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Smart Antenna Performance for Correlated Azimuth Spread and Rician K-Factor

Constantin Siriteanu, Yoshikazu Miyanaga, and Steven D. Blostein

***Abstract*—The recently-proposed superset of statistical beamforming (BF) and maximal-ratio combining (MRC) known as maximal-ratio eigencombining (MREC) promises to achieve near-optimum performance and to reduce numerical complexity. Furthermore, eigencombining can be exploited to greatly simplify the performance analysis of diversity schemes for correlated fading. This paper contributes a new MREC (BF, MRC) performance analysis and evaluation for Rician fading with lognormally-distributed K-factor, and Laplacian power azimuth spectrum with lognormally-distributed azimuth spread (AS). First, an average error probability (AEP) expression is derived for MREC for correlated Rician fading and perfectly known channel gains and eigenstructure. This AEP expression is then employed to study the effect on performance of the randomness of K and of the AS – K correlation. We find that disregarding this randomness and correlation may significantly distort performance indications.**

***Index Terms*—Azimuth spread, Rician fading, K-factor, correlation, eigencombining.**

I. INTRODUCTION

Wireless communications standards (3GPP/2, WiMAX, WiFi, etc.) specify smart antenna algorithms designed to take advantage of array and diversity gains [1, Sections 5.2, 5.3] [2] [3] [4]. However, inaccurate channel fading estimation and inadequate received-power azimuth spread (AS) [5] degrade the performance of conventional combining techniques such as maximal-ratio combining (MRC) and maximum average signal-to-noise ratio (SNR) beamforming, i.e., statistical beamforming (BF) [2] [3]. Fading type (e.g., Rayleigh, Rician) can also affect performance very significantly [1] [6] [7].

BF combines the received signal vector with the dominant eigenvector of the channel correlation matrix. Thus, BF performance degrades with increasing AS due to combiner–channel coherence loss. On the other hand, MRC combines the received signal vector with the channel gain vector, which is in practice estimated through computationally-intensive operations. MRC performs best for uncorrelated channel gains, i.e., for wide AS. However, in typical urban (TU) scenarios the base sation 'sees' a Laplacian power azimuth spectrum (PAS) with a predominantly small-to-moderate, lognormally-distributed, AS that also fluctuates orders of magnitude slower than the Doppler-shift-induced fading [5]. Therefore, BF and MRC may periodically underperform or may have oversized numerical complexity [2] [3].

Maximal-ratio eigencombining (MREC) [2] consists of projecting the received signal vector onto dominant eigenvectors of the channel gain correlation matrix — i.e., the Karhunen-Loeve Transform (KLT) — followed by MRC. Therefore, MREC is a superset of BF and MRC [3] [4] that has been promoted as able to extract the available array and diversity gains for lower complexity than MRC, and to simplify MRC performance analysis [2] [3] [4].

Our previous MREC analyses focused on Rayleigh fading [2] [3] [4]. However, many typical scenarios and environments exhibit Rician fading (microcell, macrocell; urban, suburban; outdoor, indoor; line-of-sight — LOS, NLOS) [8] [9] [10]. Furthermore, the effect of the Rician K-factor [1] [6, p. 21] [7] has been evaluated independently of the AS: for instance, evaluating combiner performance for AS values or for AS ranges while assuming Rayleigh fading (i.e., $K = 0$) effectively discounts any AS – K relationship [2] [3] [4]; on the other hand, combiner performance studies for Rician fading generally set K to typical values (e.g., averages from measurements [9]) and antenna correlation (which, in fact, is a function of the AS) to arbitrary values [6, Fig. 9.14, 316].

However, the actual K-factor is a lognormally-distributed random variable [9] [10] not independent of AS. Simplistic Rician fading modeling as superposition of a LOS wave and diffuse omnidirectional arrivals reveals a certain inverse relationship between AS and K [11, Eqn. (19)]. This expression does not apply for NLOS conditions [8] and for typical PAS, i.e., Laplacian [5] and Gaussian [12]. A different model will thus yield a different AS – K relationship (and a different performance indication). Performance evaluations less dependent on models are possible by considering instead that AS and K are correlated random variables. Measurements in actual channels [12] have revealed the following AS – K correlation values: $\rho = -0.6$ for indoor office and residential environments, $\rho = -0.3$ for TU microcells, $\rho = -0.2$ for TU macrocells [12, Table 4-5, p. 47].

