to Timing Errors

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Abstract— **Successive interference cancellation (SIC) is a low-complexity multiuser detection method for DS/CDMA systems. If the channel is AWGN or slowly varying, averaging the channel amplitude estimates over multiple bits can greatly improve performance [3]. However, error propagation associated with a hard-limiter decision rule prevents SIC from achieving the performance improvement from averaging. We propose a new soft-decision rule to be used with amplitude averaging, which combines linear and hard decision rules. Performance within 0.4 dB of the single-user bound is obtained. We also robustify the above soft-decision SIC to time delay errors.**

I. Introduction

The capacity of a code division multiple access (CDMA) system is multiple access interference (MAI) limited. CDMA multiuser detection is an effective method to suppress MAI and improve uplink system capacity. The optimal multiuser detector has exponential computational complexity [1], so low complexity suboptimal multiuser detectors have been proposed [2], including the decorrelating detector [4], MMSE detector, successive interference cancellation (SIC) [3] and parallel interference cancellation (PIC) receivers.

The SIC regenerates and cancels other users' signals before data detection of the desired user. If the amplitudes are not known at the receiver, it was shown in theory in [3] that amplitude estimation by averaging over M bits can result in a significant performance improvement over no averaging for the SIC receiver. However, if hard-limiter decision rule is used, the interference could actually be doubled if the hard decision is incorrect, due to error propagation [2].

Interference cancellation using the linear soft-decision rule has no error propagation with iterations converging to the decorrelating detector. On the other hand, hard-decision interference cancellation can completely cancel interference when the hard decisions made are correct. We propose to combine the advantages of these two decision rules. When the instantaneous estimated signal amplitude is small compared to the averaged amplitudes, linear cancellation is used. Otherwise hard-decision cancellation is employed. As a result, the SIC may take full advantage of the amplitude averaging and achieve a performance close to that of the single user bound.

Section II describes the system model. Section III proposes the decision rule and the use of time averaging in the SIC receiver. In Section V, the SIC with amplitude averaging is robustified to operate under time delay estimation errors . Section VI provides simulation results.

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II. System Model

We consider the basestation receiver for the asynchronous uplink CDMA channel with binary phase shift keying (BPSK) modulation.

User data are transmitted in blocks, with a block length M. It is assumed that the channel parameters remain constant in one block. The equivalent baseband received signal for one block is

$$r(t) = \sum_{i=1}^{M} \sum_{k=1}^{K} A_k e^{j\theta_k} b_k(i) \tilde{s}_k(t - iT - \tau_k) + n(t) \qquad (1)$$

where $A_k(i) \in \mathcal{R}$, $\theta_k(i)$, and $b_k(i) \in \{+1, -1\}$ are the kth user's received signal amplitude, phase shift and data bit for the ith time interval, $\tau_k \in [0, T)$ is the kth user's propagation delay, T is the bit duration and K is the total number of users.

In (1), the normalized signature waveform of user k is

$$\tilde{s}_k(t) = \sum_{j=0}^{N-1} c_k(j) h(t - jT_c) \qquad (2)$$

where $N = T/T_c$ is the spreading factor, $\{c_k(j)\}_{j=0}^{N-1}$ is the spreading code, T_c is the chip duration and $h(t)$ is a rectangular chip pulse with duration $[0, T_c)$.

The phase shifts and spreading codes of all users are known at the receiver. Initially, we assume that the time delays are exactly known at the receiver. In Section VI, we will consider the practical case where the time delay errors exist.

After chip-matched filtering and chip-rate sampling, the received signal corresponding to the mth observation interval is discretized and organized into a vector

$$\mathbf{r}(m) = [r(mN+1) \ \ldots \ r(mN+N)]^T \in \mathcal{C}^N. \qquad (3)$$

Concatenating the $(M+1)$ vectors into a long vector $\mathbf{r}$ of length $(M+1)N$, the received signal can be expressed as

$$\mathbf{r} = \sum_{i=1}^{M+1} \sum_{k=1}^{K} A_k b_k(i) \mathbf{d}_k(i) + \mathbf{n} \qquad (4)$$

where $\mathbf{n}$ is a zero-mean white Gaussian random vector, and $\mathbf{d}_k(i) \in \mathcal{R}^{(M+1)N}$ is the discretized signature waveform of user k for the ith bit.

