DIVERSITY ANALYSIS FOR LINEAR DISPERSION OVER TIME AND FREQUENCY

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ABSTRACT

To improve performance of orthogonal frequency division multiplexing (OFDM) for fading channels, linear dispersion coded OFDM (LDC-OFDM) was recently proposed to increase frequency and time diversity with high bandwith efficiency and performance. The performance of recently proposed LDC-OFDM was investigated through simulations. This paper analyzes the diversity aspects of LDC-OFDM system, and shows that the upper bound of the diversity order, which LDC-OFDM can achieve, is equal to the full diversity order available in the channel.

1. INTRODUCTION

Multicarrier communications systems, especially those employing orthogonal frequency division multiplexing (OFDM) [1], have been considered as primary candidates for next generation broadband transmission in frequency selective fading environments, which exhibit inter-symbol interference (ISI) [2]. Actually, orthogonal frequency division multiplexing (OFDM) has been accepted as an industrial standard for high-data-rate communications, such as Digital Television Broadcasting [3], wireless local area networks (IEEE 802.11 operating at 5 GHz [2] and ETSI BRAN's HYPERLAN 2 standards [2]). By serial-to-parallel conversion, OFDM transforms a single wideband multipath channel into multiple parallel narrowband flat fading channels, enabling simple equalization.

It is important to notice that uncoded OFDM cannot provide the same level of diversity combining effects as uncoded single-carrier systems in severe frequency-selective fading environments, since the frequency responses of channel space branches differ from one another, and hence the optimal diversity combining weights chosen for one of the OFDM subcarriers is no longer optimal for the other OFDM subcarriers. In frequency-selective fading channels, very low signal-to-noise ratio (SNR) or channel nulls often are the main reasons for lost transmitted information. Conventionally, schemes using a combination of error control coding combined with frequency interleaving across all subchannels are a class of effective techniques to mitigate the above problem at the price of reduced bandwidth efficiency, or coded OFDM (see e.g. [4, 5]).

Bandwidth efficiency is critical for high-data-rate transmission and is determined by the coding rate for OFDM. In conventional coded OFDM (COFDM) schemes [5], the coding rate usually is less than one, and achieving appropriate trade-offs between coding rate and error probability are critical design criteria. It has been shown in [6] that linear dispersion codes (LDC) may achieve a coding rate of up to one and outperform the well-known full-rate uncoded V-BLAST scheme. Linear dispersion codes (LDC) [6], which can support any configuration of transmit and receive antennas and includes both V-BLAST [7] and space-time block codes [8, 9] as special cases. LDC has been designed to optimize the mutual information between the transmitted and received space time signals [6]. Recently linear dispersion codes were proposed to help OFDM achieve not only frequency diversity but also time diversity, known as linear dispersion coded OFDM (LDC-OFDM) [10]. The newly proposed LDC OFDM method exploits diversity across both multiple subcarrier channels (frequency) and multiple OFDM blocks (time). Simulations have shown that LDC-OFDM with zero padding significantly improves error probability performance. However, in [10], no analytical insights were provided.

This paper analyzes diversity aspects of LDC-OFDM, and provides the upper bound of diversity order LDC-OFDM can achieve, which provides further insights into LDC-OFDM systems.

This paper is organized as follows. In Section 2, the related system models are introduced. The diversity aspects of LDC-OFDM are analytically discussed in Section 3.

The following notation is used in the following sections: $(\cdot)^\dagger$ denotes matrix pseudoinverse, $(\cdot)^T$ matrix transpose, $(\cdot)^H$ matrix transpose conjugate, I_K denotes identity matrix with size $K \times K$, $\mathbf{0}_{m\times n}$ denotes zero matrix with size $m \times n$, $A \otimes B$ denote Kronecker (tensor) product of matrices A and B, and $C^{m\times n}$ denotes a complex matrix of dimensions $m \times n$.

2. SYSTEM MODEL

2.1. Wideband OFDM model

During transmission, for each block of N_C IFFT transformed complex symbols, a block of P symbols are corrupted in a frequency selective, temporally flat Rayleigh fading channel with order L channel coefficients for the k-th OFDM block $\mathbf{h}^{(k)} = \left[h_0^{(k)}, ..., h_L^{(k)}\right]^T$.

A key assumption is that the channel experiences slow fading so that channel coefficients are constant over one OFDM block, considered as one channel use, while channel coefficients could change in subsequent OFDM blocks. Choosing $P \geq N_C + L$, the inter-block interference due to the previous transmitted block is eliminated by a guard interval.