In this paper we first derive, based on our previous results for Rayleigh fading [3], a new average error probability (AEP) expression for MREC, MRC, and BF, that applies for correlated Rician fading and perfectly known channel gains and eigendecomposition. This expression is then used to study, for the first time, to the best of our knowledge, the effect of K randomness and of AS – K correlation on AEP performance indications. We find that disregarding the randomness of K can greatly overestimate BF, MRC, and MREC performance.

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On the other hand, disregarding the AS – K correlation can significantly underestimate MRC and MREC performance.

Section II of this paper presents the transmitted signal, channel fading, receiver noise, K-factor, PAS, and AS models. Section III describes MREC and derives its AEP for Rician fading and perfectly-known channel gains and eigenstructure. Section IV shows numerical results for BF, MRC, and adaptive MREC AEP performance.

II. Signal, Channel, and Noise Models

A. Received Signal Model

A mobile station transmits signal through a frequency-flat Rician fading channel. At an L-element base-station antenna array the received signal vector after demodulation, matched-filtering, and symbol-rate sampling is [3]

$$\widetilde{\mathbf{y}} = \sqrt{E_s}\, b\, \widetilde{\mathbf{h}} + \widetilde{\mathbf{n}} \tag{1}$$

where b is the equiprobable transmitted symbol, and E_s is the energy transmitted per symbol. For the numerical results shown later we assume an M–PSK transmitted signal. The channel fading and receiver noise vectors, $\widetilde{\mathbf{h}}$ and $\widetilde{\mathbf{n}}$, are assumed to be mutually uncorrelated, circularly-symmetric, complex-valued, random Gaussian vectors [1, p. 39], described by $\widetilde{\mathbf{h}} \sim \mathcal{N}_c(\widetilde{\mathbf{h}}_d, \mathbf{R}_{\widetilde{\mathbf{h}}})$ and $\widetilde{\mathbf{n}} \sim \mathcal{N}_c(\mathbf{0}, N_0\,\mathbf{I})$. The distribution of the channel gain vector is completely described by the mean (i.e., the deterministic, constant, component), $\widetilde{\mathbf{h}}_d$, and the covariance matrix

$$\mathbf{R}_{\widetilde{\mathbf{h}}} \triangleq E\left\{ \left(\widetilde{\mathbf{h}} - \widetilde{\mathbf{h}}_d\right) \left(\widetilde{\mathbf{h}} - \widetilde{\mathbf{h}}_d\right)^H \right\}. \tag{2}$$

Let us assume that the (real-valued and non-negative) eigenvalues are ordered as $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L \geq 0$. The corresponding, orthonormal, eigenvectors are denoted as $\mathbf{u}_i$, $i = 1 : L$. The eigendecomposition of $\mathbf{R}_{\widetilde{\mathbf{h}}}$ is then:

$$\mathbf{R}_{\widetilde{\mathbf{h}}} = \mathbf{U}_L \mathbf{\Lambda}_L \mathbf{U}_L^H = \sum_{i=1}^{L} \lambda_i\, \mathbf{u}_i\, \mathbf{u}_i^H, \tag{3}$$

where $\mathbf{\Lambda}_L$ and $\mathbf{U}_L$ are a diagonal and a unitary matrix formed with the eigenvalues and eigenvectors of $\mathbf{R}_{\widetilde{\mathbf{h}}}$, respectively. Hereafter, these matrices are assumed to be perfectly known.

B. Statistical Model for the K-Factor in Rician Fading

Several authors have estimated from channel measurements the K-factor for the Rician distribution. Greenstein *et al.* reported in [10] results of several measurement campaigns for the downlink in four suburban areas under various conditions. It was found that channel gain samples fit well the Rician distribution [10, Fig. 1], and that the K-factor follows a lognormal distribution [10, Fig. 6], and that the K-factor median is a simple function of the season, antenna height, and antenna beamwidth [10, Eqn. (15)]. For a typical suburban macrocell scenario, Erceg *et al.* had drawn similar conclusions and proposed the following model for the K-factor (in dB), for a BS – MS distance of 1 km [9, Table II]:

$$K_{dB} = 7.87\,\psi + 8.53; \quad \psi \sim \mathcal{N}(0,1). \tag{4}$$

C. PAS and AS Models

In TU scenarios, intended-signal power arrives with azimuth angle dispersion, which is typically modeled using the Laplacian power azimuth spectrum (PAS) [5]. Laplacian PAS is parameterized by the mean angle of arrival (AoA), θ_c, and by the AS, which is (approximately) the root second central moment of the PAS [4, Eqn. (4.2), p. 136]. The correlation between two antenna elements can then be computed with [4, Eqn. (4.3), p. 136].

