Cross-layer Raptor coding for broadcasting over wireless channels with memory

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***Abstract*—Raptor codes are a class of rateless codes that have been shown to provide promising performance in erasure channels, and more recently, in noisy channels. This paper investigates the performance of application layer Raptor codes for broadcasting services over wireless channels with memory. A hybrid erasure-soft decoding algorithm is proposed as a cross-layer protocol for application layer raptor codes. These protocols relay corrupted packets into the application layer. The resulting hybrid error-erasure channels are modeled by a hierarchical Markov channel model. Capacity evaluation and simulation results show that the proposed cross-layer decoding algorithms outperform existing erasure decoding schemes significantly without any modification to the transmitter. The effects of channel memory and other parameters are also studied by simulation.**

I. Introduction

Fountain codes are a class of rateless codes whose rate is not predetermined before encoding, but is determined during decoding on-the-fly. The two most widely used fountain code implementations are the Luby Transform (LT) [1] and Raptor codes [2]. Raptor codes provide nearly optimal performance for erasure channels with linear decoding complexity [2]. In addition, Raptor codes can be "universal", meaning that the same code parameters can achieve near-optimal performance regardless of the channel erasure probability. Because of these advantages, Raptor codes are particularly well-suited to broadcast channels where both the rateless and universality properties are important. Raptor codes have been applied to Internet and wireless communications, such as multicasting, parallel downloading and peer-to-peer communications. For example, in third generation partnership program (3GPP) Multimedia Broadcast/Multimedia Services (MBMS), Raptor codes have been chosen as the forward error correction (FEC) code in the application layer for file downloading services.

To date, most of the applications of Raptor codes assume a perfect erasure channel [3]. For example, the authors in [3] investigate the application of Raptor codes to MBMS download delivery services. In their system, packets which contain errors that are not fully corrected by the physical layer turbo code are discarded. These conventional schemes can result in large numbers of dropped packets in poor channel conditions. There are also approaches that study the performance of Raptor codes over AWGN or fading channels using soft decoding [4] [5] [6]. However, soft decoding schemes are usually more complex than traditional erasure decoding schemes. In addition, Raptor codes are not originally designed as physical layer codes, and using Raptor codes directly in the physical layer would require system redesign which is expensive and not backward compatible.

Recently, two general cross-layer communication protocols, known as hybrid error-erasure protocols (HEEPs), have been applied to Reed Solomon (RS) codes and Low Density Parity Check (LDPC) codes in wireless multimedia/video transmissions [7]. These HEEPs allow corrupted packets to be relayed into the application layers. However, the protocols in [7] have not been applied to rateless Raptor codes and do not model practical physical layer channels nor the behavior of physical layer FECs. In addition, channel memory has not been considered in [7]. In this paper, we investigate performance of Raptor codes in MBMS file downloading services when different cross-layer protocols and conventional protocols are applied. The Raptor coded packets experience both packet erasures due to network congestion and packet corruptions due to fading and noise. Channel memory in both the wireline and the wireless channel have been considered.

The main contributions of the papers are as follows: first, by taking channel memory and the behavior of physical layer turbo codes into account, we model the channel experienced by the Raptor code as a hierarchical Markov model. We derive the transition probabilities based on the turbo code rate and parameters of a correlated Rayleigh fading channel. The main difference between this channel model and a regular markov-type model (such as the well known Gilbert-Elliott channel (GEC)) is the choice of channel states. Rather than a general choice of good and bad states, the three states used in this model (erasure, corrupt and correct) directly represent the results of physical layer decoding. With this model, the two cross-layer protocols we considered only differ in the availability of side information about the instantaneous channel state. Therefore, based on this model, we can easily evaluate and compare the performance of different cross-layer and conventional protocols in channels with memory. Second, we propose a hybrid erasure-soft Raptor decoding scheme to implement protocols across the physical and application layers. The decoding scheme improves system performance substantially compared to that using conventional protocols [3] with modification only required on the receiver side. The main new idea of the decoding scheme is to perform traditional erasure decoding based on the correct packet first and soft iterative decoding based on the corrupt packets afterwards. In broadcasting applications, each user/receiver also has the flexibility to choose whether to use traditional erasure decoding or the proposed hybrid decoding depending on channel conditions and individual quality of service (QoS) requirements. Therefore, the hybrid decoding provides a flexible tradeoff between performance and complexity. For example, when the channel quality is good and the physical layer code is able to correct all the errors in most packets, the receiver can recover all information using simpler traditional erasure decoding methods; when those non-corrupt packets are insufficient to decode all the source information, the receiver can collect soft information from corrupt packets to help in the decoding process. Third, we evaluate the system throughput using different turbo code rates and simulate performance of an actual Raptor code using different protocols in various channel conditions.