Let user k's time delay be decomposed into an integer p_k and fractional δ_k part as $\tau_k = (p_k + \delta_k)T_c$, where $p_k \in \{0, 1, \ldots, N-1\}$ and $\delta_k \in [0, 1)$. $\mathbf{d}_k(i)$ can be expressed as the combination of two adjacent shifted version of user spreading codes [8]

$$\mathbf{d}_k(i) = \delta_k \mathbf{c}_k(p_k+1, i) + (1-\delta_k)\mathbf{c}_k(p_k, i). \qquad (5)$$

In (5), $\mathbf{c}_k(p_k, i)$ is defined as $\mathbf{c}_k$ right-shifted by $(i–1)N+p_k$ samples, where $\mathbf{c}_k \in \mathcal{R}^{(M+1)N}$ is a $(M+1)N$ sample vector consisting of the kth user's spreading codes padded by MN zeros:

$$\mathbf{c}_k = [c_k(0)\ c_k(1)\ \ldots\ c_k(N-1)\ \ 0\ 0\ \ldots\ 0]^T. \qquad (6)$$

The received signal vectors $\mathbf{r}$ over the $(M+1)$ observation intervals provides the sufficient statistics for detecting the transmitted data bits from all the K users.

III. SIC Multiuser Detector with Amplitude Averaging

Since the amplitude of each user's received signal is not known at the receiver, it must be estimated from the received signal. In [3], it was shown in theory that amplitude estimation by averaging over M bits can reduce the noise variance by an order of M. This results in a significant BER performance improvement over a linear SIC receiver which does not use amplitude averaging. The single-user BER lower bound may be approached if the number of bits used for averaging is large enough [3].

However, if the interference canceller uses a hard-decision rule, then it will exhibit error propagation. The extent to which amplitude averaging can be exploited depends on the decision rule used. In the following, we compare some known decision rules and propose an improved decision rule.

The SIC receiver with amplitude averaging has multiple interference cancellation stages, starting from stage $v = 1$. During the *(v+1)*st stage, the SIC does the following steps (1) to (3) on user $k = 1$ first, then repeats the same steps on user $k = 2, 3, \ldots, K$:

(1) The k-th user's received signal is estimated by subtracting other users' regenerated signals from the received signal $\mathbf{r}(i)$ of (4):

$$\begin{aligned} \tilde{\mathbf{r}}_k^{v+1} &= \mathbf{r} - \sum_{l=1}^{k-1}\sum_{i=1}^{M} e^{j\theta_l}\bar{A}_l^{v+1}\hat{b}_l^{v+1}(i)\mathbf{d}_l(i) \\ &- \sum_{l=k+1}^{K}\sum_{i=1}^{M} e^{j\theta_l}\bar{A}_l^{v}\hat{b}_l^{v}(i)\mathbf{d}_l(i) \end{aligned}$$

(2) Obtain the averaged amplitude estimate by averaging the instantaneous estimate of user k's amplitudes over a block of M bits after despreading with PN sequence $\mathbf{d}_k(i)$:

$$\bar{A}_k^{v+1} = \frac{1}{M}\sum_{i=1}^{M} abs\ (Re\ (e^{-j\theta_k}(\mathbf{d}_k(i))^H\ \tilde{\mathbf{r}}_k^{v+1}))$$

where $abs(\ )$ and $Re(\ )$ take the absolute value and the real value, respectively.

(3) For each bit in the block, $i = 1, \ldots, M$, obtain the soft data bit estimate and make a data bit decision. For the ith bit, the soft data bit estimate is normalized with respect to the averaged amplitude $\bar{A}_k^{v+1}$:

$$\tilde{b}_k^{v+1}(i) = \text{Re}\ (e^{-j\theta_k}(\mathbf{d}_k(i))^H\ \tilde{\mathbf{r}}_k^{v+1})/\bar{A}_k^{v+1},$$

and then the data bit decision is made by the decision rule $f_{dec}(\cdot)$:

$$\hat{b}_k^{v+1}(i) = f_{dec}(\tilde{b}_k^{v+1}(i)).$$

The above multistage SIC is terminated after a desired number of cancellation stages. Some possible decision rules $f_{dec}(\cdot)$ are shown in Fig. 1.