Denote $x_p^{(k)}, p = 1..., N_C$ be the channel symbol transmitted on the p-th subcarrier during the k-th OFDM block. Unlike zero-padding OFDM in [10], at the transmitter, a guard interval is added to each OFDM block in the form of cyclic prefix (CP). The received signals are suffered additive complex Gaussian noise. After FFT processing, the received channel symbol sample $y_p^{(k)}$ is

$$y_p^{(k)} = \sqrt{\rho} H_p^{(k)} x_p^{(k)} + v_p^{(k)}, p = 1, ..., N_c \qquad (1)$$

where $H_p^{(k)}$ is the p-th subcarrier channel gain during the k-th OFDM block, and

$$H_p^{(k)} = \sum_{l=o}^{L} h_l^{(k)} e^{-j(2\pi/N_c)l(p-1)},$$

or

$$H_p^{(k)} = [\mathbf{w}_p]^T \mathbf{h}^{(k)}, \qquad (2)$$

where

$$\mathbf{w}_p = \left[1, \omega^{p-1}, \omega^{2(p-1)}, \cdots, \omega^{L(p-1)}\right],$$

and

$$\omega = e^{-j(2\pi/N_c)}.$$

The additive noise is circularly symmetric, zero-mean, complex Gaussian with variance N_0. The additive noise is assumed statistically independent for different k. ρ is normalized signal to noise ratio (SNR).

2.2. LDC-OFDM system

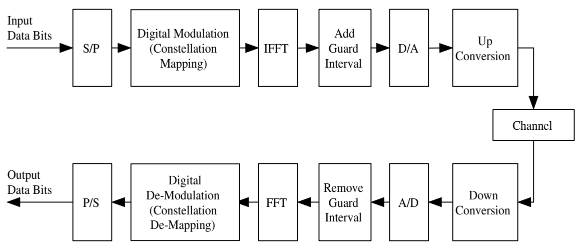

Fig. 1. Conventional OFDM system model

The differences between conventional OFDM and the newly proposed LDC-OFDM [10] are illustrated in Figures 1 and 2. The block sizes of signal transmission and reception of these two systems differ. The parts shared by both the OFDM and LDC-OFDM systems are from the IFFT operation in the transmitter to the FFT operation in the receiver. In both OFDM and LDC-OFDM systems, a guard interval is added into each OFDM block.

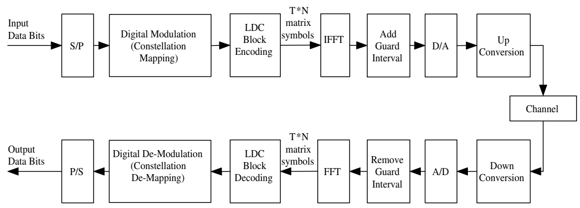

Fig. 2. Proposed LDC-OFDM system model

2.3. LDC-OFDM codeword

There are N_C subcarriers in one OFDM block. One LDC-OFDM block, illustrated in Figure 3, consists of T adjacent OFDM blocks. One LDC-OFDM block, consists of T adjacent OFDM blocks. An LDC-OFDM system includes D LDC codewords, each with LDC matrices occupying $N_{F(i)}$ subcarriers and T OFDM blocks $\in C^{T \times N_{F(i)}}$, $i = 1, ..., D$, with $\sum_{i=1}^{D} N_{F(i)} = N_C$.

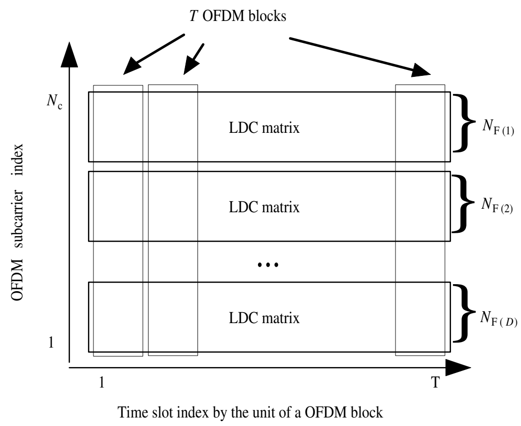

Fig. 3. LDC-OFDM blocks in the time-frequency plane

3. DIVERSITY PROPERTIES OF LDC-OFDM

Since LDC-OFDM includes all LDC coding properties within $T \times N_{F(i)}$ block in a LDC-OFDM codeword, in this analysis, we consider single $T \times N_{F(i)}$ block $\mathbf{C}^{(i)}, i = 1, ..., D$ in a LDC-OFDM codeword. The block $\mathbf{C}^{(i)}$ is created after encoding all the i-th LDC codewords within a LDC-OFDM codeword.