For the numerical results shown later the 'TU-32' scenario described in [5, Table I] has been considered. Then, the base-station AS, measured in degrees, is well modeled as a lognormally-distributed random variable [5, Eqn. (9), Table II]:

$$\text{AS} = 10^{0.47\,\chi + 0.74}; \quad \chi \sim \mathcal{N}(0,1). \tag{5}$$

III. MREC, MRC, and BF

A. MREC for Perfectly Known Channel

We summarize below from [3, Section III.A.1] the steps of maximal-ratio eigencombining (MREC) of order $N = 1 : L$, denoted hereafter as MREC$_N$:

(1) The $L \times N$ matrix $\mathbf{U}_N \triangleq [\mathbf{u}_1\, \mathbf{u}_2\, \ldots\, \mathbf{u}_N]$ transforms the signal vector from (1) into

$$\mathbf{y} = \sqrt{E_s}\, b\, \mathbf{h} + \mathbf{n}, \tag{6}$$

where

$$\mathbf{y} \triangleq \mathbf{U}_N^H\, \widetilde{\mathbf{y}}, \quad \mathbf{h} \triangleq \mathbf{U}_N^H\, \widetilde{\mathbf{h}}, \quad \mathbf{n} \triangleq \mathbf{U}_N^H\, \widetilde{\mathbf{n}}. \tag{7}$$

This is the well-known Karhunen-Loeve Transform (KLT). The elements of the N-dimensional vectors $\mathbf{y}$ and $\mathbf{h}$ are hereafter referred to as *eigenbranches* and *eigengains*, respectively. Our assumptions about the fading and noise imply that: 1) $\mathbf{h} \sim \mathcal{N}_c(\mathbf{h}_d, \mathbf{\Lambda}_N)$, where $\mathbf{h}_d \triangleq \mathbf{U}_N^H\, \widetilde{\mathbf{h}}_d$; 2) the eigengains are independent; 3) $\mathbf{n} \sim \mathcal{N}_c(\mathbf{0}, N_0\, \mathbf{I}_N)$.

(2) For perfectly known channel gains and eigenstructure, the transformed signal vector is linearly combined, based on the maximal-ratio combining criterion [6], with

$$\mathbf{w}_{\text{MREC}} = \mathbf{h}. \tag{8}$$

By definition, the post-KLT K-factors are the ratios of the powers in the deterministic and the random components of the eigengains, i.e.:

$$K_i = \frac{|h_{d,i}|^2}{\lambda_i} = \frac{|\mathbf{u}_i^H\, \widetilde{\mathbf{h}}_d|^2}{\lambda_i}. \tag{9}$$

These eigengain K-factors depend on the angle between the deterministic channel component and the channel eigenvectors as well as on the corresponding eigenvalues (and thus on the AS). The relation between K_i, AS, and the direction of $\widetilde{\mathbf{h}}_d$ will be investigated in future work.

B. MREC Analysis for Perfectly Known Channel

The following analysis is an extension to Rician fading of the analysis for Rayleigh fading from [3]. However, we consider only the perfectly-known channel case herein. The case of estimated channel will be analyzed in future work.

1) Optimum Eigencombining — MREC: For the post-KLT signal vector in (6) the optimum weight vector from (8) yields the symbol-detection SNR

$$\gamma = \frac{E_s}{N_0}\sum_{i=1}^{N}|h_i|^2 = \sum_{i=1}^{N}\gamma_i, \tag{10}$$

i.e., the sum of the individual SNRs. The average of γ_i is:

$$\Gamma_i = \frac{E_s}{N_0}\left(|h_{\mathrm{d},i}|^2 + \lambda_i\right). \tag{11}$$

The property in (10) indicates that this eigencombining method maximizes the symbol-detection SNR. Therefore, it has been referred to as *MREC* [3].