The hard-limiter decision rule [5] of Fig.1(a) utilizes only the sign of the soft data bit estimate, $\hat{b}_k^{j+1}(i) = sign(\tilde{b}_k^{j+1}(i))$. If the soft estimate has small magnitude, and a wrong hard data bit decision is made, the interference is increased. This may cause errors to propagate to the following stages.

The linear decision rule [3] [6] of Fig. 1(b) does not make hard bit decisions. In the linear SIC the ith amplitude and data bit are estimated as a composite signal $\hat{b}_k(i)\hat{A}_k(i)$ [3] [6]. This is equivalent to estimating the instantaneous amplitude in a bit-by-bit fashion. The linear SIC converges to the decorrelating detector as the number of interference cancellation stages tends to infinity [6], so its performance is limited by the same noise enhancement that is occurring in the decorrelating detector [4]. This noise enhancement is due to the noisy instantaneous amplitude estimates used in the interference cancellation.

The limiter in the unit-clipper decision rule [5] [7] of Fig. 1(c) improves performance over the linear SIC. However, the unit-clipper cancels only part of the noise. The noise below the amplitude limit is not cancelled.

Based on the advantages of the above decision rules, we propose the new soft bit decision rule shown in Fig. 1(f):

$$\hat{b} = f_{dec}(\tilde{b}) = \begin{cases} 1, & \tilde{b} > c \\ \tilde{b}, & \tilde{b} \in [-c, c] \\ -1, & \tilde{b} < -c \end{cases} \qquad (7)$$

where the threshold c should satisfy $0.0 \le c \le 1.0$. The effect of the choice of c on the performance of the SIC will be investigated in Section VI.

Our decision rule makes a linear soft bit decision when the value of the normalized soft bit estimate is small, and so will benefit from the convergence property of the linear SIC. Otherwise, it makes a hard bit decision, which cancels the interference completely with a high probability as will be shown in the following section.

IV. Steady-state Performance Analysis

We analyze the steady-state performance of the proposed SIC detector after convergence.

The residual interference is assumed to be Gaussian-distributed, and the interference introduced by individual users can be assumed to be mutually independent [12]. Let the interference variance from one bit of user k be σ_k^2. The total interference and noise variance is the sum of the K users' interference variances and the channel noise variance σ_N^2, i.e., $\sigma^2 = \sum_1^K \sigma_k^2 + \sigma_N^2$.

For the multistage linear SIC detector, σ^2 is the solution to [12]:

$$\sigma^2 = K\frac{\sigma^2}{N} + \sigma_N^2 \tag{8}$$

The result is $\sigma^2 = \frac{N}{N-K}\sigma_N^2$. For a spreading factor of $N = 31$, with $K = 20$ users, the performance loss relative to the single-user performance bound is 4.5 dB.

For the proposed decision rule Fig. 1(f), user k's decision region can be divided into three regions: hard-decision (cA_k, ∞), linear decorrelator $[-cA_k, cA_k]$ and bit-error $(-\infty, -cA_k)$. It can be shown that conditioned on its amplitude A_k, the interference variance contribution from user k is comprised of three terms, corresponding to the three regions, weighted by their probabilities:

$$\begin{aligned}\sigma_k^2(A_k) &= \left[1-Q\left(\frac{(1-c)A_k}{\sigma}\right)\right]\frac{\sigma^2}{MN} + \left[Q\left(\frac{(1-c)A_k}{\sigma}\right)\right.\\ &\quad \left. -Q\left(\frac{(1+c)A_k}{\sigma}\right)\right]\frac{\sigma^2}{N} + Q\left(\frac{(1+c)A_k}{\sigma}\right)\frac{(2A_k)^2}{N} \end{aligned}\tag{9}$$

where Q is the error function.

