Optimization of Rateless Coding for Multimedia Multicasting

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Abstract—Fountain codes are a class of recently proposed efficient erasure error-control codes. However, the original design of fountain codes, despite being highly efficient for erasure channels, has very poor progressive decoding performance. The purpose of this paper is to design and optimize unequal error protection (UEP) fountain codes to maximize the throughput of multimedia communication systems while providing different levels of guaranteed QoS for heterogenous users. A random interleaved rateless code design is proposed to solve the formulated constraint optimization problem. Numerical results demonstrate the advantage of the proposed design with optimal parameters compared to existing UEP designs.

I. INTRODUCTION

Fountain codes, also known as rateless codes, are a class of recently proposed efficient erasure error-control codes. Fountain codes are rateless in the sense that the transmitter can generate, as needed, potentially infinitely many encoded symbols. Fountain codes are universally capacity approaching for binary erasure channels (BEC) [1]. Therefore, they are particularly suitable for multicasting to multiple users with different receiving capabilities and channel erasure rates. As one of the most successful implementations of the rateless fountain codes, Raptor codes [2] have been standardized as application layer forward error correction (FEC) codes for file downloading services in the third generation partnership program (3GPP) Multimedia Broadcast/Multicast Services (MBMS) [3].

Real time media streaming applications, while benefitting from fountain codes, pose challenges because of delay constraints. Unlike multicasting bulk file data, the number of encoded symbols the fountain encoder produces for each data block is limited in video multicasting due to delay requirements of end users. On the other hand, multimedia streaming services do not require reconstruction of all the source data in each block.

In addition, the source data of multimedia streams are usually not created with equal importance. Users can view/play-back multimedia content at a basic quality when the most import bits are recovered and play-back quality is progressively increased with newly-recovered less important bits. However, the original design of Raptor codes is optimized in order to recover all of the source data block with minimum overhead. Experimental results show that traditional Raptor

codes have very steep error curves [1], meaning that very few bits are recoverable until the number of successfully received symbols exceeds the number of source information symbols in each block. Therefore, multimedia streaming applications demand unequal error protection (UEP) Raptor codes that take priority of the source data into consideration. In particular, it is required that more important data are more likely to be decoded when users collect fewer than enough packets to fully decode the data.

In the literature, a few techniques have been proposed for unequal error protection design of Raptor codes. In [4], the UEP Raptor codes are designed by non-uniformly choosing message bits to encode each symbol, and the design improves performance of more important bits at the expense of slightly decreased overall performance. However, these existing UEP Raptor code designs have not been optimized for the application of multimedia multicasting and do not give users Quality of Service (QoS) guarantees. In [5], expanding window fountain (EWF) codes have been applied for multicasting data. The idea of EWF codes is to encode each symbol based on only source symbols inside a window. The windows are pre-designed in an overlapping and expanding manner such that any larger window contains all the symbols inside a smaller window. However, the designs in [4] and [5] have inherent disadvantages: both alter the overall degree distribution and therefore change code behavior significantly. It is well known that both performance and complexity are sensitive to the choices of degree distribution. Therefore, without re-optimizing the degree distributions, the above designs may worsen code behavior.

In this paper, we consider the application of the UEP Raptor codes in multimedia multicasting. We formulate a general problem of UEP Raptor code design for video or image multicasting over different classes of users with different receiving capabilities. The problem addressed here is different from that in [5]. In [5], the authors focus on, for a given transmission overhead, optimizing the EWF code to maximizing the overall video/image quality of end users. In this paper, we focus on optimizing the UEP Raptor codes to minimize the cost at the server, i.e., the required transmission overhead, subject to providing users with QoS guarantees. We also propose a simpler and more modular UEP Raptor code design which encodes different layers separately and is convenient to analyze. The main advantage of the proposed layered design compared to