The eigenbranch SNRs from (10) are independent random variables with non-central χ^2 distributions described by probability density function [6, Eqn. (2.16), p. 21]

$$p(\gamma_i) = \frac{(K_i+1)\, e^{-\left[K_i + \frac{(K_i+1)\,\gamma_i}{\Gamma_i}\right]}}{\Gamma_i}\, I_0\left(\sqrt{\frac{4\,K_i\,(K_i+1)\,\gamma_i}{\Gamma_i}}\right)$$

and moment generating function (MGF) [6, Eqn. (2.17), p. 21]

$$M_{\gamma_i}(s) = E\{e^{s\,\gamma_i}\} = \frac{K_i+1}{(K_i+1)-s\,\Gamma_i}\exp\left[\frac{K_i\,s\,\Gamma_i}{(K_i+1)-s\,\Gamma_i}\right].$$

Based on the SNR property (10), we can express the MREC average error probability (AEP) using the method from [6, Chapter 9], as follows. For MREC$_N$ and M-PSK transmitted signals, the symbol error probability conditioned on γ can be written as [6, Eqn. 8.22]

$$P_{e,N}(\gamma) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\exp\left\{-\gamma\,\frac{g_{\mathrm{PSK}}}{\sin^2\phi}\right\}d\phi, \tag{12}$$

where $g_{\mathrm{PSK}} = \sin^2 \pi/M$. Then, the AEP is [6]

$$P_{e,N} \triangleq E\{P_{e,N}(\gamma)\} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} M_\gamma\left(-\frac{g_{\mathrm{PSK}}}{\sin^2\phi}\right)d\phi, \tag{13}$$

where $M_\gamma(s)$ is the MGF of γ. Using (10) and the independence of γ_i, $i = 1:N$, Eqn. (13) becomes

$$P_{e,N} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\prod_{i=1}^{N} M_{\gamma_i}\left(-\frac{g_{\mathrm{PSK}}}{\sin^2\phi}\right)d\phi. \tag{14}$$

This equation and the expression shown above for $M_{\gamma_i}(s)$ yield the symbol-AEP expression for MREC$_N$ as

$$P_{e,N} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\prod_{i=1}^{N}\frac{(K_i+1)\, e^{-\frac{K_i \frac{g_{\mathrm{PSK}}}{\sin^2\phi}\Gamma_i}{(K_i+1)+\frac{g_{\mathrm{PSK}}}{\sin^2\phi}\Gamma_i}}}{(K_i+1)+\frac{g_{\mathrm{PSK}}}{\sin^2\phi}\Gamma_i}\,d\phi \tag{15}$$

which depends on modulation constellation size, M, MREC order, N, antenna correlation (i.e., also AS), and K-factor. The derived AEP expression suits well our goal of evaluating the effect of the AS – K correlation on MREC performance.

2) Relation to BF and MRC: For $N = 1$, MREC becomes BF, and then (15) is the BF AEP expression. For $N = L$, MREC is equivalent to MRC because the symbol-detection SNRs are equal [4, Section 3.9]. Thus, (15) with $N = L$ describes MRC performance.

C. Optimum Order Selection for MREC

For Rayleigh fading, we previously adapted the MREC order to AS using the *bias-variance tradeoff* criterion (BVTC) [3, Eqn. (31)] and found that BVTC-based MREC can attain MRC-like performance for a fraction of the MRC complexity [2] [3]. These benefits are due to the fact that the BVTC balances the loss incurred by removing the weakest $(L-N)$ intended-signal contributions against the residual-noise contribution.

For Rician fading the BVTC expression from [3, Eqn. (31)] becomes:

$$\min_{N=1:L}\left[E_s\cdot\sum_{i=N+1}^{L}\left(\lambda_i + \left|h_{\mathrm{d},i}\right|^2\right) + N_0\cdot N\right] = \min_{N=1:L}\left[E_s\cdot\sum_{i=N+1}^{L}\lambda_i\,(1+K_i) + N_0\cdot N\right]. \tag{16}$$

The K-factor for the ith eigenbranch, i.e., K_i defined in (9), is determined by both the eigenvalue, λ_i, and by the angle between the ith channel eigenvector, $\mathbf{u}_i$, and the deterministic component $\tilde{\mathbf{h}}_{\mathrm{d}}$. Therefore, BVTC for Rician fading requires reordering decreasingly the terms $\lambda_i\,(1+K_i)$, $i = 1:N$, before (16) is employed to compute the MREC order, N.