If the received user signals have unequal powers, we may assume that the received amplitudes A_k are uniformly distributed between A and AX, where $A = \min\{A_1, \ldots, A_K\}$ is the amplitude of the weakest user, and $X > 1$ is the ratio of $\max\{A_1, \ldots, A_K\}/A$. The average interference variance contribution from user k can be obtained by averaging (9) over the distribution of A_k, which is uniform in $[A, AX]$.

Denote

$$\begin{aligned} f(b) &= E_{A_k}\left[Q\left(\frac{bA_k}{\sigma}\right)\right] = \frac{1}{A(X-1)}\left[AXQ\left(\frac{bAX}{\sigma}\right)\right.\\ &\quad \left. -AQ\left(\frac{bA}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi}b}\left(e^{-\frac{b^2A^2}{2\sigma^2}} - e^{-\frac{b^2(AX)^2}{2\sigma^2}}\right)\right], \end{aligned}\tag{10}$$

and by using the approximation $Q(t) \approx \frac{1}{\sqrt{2\pi}t}e^{-t^2}$, denote

$$\begin{aligned} g(b) &= E_{A_k}\left[A_k^2 Q\left(\frac{bA_k}{\sigma}\right)\right] = \int_A^{AX} A_k^2 Q\left(\frac{bA_k}{\sigma}\right) dA_k \\ &\approx \frac{\sigma^3}{\sqrt{2\pi}b^3}\frac{1}{A(X-1)}\left(e^{-\frac{b^2A^2}{2\sigma^2}} - e^{-\frac{b^2(AX)^2}{2\sigma^2}}\right). \end{aligned}\tag{11}$$

Substituting (10) and (11) into (9), the total interference σ^2 for all K users including the channel noise variance σ_N^2 is the solution to

$$\begin{aligned} \sigma^2 &= \sum_{k=1}^K E_{A_k}[\sigma_k^2(A_k)] + \sigma_N^2 \approx \left\{[1-f(1-c)]\frac{\sigma^2}{MN}\right.\\ &\quad \left. +[f(1-c)-f(1+c)]\frac{\sigma^2}{N} + \frac{4}{N}g(1+c)\right\}K + \sigma_N^2. \end{aligned}\tag{12}$$

Alternatively, if the received user powers are all equal (perfect power control), i.e., $A_k = A$ for $k = 1, \ldots, K$, then (9) need not be averaged. Instead of (12), the total interference and noise variance is given by the solution to:

$$\sigma^2 = K\sigma_k^2(A) + \sigma_N^2. \tag{13}$$

V. Robustification to time delay errors

Until this point, the time delays of all users are assumed to be exactly known at the receiver. It has been shown, however, that multiuser detectors designed for perfect time delay conditions will have a large performance degradation with time delay errors [9]. We have proposed a robust multiuser detection method for the linear SIC with time delay errors in [10]. It can be also applied here to the soft-decision SIC.

Denote the estimated time delay of the kth user as $\hat{\tau}_k = (\hat{p}_k + \hat{\delta}_k)T_c$. It is assumed that all users are estimated to within $\pm 0.5T_c$ of the true time delays, i.e., $|\hat{\tau}_k - \tau_k| \leq 0.5T_c$.

Since the chip-rate sampling time instants are arbitrarily chosen, the relative position of the estimated and true time delays can be divided into two cases: delays occurring (i) in the same sampling interval and (ii) in two adjacent sampling intervals.

For the first case, the true and estimated delays have the same integer part, i.e, $p_k = \hat{p}_k$ for $1 \leq k \leq K$. The kth user's discretized signature waveform for the ith interval $\mathbf{d}_k(i)$ in (5) can be expressed as the weighted sum of two signals $\hat{\mathbf{d}}_k(i)$ and $\Delta\mathbf{d}_k(i)$:

$$\begin{aligned} \mathbf{d}_k(i) &= \left[\hat{\delta}_k\mathbf{c}_k(p_k+1, i) + (1-\hat{\delta}_k)\mathbf{c}_k(p_k, i)\right] \\ &\quad +(\delta_k - \hat{\delta}_k)\left[\mathbf{c}_k(p_k+1, i) - \mathbf{c}_k(p_k, i)\right] \\ &\overset{\text{def}}{=} \hat{\mathbf{d}}_k(i) + (\delta_k - \hat{\delta}_k)\Delta\mathbf{d}_k(i). \end{aligned}\tag{14}$$

We denote the second vector $\Delta\mathbf{d}_k(i)$ as the error vector.