The rest of the paper is organized as follows: In Section II , we first describe the overall MBMS system model and the application of cross-layer protocols. We then describe the channel modeling process and derive the transition probabilities based on the physical layer parameters. In Section III , we evaluate the application layer capacity and maximum system throughput when different turbo code rates are used. In Section IV, after providing some background information on Raptor codes, we then describe the proposed hybrid erasure-soft decoder for different cross-layer protocols. Section V shows the simulation results of Raptor codes using conventional and cross-layer protocols in various channel conditions.

II. System and channel models

A. System model and cross-layer protocols

Two layers of FEC have been used in MBMS: turbo codes in the physical layer and Raptor codes in the application layer [3]. Since the protocol stack in MBMS systems is rather complex to present here, a much simpler two layer model is considered. In our model, the information data are first segmented into data-bearing packets. Multiple data packets are coded by a Raptor code where each packet is considered as a symbol (a vector of binary bits) of a Raptor code. Cyclic Redundancy Checks (CRC) and packet header information are then appended to each output packet to form the transmitted packets. Each packet is further protected by a physical layer code (turbo code), modulated by BPSK and transmitted over the physical channel.

The packets experience a hybrid type of channel where transmitted packets can be lost due to network congestion. Packets that are not lost are still subjected to channel fading and noise. When the packet is not lost, the receiver first demodulates and decodes data using the turbo decoder. The correctness of the turbo decoder output is checked by the CRC embedded in each packet. In the current MBMS standard, the conventional (CON) scheme is used where the entire packet is dropped if the CRC fails. Therefore only packets that do not contain any errors are forwarded to the Raptor decoder. Two general cross-layer protocols, known as cross-layer design (CLD) and cross-layer design with side information (CLDS),

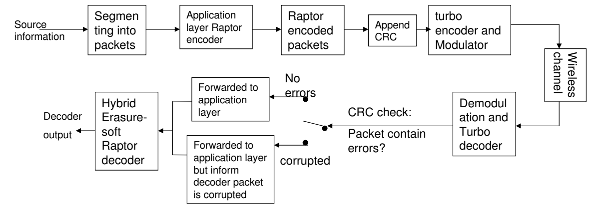

Fig. 1. system and coding structures

are summarized in [7]. To apply CLD protocols, the CRC information is simply ignored and all the turbo decoder outputs are forwarded to the Raptor decoder. To apply CLDS protocols, all the outputs of the turbo decoder are forwarded to the Raptor decoder along with the side information provided by the CRC check indicating whether the packet is corrupted. A block diagram of a system using CLDS protocols are illustrated in Fig. 1.

B. Channel modeling

Packets are lost in bursts when network congestion is severe. Therefore a more accurate model for packet loss should take channel memory into account. We model the behavior of packet losses as a GEC, which is a well known two state markov model for modeling channels with memory (Fig. 2). The transition between the two states form a binary Markov

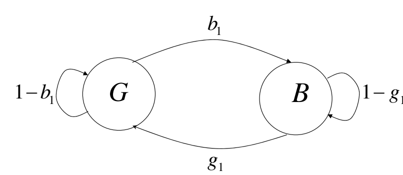

Fig. 2. Structure of the Gilbert Elliott channel

process. In the bad state (erasure state), the packet loss probability is 1 while in the good state (non-erasure state), the packet loss probability is 0. Let g_1 and b_1 represent transition probabilities from bad state to good state, and from good state to bad state, respectively. The average packet loss rate $\lambda = b_1/(g_1 + b_1)$. The channel memory is defined as $\mu_1 = 1 - g_1 - b_1$ [8]. The two parameters λ and μ_1 can determine g_1 and b_1 and the packet loss behavior.