IV. Numerical Results

Our numerical experiments employed the following parameter settings: QPSK transmitted signal; uniform linear array with $L = 5$ elements and normalized interelement distance $d_{\mathrm{n}} = 1$, i.e., physical distance equals half of the carrier wavelength; mean angle of arrival (AOA) $\theta_{\mathrm{c}} = 0$, which corresponds to the direction perpendicular to the antenna array; Laplacian PAS with lognormally-distributed AS described by (5); lognormally-distributed K-factor described by (4), unless stated otherwise; the channel gains have unit variance (so that the bit-SNR is given by $\gamma_{\mathrm{b}} = E_{\mathrm{s}}/N_0$).

Our numerical experiments employed the following procedure: 1) a batch of $10,000$ independent AS samples from the lognormal distribution from (5) was generated and stored; its mean and standard deviation are $9.67°$ and $12.8°$, respectively, and Prob$(AS < 20°) = 0.84$; a commensurate batch of K-factor samples was also generated, using (4), but was only employed in some of the simulations; when necessary, AS – K correlation was introduced; 2) for γ_{b} values shown on the abscissa in the figures described subsequently, the AEP was computed using (15) for each stored AS value or AS–K pair; 3) the mean AEP was computed by averaging over the AS samples or the (AS, K) pairs of samples.

Fig. 1 shows the mean AEP for SISO (single-input, single-output), BF, BVTC-based adaptive MREC, and MRC, for Rayleigh fading, i.e., for $K = 0$. Note that BVTC MREC

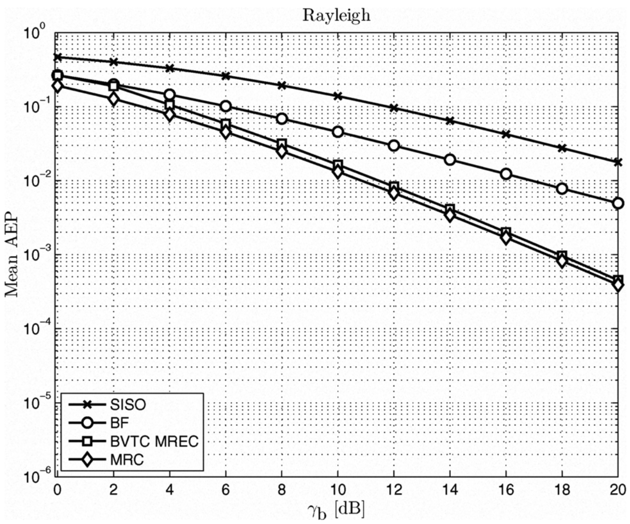

Fig. 1. Average (over noise, fading, and AS) error probability vs. bit-SNR, for SISO, BF, BVTC MREC, and MRC, for perfectly known Rayleigh fading channel gains.

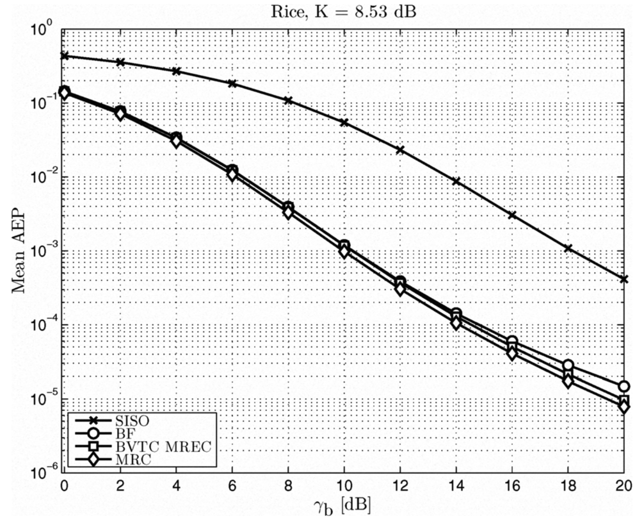

Fig. 2. Average (over noise, fading, and AS) error probability vs. bit-SNR, for SISO, BF, BVTC MREC, and MRC, for perfectly known Rician fading channel gains with $K = 8.53$ dB.

can attain near-MRC performance. Our previous numerical complexity comparisons between MRC and BVTC MREC for Rayleigh fading and estimated channel have indicated that BVTC MREC can also yield significant numerical complexity savings compared to MRC [2] [3].