For the second case, without lost of generality, we may let $\hat{p}_k = p_k - 1$. The kth user's discretized signature waveform for the ith interval $\mathbf{d}_k(i)$ in (5) can be expressed as the weighted sum of three signals $\hat{\mathbf{d}}_k(i)$, $\Delta\mathbf{d}_k(i)$ and $\mathbf{c}_k(p_k+1, i)$:

$$\begin{aligned} \mathbf{d}_k(i) &= (1-\delta_k)\left[(\hat{\delta}_k\mathbf{c}_k(p_k, i) + (1-\hat{\delta}_k)\mathbf{c}_k(p_k-1, i)\right] \\ &\quad -(1-\delta_k)\hat{\delta}_k[\mathbf{c}_k(p_k, i) - \mathbf{c}_k(p_k-1, i)] + \delta_k\mathbf{c}_k(p_k+1, i) \\ &\overset{\text{def}}{=} (1-\delta_k)\hat{\mathbf{d}}_k(i) - (1-\delta_k)\hat{\delta}_k\Delta\mathbf{d}_k(i) + \delta_k\mathbf{c}_k(p_k+1, i). \end{aligned}\tag{15}$$

We denote the third vector, $\mathbf{c}_k(p_k+1, i)$, as the guard vector.

Since the receiver cannot determine whether the estimated and true time delays are in the same sampling interval, the robust SIC detector uses (15) to cancel two residual MAI terms for each user, corresponding to the error vector and the guard vector. If the estimated and true time delays are in the same sampling interval, then the estimated signal corresponding to the guard vector will contribute noise terms only, i.e., the negative effect of using (15) instead of (14) is noise enhancement.

At each stage, the M error vectors of each user are combined into a long error vector based on the tentative data bit decisions, $\hat{b}_k(i)$, as

$$\mathbf{e}_k = \sum_{i=1}^{M} \Delta\mathbf{d}_k(i)\hat{b}_k(i). \tag{16}$$

Similarly the M guard vectors are combined into a long guard vector:

$$\mathbf{g}_k = \sum_{i=1}^{M} \mathbf{c}_k(p_k+1, i)\hat{b}_k(i). \tag{17}$$

Denote the long error and guard vectors of the lth user at the vth iteration by $\mathbf{e}_l^v$ and $\mathbf{g}_l^v$, and their amplitude estimates by $\hat{f}_l^v$ and $\hat{h}_l^v$, respectively.

At the $(v+1)$st stage, the estimated amplitude of the error vector is updated as:

$$\hat{f}_k^{v+1} = \frac{1}{M}(\mathbf{e}_k^{v+1})^H(\breve{\mathbf{r}}_k^{v+1}), \tag{18}$$

where

$$\breve{\mathbf{r}}_k^{v+1} = \mathbf{r} - \sum_{l=1}^{k-1}\left[\hat{f}_l^{v+1}\mathbf{e}_l^{v+1} + \hat{h}_l^{v+1}\mathbf{g}_l^{v+1} + \sum_{i=1}^{M} e^{j\theta_l}\bar{A}_l^{v+1}\hat{b}_l^{v+1}(i)\mathbf{d}_l(i)\right] - \sum_{l=k+1}^{K}\left[\hat{f}_l^{v}\mathbf{e}_l^{v} + \hat{h}_l^{v}\mathbf{g}_l^{v} + \sum_{i=1}^{M} e^{j\theta_l}\bar{A}_l^{v}\hat{b}_l^{v}(i)\mathbf{d}_l(i)\right]. \tag{19}$$

The estimated amplitude of the guard vector $\hat{h}_l^{v+1}$ is obtained similarly.