the design in [5] and others is that it allows direct application of existing high performance Raptor codes, such as the Raptor code used in the 3GPP standard [3], without degrading their performance. In addition, the receiver can decode each layer separately and therefore decoding complexity is significantly reduced. Using the proposed layered UEP Raptor codes, the original formulated problem is transformed into a convex optimization problem and can be further simplified based on Raptor code properties. The optimal configuration parameters are then obtained analytically. In [5], the algorithm needs to search and compare different parameter sets in order to find the optimum, a complex task when the number of user classes is large. Finally, we provide examples as well as comparisons with existing UEP Raptor code schemes. It should be noted that the number of bits in each source layer is given and no attempt is made to re-allocate source bits to layers. This allocation is a major focus in [6], which uses a different packetization structure and addresses a different aspect of the multimedia multicasting problem.

The rest of the paper is organized as follows: Section II describes the system setup and channel models; Section III presents the proposed UEP Raptor code design and problem formulation; Section IV discusses the solution illustrated by an example; Section V provides analytical results and comparisons to other UEP schemes.

II. SYSTEM SETUP AND CHANNEL MODELS

In the system under consideration, a multimedia server first compresses video or images and then encodes the source information using a UEP Raptor code. The encoded symbols are multicast over a wireless lossy packet network. Let K represent the number of input symbols in a Raptor code source block. The server transmits $(1+\varepsilon) \times K$ encoded symbols to end users in each block before moving to the next source block, where ε is the transmission overhead. Based on the importance of each source symbol, the Raptor code source block is divided into L layers, where layer 1 contains the most important bits and layer L contains the least important bits. For example, for a compressed video or image file, layer 1 can represent the base layer, layer 2 can be the first enhancement layer etc. Let S_l represent the number of bits in layer l. Thus we have $S_1 + S_2... + S_L = K$.

The users that subscribe to the multimedia streaming services are categorized into different classes due to different reception capabilities. Assume that there are J classes of users: Class 1, Class 2, ..., and Class J. For class j users, the reception capability δ_j is defined as the proportion of symbols that the receiver can successfully receive compared to the number of transmitted symbols. Therefore, in each transmission session, the number of successfully received encoded symbols for each user in class j is $\delta_j \times (1+\varepsilon) \times K$. In addition, without loss of generality, we assume that class 1 has the lowest channel quality and that class J has the highest channel quality. Hence $\delta_1 < \delta_2... < \delta_J$. The reception capabilities are determined by the channel quality between server and receiver. For example, a class 1 user may represent

a mobile cell phone that has limited reception quality due to small size and power restrictions, while a class 2 user may represent an automobile equipped with a large antenna and hence has better reception quality.

Traditionally, to accommodate users with very low reception capabilities, the transmitter has to either transmit a large number of repair symbols which results in very low throughput or to ignore user classes with the worst channel conditions. Therefore, the objective of the UEP Raptor code design is to provide different levels of QoS guarantees corresponding to different user classes while minimizing transmission overhead. Ideally, the UEP Raptor coder would provide a progressive increase in QoS guarantees as the channel condition of users improves. We consider the outage QoS guarantee where each class of users is guaranteed to be able to recover a given portion of source data with relatively high probability. For each transmission block, let $PSNR_j$ represent the peak signal-to-noise ratio ($PSNR$) of the successfully recovered source data of a class j user. Then the outage QoS guarantee can be represented by $Prob(PSNR_j \geq \gamma_j) \geq P_j$ for $j = 1, 2, ...J$ where γ_j denotes a threshold and target outage probabilities $1 - P_j$ $(0 < P_j < 1, j = 1, 2, ..., J)$ are constants. Note that we use $PSNR$ as a measurement of reconstructed signal quality of video/image content. However, the same formulation is easily applicable to other QoS measurements such as end-to-end distortion.