For Rician fading, the direction of the deterministic component of the channel gain vector, $\mathbf{h}_d$, has not yet considered statistically, to the best of our knowledge. Nevertheless, it makes intuitive sense that this component should correspond to the mean AOA [12, Section 3.6, p. 35]. So, we set the deterministic component of the channel gain vector as proportional with the array response for a wave arriving from $\theta_c = 0$, i.e., $\widetilde{\mathbf{h}}_d \propto [1\ 1\ \ldots\ 1]^T$.

Now, for the same AS batch as above, we consider Rician fading with $K = 8.53$ dB, i.e., the mean of the distribution from (4). Then, Fig. 2 reveals that the deterministic component of the channel gain improves very significantly the performance for all combiners (compared to the Rayleigh fading results from Fig. 1). SISO performance improves significantly, due to steeper high-SNR slope caused by the high K value. BF yields significant array gain because its weight vector, which is proportional to $\mathbf{u}_1$, lines up with $\widetilde{\mathbf{h}}_d$. For the diversity schemes (BVTC MREC and MRC) the above figures also reveal a tremendous array gain for Rician vs. Rayleigh fading (but similar AEP slopes).

Fig. 3 shows the BF, MRC and MREC mean-AEP performance for random K-factor with the distribution from (4) but uncorrelated with the AS. Comparing this figure with Figs. 1 and 2 indicates that Rician fading with random and uncorrelated AS and K yields better performance than for Rayleigh fading, and far worse performance than for Rice fading with fixed $K = 8.53$ dB. Clearly, our assumptions about the statistics of K have a significant impact on performance indications.

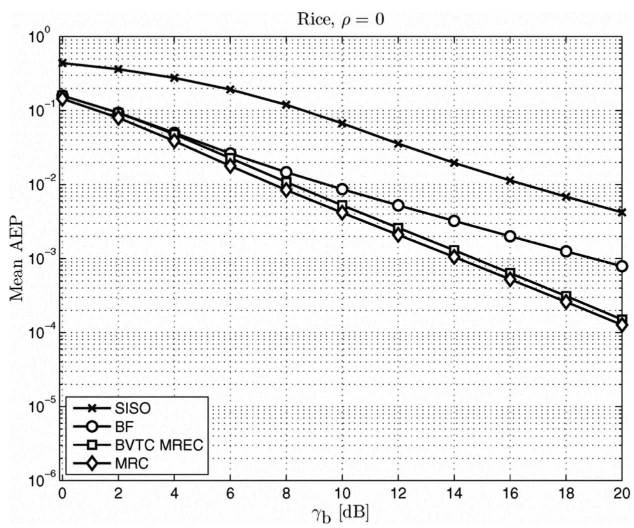

Fig. 3. Average (over noise, fading, AS and K) error probability vs. bit-SNR, for BF, BVTC MREC, and MRC, for perfectly known Rician fading channel gains, for random and uncorrelated AS and K-factor.

Fig. 4 shows mean-AEP performance for correlated AS and K with correlation $\rho = -0.6$ (i.e., the upper limit of values measured in [12]). SISO and BF performance does not change significantly. On the other hand, Figs. 4 and 3 suggest that, by discounting the AS – K correlation, the performance of diversity schemes will appear poorer.