For simplicity, in block $\mathbf{C}^{(i)}$, consider the case that the subcarrier indices chosen from all the OFDM blocks are the same over time, and denote subcarrier indices

$$\begin{array}{l} \{p_{n_{F(i)}}^{(k)}, n_{F(i)} = 1, ..., N_{F(i)}, \\ i = 1, ..., D, k = 1, ..., T\} \end{array}.$$

Denote the block $\mathbf{C}^{(i)}$ in matrix form as

$$\mathbf{C}^{(i)} = \begin{bmatrix} c_{p_{1_{(i)}}}^{(1)} & c_{p_{2_{(i)}}}^{(1)} & \cdots & c_{p_{N_{F(i)}}}^{(1)} \\ c_{p_{1_{(i)}}}^{(2)} & c_{p_{2_{(i)}}}^{(2)} & \cdots & c_{p_{N_{F(i)}}}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p_{1_{(i)}}}^{(T)} & c_{p_{2_{(i)}}}^{(T)} & \cdots & c_{p_{N_{F(i)}}}^{(T)} \end{bmatrix}. \quad (3)$$

We may express the system equation for block $\mathbf{C}^{(i)}$ as

$$\mathbf{R}^{(i)} = \sqrt{\rho}\mathbf{M}^{(i)}\mathbf{H}^{(i)} + \mathbf{V}^{(i)}, \quad (4)$$

where received signal vector $\mathbf{R}^{(i)}$ and noise vector $\mathbf{V}^{(i)}$ are of size of $N_{F(i)}T \times 1$, the i-th LDC coded channel symbol matrix is of size $N_{F(i)}T \times N_{F(i)}T$, $\mathbf{M}^{(i)}$ is of size $N_{F(i)}T \times N_{F(i)}T$, $c_{p_{n_{F(i)}}}^{(k)}, n_{F(i)} = 1, ..., N_{F(i)}$, is the channel symbol of the k-th OFDM block, the $p_{n_{F(i)}}$-th subcarrier in the i-th LDC codeword, and

$$\begin{aligned} &\mathbf{M}^{(i)} = \\ &diag(c_{p_{1_{(i)}}}^{(1)}, ..., c_{p_{N_{F(i)}}}^{(1)}, ..., c_{p_{1_{(i)}}}^{(T)}, ..., c_{p_{N_{F(i)}}}^{(T)}), \end{aligned} \quad (5)$$

where $i = 1, ..., D$.

The channel $\mathbf{H}^{(i)}$ is of size $N_{F(i)}T \times 1$, and

$$\mathbf{H}^{(i)} = \begin{bmatrix} H_{p_{1(i)}}^{(1)}, H_{p_{2(i)}}^{(1)}, ..., H_{p_{N_{F(i)}}}^{(1)}, \\ ..., H_{p_{1(i)}}^{(T)}, H_{p_{2(i)}}^{(T)}, ..., H_{p_{N_{F(i)}}}^{(T)} \end{bmatrix}^T \quad (6)$$

and $H_{p_{n_{F(i)}}}^{(k)}$ is the path gain of of k-th OFDM block , the $p_{n_{F(i)}}$-th subcarrier for block $\mathbf{C}^{(i)}$. Thus

$$H_{p_{n_{F(i)}}}^{(k)} = \left[\mathbf{w}_{p_{n_{F(i)}}}\right]^T \mathbf{h}^{(k)}, \quad (7)$$

where $\mathbf{w}_p$ and $\mathbf{h}^{(k)}$ has been defined in Section 2.

Consider a pair of matrices $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ corresponding to two different time-frequency (TF) blocks $C^{(i)}$ and $\tilde{C}^{(i)}$. Then the upper bound pairwise error probability [11] between $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ is

$$\begin{aligned} &P\left(\mathbf{M}^{(i)} \to \tilde{\mathbf{M}}^{(i)}\right) \le \\ &\begin{pmatrix} 2r-1 \\ r \end{pmatrix} \left(\prod_{a=1}^{r} \gamma_a\right)^{-1} (\rho)^{-r} \end{aligned} \quad (8)$$

where r is the rank of

$$\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \mathbf{R}_{\mathbf{H}^{(i)}} \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^H,$$

and $\mathbf{R}_{H^{(i)}} = E\left\{\mathbf{H}^{(i)}\left[\mathbf{H}^{(i)}\right]^H\right\}$ is the correlation matrix of vector $\mathbf{H}^{(i)}$, $\gamma_a, a = 1, ..., r$ are the non-zero eigenvalues of

$$\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \mathbf{R}_{\mathbf{H}^{(i)}} \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^H.$$

Then the corresponding rank and product criteria are

1. Rank criterion: the minimum rank of

   $$\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \mathbf{R}_{\mathbf{H}^{(i)}} \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^H$$

   over all pairs of different matrices $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ and should be as large as possible.

2. Product criterion: the minimum value of the product $\prod_{a=1}^{r} \gamma_a$ over all pairs of different $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ should be maximized.