The physical layer wireless channel is assumed to be correlated Rayleigh fading with Doppler frequency f_d and average received SNR $\bar{\gamma}$. The"water-fall" region of the turbo code is narrow [3] and has the following property: for a given rate R_{turbo}, there exists a SNR threshold γ_t such that when the channel SNR $\gamma > \gamma_t$, the turbo decoder almost always decodes the information correctly; and when $\gamma < \gamma_t$, the decoder almost always fails (due to errors in the decoder output). The cutoff rate of the turbo codes satisfies $R_{turbo}(\gamma_t) = 1 - log(1 + exp(-\gamma_t))$ [3]. Hence for a given turbo code rate R_{turbo},

$$\gamma_t = -ln(2^{1-R_{turbo}} - 1). \quad (1)$$

To model the correlated fading channels combined with the two cross-layer protocols, a good state is used to represent the case when the instantaneous channel SNR $\gamma > \gamma_t$, while a bad state represents the case when $\gamma < \gamma_t$. In the good state (correct state), the turbo code always decodes the information correctly and the CRC check is satisfied. In the bad state (corrupt state), there are errors present in the turbo decoder output and the CRC check fails.

To match the two-state GEC to the correlated Rayleigh fading, the steady-state probability $\pi_b = \int_0^{\gamma_t} f_\gamma(\gamma)d\gamma = 1 - exp(-\gamma_t/\bar{\gamma})$ where $f_\gamma(\gamma) = \frac{1}{\bar{\gamma}}exp(-\gamma/\bar{\gamma})$ is the PDF of the instantaneous SNR of Rayleigh fading channels. Next, by matching the average time of the fading amplitude below the threshold to the average time of the GEC staying in the bad state, it can be shown that [9],

$$g_2 = \frac{\sqrt{\gamma_t/\bar{\gamma}} f_d T \sqrt{2\pi}}{exp(\gamma_t/\bar{\gamma}) - 1} \quad (2)$$

$$b_2 = \sqrt{\gamma_t/\bar{\gamma}} f_d T \sqrt{2\pi} \quad (3)$$

where g_2 and b_2 are the transition probabilities of the GECs, f_d is the Doppler frequency and T is the packet duration and $f_d T$ is the normalized Doppler frequency.

The overall channel for the application layer Raptor codes can be represented as a hierarchical markov channel model (Fig. 3). At the higher level, the channel can be in the erasure

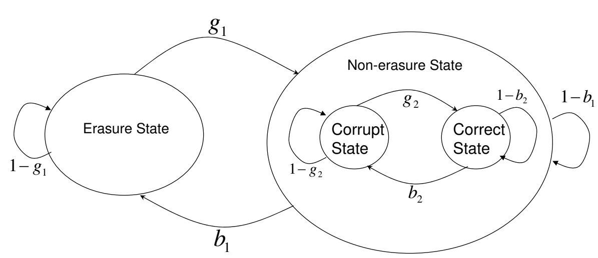

Fig. 3. Hierarchical markov model

state (packet loss) or the non-erasure state with transition probabilities g_1 and b_1, respectively where the erasure probability is 1 in the erasure state and 0 in non-erasure state. Conditional on the event that the packet is not erased, the channel is a GEC with transition probabilities g_2 and b_2, where the error probabilities in the good (correct) state and bad (corrupt) state are 0 and ε, respectively, where ε is also termed as the packet corruption level. Note that g_1 and b_1 are independent of g_2 and b_2 because the packet loss in this channel model is caused by network congestion as opposed to packet header corruption used as in [7].