The SIC in Section III can be robustified by further subtracting the estimated signals due to timing errors, $\sum_{l=1}^{k-1}(\hat{f}_l^{v+1}\mathbf{e}_l^{v+1} + \hat{h}_l^{v+1}\mathbf{g}_l^{v+1}) + \sum_{l=k+1}^{K}(\hat{f}_l^{v}\mathbf{e}_l^{v} + \hat{h}_l^{v}\mathbf{g}_l^{v})$, from $\tilde{\mathbf{r}}_k^{v+1}$ in step (1).

VI. Numerical and Simulation Results

Throughout the simulations, Gold code sequences of length $N = 31$ and a block size of $M = 9$ are used. The channel is additive white Gaussian noise (AWGN). The number of users is $K = 20$ to account for a highly-loaded system. The signal-to-noise ratio (SNR) is defined with respect to the user of interest, user 1. The near-far ratio is defined as the power ratio between the strongest user and user 1, which is fixed at 10 dB. All other users have a amplitude uniformly distributed between those of the strongest user and the weakest user.

Fig. 2 compares the bit error probability (BER) performance of the proposed SIC detector with thresholds $c = 0.0$, 0.5 and 1.0, where $c = 0.0$ is the special case equivalent to

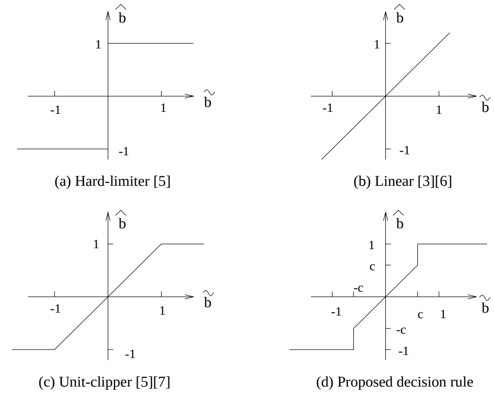

Fig. 1. The decision rules for SIC CDMA detectors.

the hard-limiter, and $c = 1.0$ is the special case equivalent to the unit-clipper. At a BER of 10^{-3}, the proposed SIC with $c = 0.5$ has a performance loss of about 0.4 dB compared to the single-user BER curve. The hard-limiter also exhibits an error floor due to error propagation.

In Fig. 3, the SNR loss to the single user bound as a function of the thresholds at SNR = 10 dB is calculated numerically using (12) and (13). At a near-far ratio of 10 dB, the SNR loss to single user performance is 0.35 dB and 1.93 dB for thresholds $c = 0.5$ and 1.0, respectively, which is very close to the simulation results of 0.4 dB and 2.1 dB losses in Fig. 2.

We investigate the case of time delay errors, which are modeled as zero-mean Gaussian random variables truncated to be within the interval $\pm 0.5T_c$. In Fig. 4, the standard deviation of the time delay error is $\sigma = 0.1T_c$, which is typical of the current time delay estimation methods for CDMA [8]. Our robustified soft-decision SIC performs within 1.2 dB of single user performance.

In Fig. 5, results for the extreme case of $\sigma = 0.5T_c$ are shown. Our robustfied SIC performs almost the same as the decorrelating detector with exact time delays under this severe condition.

VII. Conclusion

An improved SIC detector with a new soft-decision rule is proposed. This new soft-decision rule combines the advantages of the unit-clipper and the hard-limiter decision rules. By using time-averaged amplitude estimation, the noise in the amplitude estimate can be greatly reduced. A BER performance within 0.4 dB of the single-user bound is achieved. This soft-decision SIC with time averaging can also be made robust to time delay estimation errors. Currently we are analyzing our proposed decision rule performance in multipath fading channels.