Our observations on the previous figures are confirmed in Fig. 5. The mean AEP is shown for SISO, BF, BVTC MREC (MRC performs negligibly better than BVTC MREC) for Rayleigh fading as well as for Rician fading, for both fixed K as well as for random K, for $\rho = 0, -0.3, -0.6, -1$. Notice first that BVTC MREC can indeed take advantage of the diversity gain and thus greatly outperforms BF. Then, as-

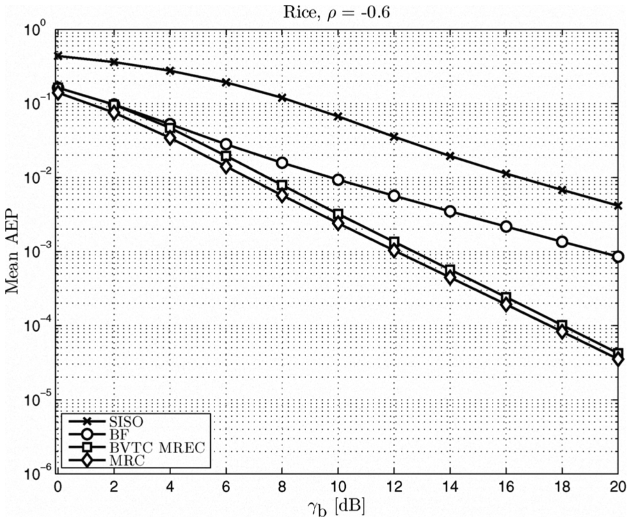

Fig. 4. Average (over noise, fading, AS and K) error probability vs. bit-SNR, for SISO, BF, BVTC MREC, and MRC, for perfectly known Rician fading channel gains, for random and correlated AS and K-factor, with $\rho = -0.6$.

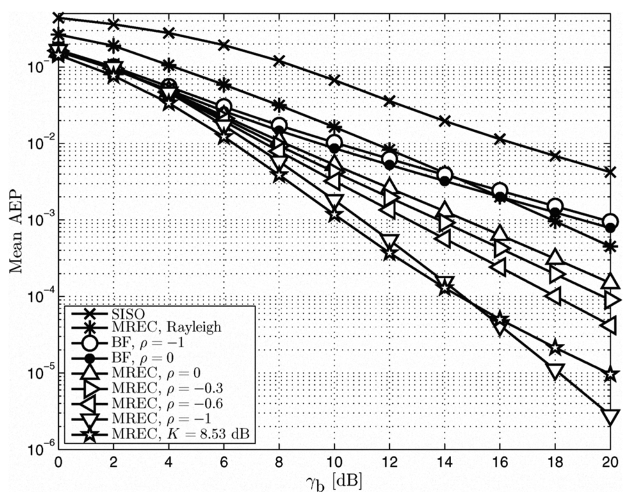

Fig. 5. Average (over noise, fading, AS and, unless otherwise stated, over K) error probability vs. bit-SNR, for SISO, BF, and BVTC MREC, for perfectly known channel gains with Rayleigh and Rician fading.

suming Rayligh fading may yield significantly underestimated performance indications compared to Rician fading for random K. Furthermore, Rician fading with K fixed to 'typical' values may yield significantly overestimated performance indications. Finally, indicated performance improves with increasing AS – K correlation magnitude. The reason is that the increasing inverse correlation means that there will be fewer (AS, K) sample pairs for which both AS and K are have low values.

V. Summary and Conclusions

We have shown a new analysis of maximal-ratio eigencombining (MREC) that applies for SISO, statistical beamforming (BF), maximal-ratio combining (MRC), correlated Rice fading, for perfectly-known channel gains and eigenstructure. We have used the derived average error probability (AEP) expression to compare SISO, BF, MRC, and MREC performance for correlated Rice fading and perfectly-known channel. Adaptive MREC is shown capable to approach MRC-like performance. Future work will evaluate also the complexity savings of MREC vs. MRC for Rician fading and estimated channel. We have also looked at the effect of the correlation between the azimuth spread (AS) and the Rician K-factor on the performance displayed by SISO, BF, MRC, and MREC. For random AS, setting $K \neq 0$ may produce overly-optimistic results for all combiners. On the other hand, for AS – K correlation values recently reported based on measurements, e.g., $\rho = [-0.6, -0.3]$, MREC and MRC display $1 - 3$ dB better performance at $2 \cdot 10^{-4}$ mean-AEP compared to $\rho = 0$ (i.e., the value typically considered in previous assessments). Future work will attempt to evaluate the effect of the direction of the deterministic channel component on performance, and will assess OFDM performance for realistic correlation between coherence bandwidth, AS, and K-factor.

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