III. Capacity and System Throughput Evaluation

The capacity of CON, CLD and CLDS protocols for memoryless channels have been summarized in [7]. Let δ and λ represent the packet dropping rate in the CON and CLD schemes, respectively. Let p represent the probability that an error occurs in a data bit in an unerased packet and let ε represent the conditional probability that an error occurs in a random data bit in an unerased packet given that the CRC fails. The capacity of the three schemes for memoryless channels can be easily obtained as [7],

$$C_{CON}^{NM} = 1 - \delta \quad (4)$$

$$C_{CLD}^{NM} = (1-\lambda)(1-h_b(p)) \quad (5)$$

$$C_{CLDS}^{NM} = (1-\delta) + (\delta-\lambda)(1-h_b(\varepsilon)) \quad (6)$$

where the superscript NM represents no memory and $h_b()$ is the binary entropy function, defined as $h_b(p) = -plogp - (1-p)log(1-p)$, and

$$p = (\delta - \lambda)\varepsilon/(1-\lambda). \quad (7)$$

In our channel model, CLD and CLDS schemes only differ by the availability of instantaneous channel state at the receiver. In [8], it is shown that channel memory does not increase the capacity of erasure channels but increases the capacity of general GECs when the instantaneous channel state is unknown to the receiver. We can conclude that the application layer capacities of our model, C_{CON}, C_{CLD} and C_{CLDS}, satisfy,

$$C_{CON} = C_{CON}^{NM} \quad (8)$$

$$C_{CLD}^{NM} < C_{CLD} < C_{CLDS}^{NM} \quad (9)$$

$$C_{CLDS} = C_{CLDS}^{NM} \quad (10)$$

However, since the Raptor decoder does not attempt to estimate side information of the instantaneous channel state, the performance of Raptor codes over these communication schemes is still bounded by the capacity for the case of no memory. Eqs. (4), (5) and (6) can be used to evaluate C_{CON}^{NM}, C_{CLD}^{NM} and C_{CLDS}^{NM}, where $\delta = 1 - (1-\lambda)\frac{g_2}{b_2+g_2}$.

The application layer capacity provides a bound to the performance of Raptor codes. However, the application layer capacity does not taken into account the extra protection bits used in the physical layer to protect the information bits. To compare the system performance using different turbo code rates, we use the maximum system throughput which is equal to $C \times R_{turbo}$, where C is the application layer capacity. Fig. (4) shows a comparison of the maximum achievable system throughput of the three schemes. Various channel SNRs and two different turbo code rates are used. It is quite obvious that the proposed hybrid scheme using the CLDS protocol can achieve much higher throughput over most of the SNR range. When the channel SNR is very high, the difference becomes negligible. We can also observe that a higher turbo code rate is preferable in most of case other than the extremely low SNR regime. [3] reaches a similar conclusion about the tradeoff between the turbo code rate and Raptor code rate, but the results are only limited to the CON scheme.

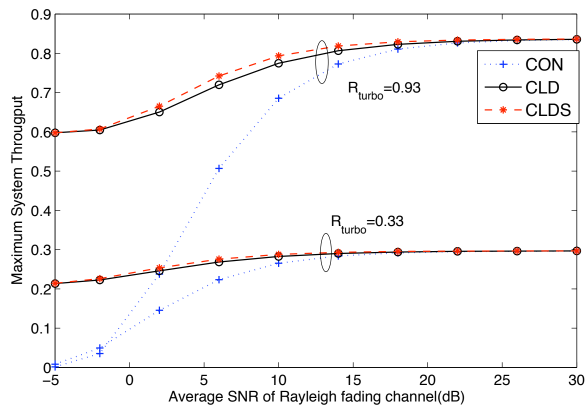

Fig. 4. Maximum system throughput under different channel SNR. ($\lambda = 0.1$, $\mu_1 = 0.9$, $fdT = 0.01$, $\varepsilon = 0.05$)