For simplicity, denote

$$\mathbf{\Lambda}^{(i)} = \left(\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \mathbf{R}_{\mathbf{H}^{(i)}} \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^H\right)$$

Now we need derive the matrix form of $\mathbf{R}_{\mathbf{H}^{(i)}}$. Denote

$$\mathbf{W}^{(i)} = \left[\mathbf{w}_{p_{1_{(i)}}}, \cdots, \mathbf{w}_{p_{N_{F(i)}}}\right]^T \quad (9)$$

and

$$\mathbf{h} = \left[\left[\mathbf{h}^{(1)}\right]^T, \cdots, \left[\mathbf{h}^{(T)}\right]^T\right] \quad (10)$$

thus

$$\mathbf{H}^{(i)} = \left(\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right) \mathbf{h}. \quad (11)$$

Then, we have

$$\begin{aligned} &\mathbf{R}_{\mathbf{H}^{(i)}} = E\left\{\left(\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right) \mathbf{h} \left[\left(\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right) \mathbf{h}\right]^H\right\} \\ &= \left[\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right] E\left\{\mathbf{h} \left[\mathbf{h}\right]^H\right\} \left[\mathbf{I}_T \otimes \left[\mathbf{W}^{(i)}\right]^H\right] \\ &= \left[\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right] \mathbf{\Phi} \left[\mathbf{I}_T \otimes \left[\mathbf{W}^{(i)}\right]^H\right] \end{aligned}, \quad (12)$$

where $\mathbf{\Phi} = E\left\{\mathbf{h}\left[\mathbf{h}\right]^H\right\}$

A well-known linear algebra results is that

$$rank\left(\mathbf{AB}\right) \le \min\left\{rank\left(\mathbf{A}\right), rank\left(\mathbf{B}\right)\right\}. \quad (13)$$

Applying (13), we have

$$\begin{aligned} &rank\left(\mathbf{\Lambda}^{(i)}\right) \le \\ &\min\left\{rank\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right), rank\left(\mathbf{R}_{\mathbf{H}^{(i)}}\right)\right\} \end{aligned}. \quad (14)$$

Thus it is desired to maximize

$$\min\left\{rank\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right), rank\left(\mathbf{R}_{\mathbf{H}^{(i)}}\right)\right\}.$$

We know maximum of rank of $\mathbf{\Phi}$ is $T(L+1)$. To maximize the rank of $\mathbf{R}_{H^{(i)}}$, we need to maximize the rank of matrix $\mathbf{W}^{(i)}$ of size $N_{F(i)} \times (L+1)$. Thus we need to choose $N_{F(i)} \geq L+1$. When $p_{n_F}^{(i)} = p_1^{(i)} + b(n_F - 1), n_F = 1, ..., N_{F(i)}, N_{F(i)} \geq L+1$, where $p_{n_F}^{(i)} \leq N_C$ and b is a positive integer, $\mathbf{W}^{(i)}$ could achieve maximum rank $L+1$, then $\mathbf{R}_{H^{(i)}}$ has the potential to achieve the maximal rank of $T(L+1)$, only if $Rank(\mathbf{\Phi}) = T(L+1)$. That is to say, channels need to be full rank jointly in frequency and time domains.

We know $\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}$ is of size $N_{F(i)}T \times N_{F(i)}T$. Thus

$$rank\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \leq N_{F(i)}T, \tag{15}$$

and

$$N_{F(i)} \geq L+1. \tag{16}$$

This above analysis has revealed that, instead of using all available subcarriers, using proper frequency-time (FT) block design, which usually is a much smaller block, could achieve diversity order up to $T(L+1)$, and the necessary condition that FT block design achieve a certain diversity order is that the rank of channel correlation matrix is equal to the diversity order of the FT block.

FT block based LDC-OFDM was proposed across multiple time varying OFDM blocks and multiple subcarriers, thus have the potentials to achieve diversity order $T(L+1)$. However, in practice, the diversity order achieved is based on specific LDC design chosen. Originally, Hassibi and Hochwald did not consider diversity order as design criterion [6]. Heath and Paulraj considered both capacity and error probability as criterion [12]; in other words, they started to discuss diversity aspects, however, they only consider constant channel coefficients over time within an entire LDC codeword. This paper provides a more general analysis that considers correlation across parallel frequency channels (OFDM subcarriers) as well as across time channel uses (OFDM blocks).

4. CONCLUSION

Exploiting both frequency and time diversity available in frequency selective wideband OFDM channels, the proposed LDC-OFDM has high transmission bandwidth efficiency and improved BER. This paper has analyzed the upper bound diversity order that LDC-OFDM can achieve, which provides an insight into linear dispersion over time and frequency. Properly designed LDC-OFDM could utilize full available time and frequency diversity in communications channels.

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