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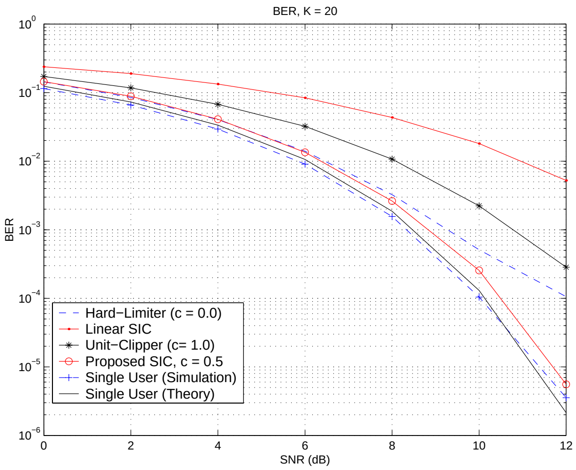

Fig. 2. Bit error rate (BER) of user 1 for proposed SIC detector (threshold $c = 0.5$) and other SIC detectors. $K = 20$ users. Near-far ratio = 10 dB.

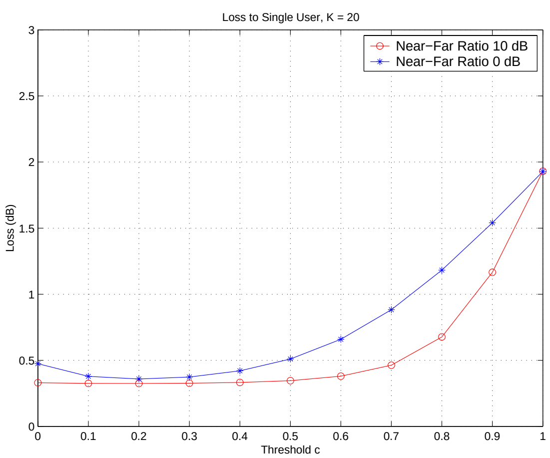

Fig. 3. The SNR loss for the proposed SIC detector relative to the single user detector as a function of the thresholds $0.0 \leq c \leq 1.0$. $K = 20$ users. SNR = 10 dB.

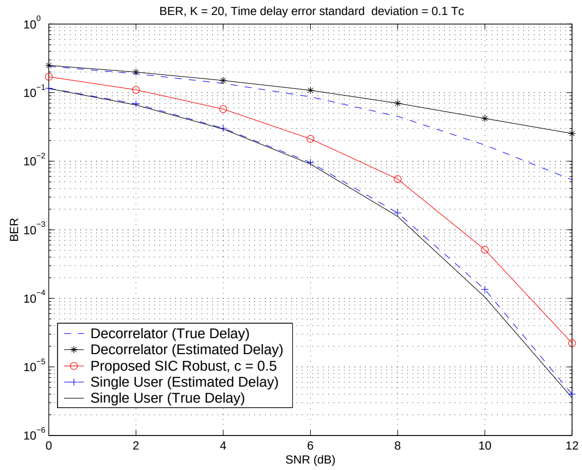

Fig. 4. Bit error rate (BER) of user 1 for robustified SIC detector under time delay errors. $K = 20$ users. Near-far ratio = 10 dB. The threshold is $c = 0.5$. The time delay has an error of $\sigma = 0.1T_c$.

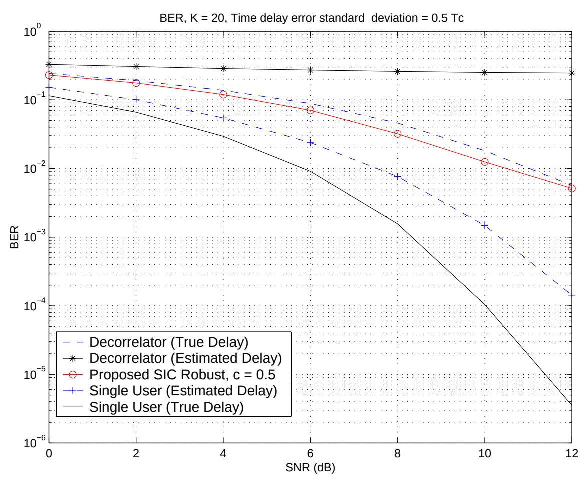

Fig. 5. Bit error rate (BER) of user 1 for robustified SIC detector under time delay errors. $K = 20$ users. Near-far ratio = 10 dB. The threshold is $c = 0.5$. The time delay has an error of $\sigma = 0.5T_c$.

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