IV. Raptor codes and the hybrid erasure-soft decoder

The first practical realization of fountain codes is known as the class of Luby Transform (LT) codes [1] that encode k information symbols $(x_1, x_2, ..., x_k)$ into a potentially infinite number of output symbols $(z_1, z_2, z_3, ...)$. The encoding process is performed by first sampling a probability distribution Ω; a degree of d distinct information symbols are then chosen uniformly at random from the k input symbols. The value of each output symbol is the modulo 2 bit-wise summation of the d chosen input symbols. The output bit stream is generated independently until the transmitter receives an acknowledgement (ACK) of successful decoding from the receiver of successful decoding or until a predesigned code rate is achieved. The degree distribution Ω is usually described by its generating polynomial $\Omega(x) = \Sigma_{i=1}^{k} \Omega_i x^i$, where Ω_i represents the probability that value i is chosen. Shokrollahi [2] extended the idea of LT codes to Raptor codes to reduce the decoding complexity to be linear for the binary erasure channel (BEC). A Raptor code with parameters (k, C, Ω) is constructed by concatenating a block code C with a LT code with degree distribution Ω. To encode a Raptor code, the precoder C first encodes k information symbols into $\tilde{k}$ intermediate symbols. The output symbol streams are then generated by applying the inner LT code on the $\tilde{k}$ intermediate symbols.

Decoding of the Raptor codes for a binary symmetric channel (BSC) can be performed iteratively using Belief Propagation (BP) algorithms over the Tanner graph of the Raptor code [5]. For the BEC, the BP algorithm can be significantly simplified, which allows for linear decoding complexity of Raptor codes [2]. In this paper, we term the decoding method for BEC as erasure decoding, and the iterative decoding that uses soft information as soft decoding. Note that the complexity of erasure decoding is much simpler than that of soft decoding.

To implement the cross-layer protocol for Raptor coding in the MBMS system, we propose a hybrid erasure-soft decoder. The hybrid decoder works according to the follow steps:

Step 1) The Tanner graph of the Raptor code is constructed as shown in Fig. 5. For each LT encoded symbol, a corresponding check node is added to form a Tanner graph of the LT code. In the final Tanner graph, there are two types of variable nodes (input and output) and two types of check nodes (LDPC and LT).

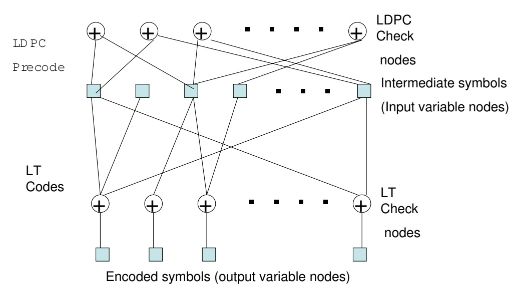

Fig. 5. Tanner graph of Raptor code

Step 2) The LLRs of the variable nodes are initialized. The initial LLRs for the input variable nodes are all set to 0 because they have not been transmitted. For all the output variable nodes connected to a packet that are lost or discarded, the initial LLRs should also be set to 0.

In the CLD scheme, since the CRC is turned off, the Raptor decoder does not know whether the channel is in the correct state or the corrupt state, i.e, the decoder does not know the instantaneous channel state for the lower level GEC. Therefore, the decoder treats the channel as a BSC with crossover probability p at non-erasure states, where p is given by (7). Hence the decoder will set the initial LLRs of output symbols to 0 for the erasure state and $(-1)^y ln((1-p)/p)$ [10] for the non-erased state, where $y \in [0, 1]$ is the decoder output of the physical layer code.

In the CLDS scheme, the receiver knows which state the current channel is in. Therefore, the decoder will set the initial LLR to 0 for the erasure state, $(-1)^y \infty$ for the correct state, and $(-1)^y ln((1-\varepsilon)/\varepsilon)$ for the corrupt state.

Step 3) The decoder eliminates all the nodes and edges that are associated with encoded symbols that are in the erasure state since they provide zero reliability.

Step 4) Based on the value of all the encoded symbols in the correct state, the decoder performs erasure decoding on the decoding Tanner graph. Any information symbols that can be decoded and any edges associated with these decoded nodes are removed from the graph. In the CLD scheme, this step is not performed because the receiver does not identify the correct states using the CRC.

Step 5) Iterative BP decoding based on LLRs from the corrupt state is performed on the remaining graph. Because the number of edges remaining is smaller than that of the original decoding graph, the decoding complexity of the hybrid scheme is simpler than a traditional iterative decoding scheme. The updating equation for the BP algorithm is the same as that used for LDPC codes [10].

V. Simulation results

To simulate the actual performance of Raptor codes, the Raptor code described in [5] is used. The pre-code of this Raptor code is a left regular and right Poisson LDPC code with rate 0.95, and the variable nodes of this LDPC code have constant degree $= 4$. The code dimension $k = 9500$ and the inner LT codes use the degree distribution,

$$\Omega(x) = 0.007969x + 0.493570x^2 + 0.166622x^3 \\ +0.072646x^4 + 0.082558x^5 + 0.056058x^8 + 0.037229x^9 \\ +0.055590x^{19} + 0.025023x^{65} + 0.003135x^{66}. \quad (11)$$

Fig. 6 depicts the performance of Raptor codes of the three communication schemes in channels with memory for different Raptor code rates. It can be seen that the CLDS and CLD schemes perform significantly better than the CON scheme. To achieve an average BER of 10^{-2}, the difference between the number of Raptor coded symbols that needs to be generated in the CON scheme and the CLDS scheme is approximately 21% when the corruption level $\epsilon = 0.02$ and 18% when $\epsilon = 0.05$. The CLDS scheme performs slightly better than the CLD scheme and the difference between their performance increases as the corruption level increases. It can be seen that the gap between the CLD and CLDS schemes is 0.6% for $\epsilon = 0.02$ and 1.5% for $\epsilon = 0.05$ to achieve a BER of 10^{-2}. The Raptor codes require less than 12% overhead for CLD and CLDS schemes to achieve a BER of 10^{-2} compared to their own capacity bounds evaluated by (5) and (6). It can also be observed that the performance curve of the Raptor code is very steep. Therefore, the rateless property of Raptor codes is very important to provide the flexibility of different code rates to accommodate different channel conditions.

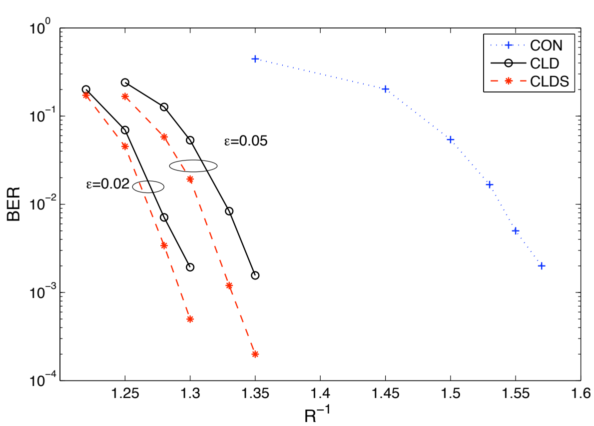

Fig. 6. Raptor code over hybrid error-erasure channel. ($\lambda = 0.1$, $\mu_1 = 0.9$, $R_{turbo} = 0.93$, $\bar{\gamma} = 10dB$, $f_dT = 0.01$)

Fig. 7 shows the effect of channel memory caused by fading correlation on the three different schemes. It can be observed that memory decreases the performance of cross-layer schemes. However, the cross-layer protocols are quite robust to fading correlation as the effect of memory is only significant for CLD and CLDS schemes when the normalized Doppler frequency is below 0.01. This can be explained by the fact that packet corruption only results in a small probability of error for a particular bit inside a packet.

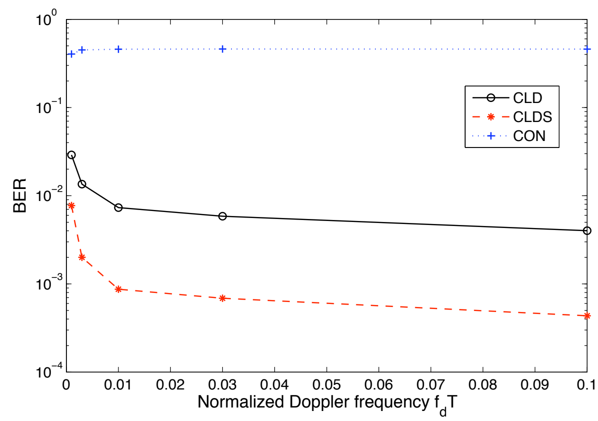

Fig. 7. The effect of channel memory. ($\lambda = 0.05$, $\mu_1 = 0$, $R_{turbo} = 0.93$, $\bar{\gamma} = 10dB$, $\varepsilon = 0.05$)

Figs. 8 and 9 show the influences of channel SNR and corruption level ε. An increase of average SNR decreases the average number of corrupt states, and hence improves the performances of all three schemes. It can be observed that the performances of the CLD and CLDS schemes are less sensitive to SNR than the CON scheme. The differences between the SNR requirement to achieve BERs of 10^{-1} and 10^{-2} is approximately 7dB for CLD and CLDS, and 5dB for CON. This also shows that the combination of an application layer Raptor code and a physical layer code is very robust to variations in channel quality, as a significant drop in channel SNR can be compensated by a slightly lowered Raptor code rate. The corruption level also has a significant impact on the performance of CLD and CLDS schemes. As shown in Fig. 9, for the same Raptor code rate and with all the other parameters equal, the performances of CLD and CLDS are reasonable at a corruption level of 0.005 (BER below 10^{-2}) but very poor at a corruption level of 0.1. The change of corruption level does not change the performance of CON since it does not change the average number of corrupt states.

VI. Conclusion

The paper proposed a hybrid erasure-soft decoding scheme for application layer Raptor codes used in broadcasting services with cross-layer protocols. By taking channel memory into account, the composite channel is modeled by a hierarchical Markov model which includes erasure, correct and corrupt states. For this channel model, the CLD and CLDS schemes differ only by the availability of side information

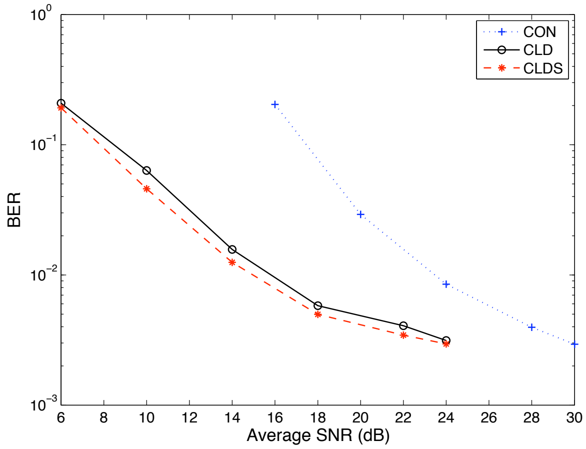

Fig. 8. Performance over different SNR. ($\lambda = 0.1$, $\mu_1 = 0.9$, $R_{turbo} = 0.93$,$\varepsilon = 0.02$, $R^{-1} = 1.25$, $fdT = 0.01$)

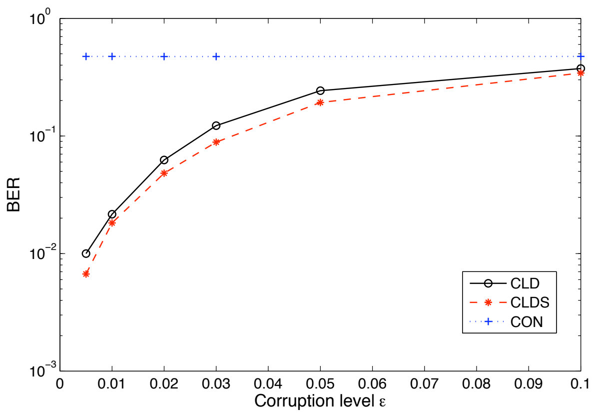

Fig. 9. The impact of corruption level ($\lambda = 0.1$, $\mu_1 = 0.9$, $R_{turbo} = 0.93$, $\bar{\gamma} = 10dB$, $R^{-1} = 1.25$, $fdT = 0.01$)

about instantaneous channel state. The proposed cross-layer decoding schemes outperform conventional (CON) scheme using erasure decoding significantly. The difference in overhead to achieve the same BER for the CLDS and CON schemes is typically around 20%. Channel correlation decreases the performance of Raptor codes for all three schemes and the impact is significant when the normalized Doppler frequency is small. The effect of the choice of turbo code rate on the system throughput is also discussed.

ACKNOWLEDGMENT

This work was supported by the Natural Sciences and Engineering Research Council of Canada Strategic Project STPSC 356826.

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