III. PROPOSED DESIGN AND PROBLEM FORMULATION

We propose a random interleaved UEP Raptor code design. In this design, we separately encode each layer and then interleave them. The encoding is performed in the following way: for each Raptor encoded symbol y, we first pick a layer number l according to probability ρ_l where $\rho_l, l = 1, 2, ..., L$, are a set of probabilities that satisfy $\sum_{l=1}^{L} \rho_l = 1$. Then, the encoded output symbol is generated by a Raptor encoder with code dimension S_l, degree distribution $\Omega_l(x)$, and precode C_l and input symbols that are limited to only symbols within layer l. Therefore, the overall encoded data stream is an interleaved stream of Raptor encoded symbols from each layer (Fig. 1).
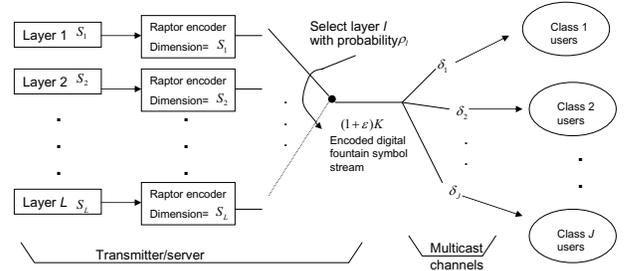

Fig. 1. Diagram of the layered UEP Raptor codes

The objective of our UEP Raptor code design can be summarized as the following problem:

Problem P1.0: find a set of probabilities $\rho_i, i = 1, 2, ..., L$, to minimize transmission overhead ε, subject to

$\sum_{i=1}^{L} \rho_i = 1$, such that for a class j user with reception quality δ_j, $Prob(PSNR_j \geq \gamma_j) \geq P_j$ for $j = 1, 2, ...J$. The aim is to effectively allocate coding rates across the layers through optimization of the probabilities.

The reconstruction quality of a video or image source coder is usually progressive, e.g, determined mainly by the bit errors in the lowest layer encountered in the recovery process. Let $q_l, l = 1, 2, ..., L$, represent the $PSNR$ that the receiver can achieve when layer 1 to layer l are successfully recovered in the Raptor decoding process. We thus have $q_1 \leq q_2 \leq q_3... \leq q_L$. The values of q_l are determined by the design of the source coder. Let $g_j \in 1, 2, ..., L$ ($j = 1, 2, ...J$) be the minimum index that satisfies $q_{g_j} \geq \gamma_j$. In order to satisfy $PSNR_j \geq \gamma_j$, the users in class j require the Raptor decoder to successfully decode at least layers 1 to g_j. For a UEP Raptor code design, let $P_e(l, j)$ represent the probability that the decoder fails to fully decode layer l given the transmission overhead ε and the reception quality δ_j. Then QoS requirements of end users can be simplified to

$$\prod_{l=1}^{g_j}(1 - P_e(l, j)) \geq P_j \quad j = 1, 2, ..., J. \tag{1}$$

The key in designing UEP Raptor codes formulated according to Problem 1.0 is to find the relationship between the UEP Raptor code design parameters and $P_e(l, j)$.

In the above general problem, L and J are arbitrary positive integers and we make no assumption that the QoS requirements of the J classes of users are monotonically increasing as their channel qualities increase [1]. However, we can match the total of J classes of users to the L layers according to their QoS requirements by the following process: for any pair of classes i and k where $i < k$, if class i has similar or higher QoS constraint than class k (i.e., $g_i \geq g_k$), we drop class k and its QoS constraint from the original problem and form a new problem dealing with one less class. The new problem is equivalent to the original problem because if the QoS constraint of class i is satisfied, the less demanding QoS constraint of class k, which has better channel quality than class i ($\delta_i < \delta_k$), is automatically satisfied. On the other hand, if there exists a layer l where there is no corresponding user class (i.e., no j exists such that $g_j = l$), we can group layers l and $l+1$ to form a new layer l' with dimension $S_{l'} = S_l + S_{l+1}$, since rate allocation among layer l and layer $l+1$ is irrelevant to the users' QoS requirements.

The above mapping process transforms the original problem into an equivalent problem with only constraints from a subset of the most demanding users. The new problem has a new set of L' layers and a new set of J' user classes, with $L' = J'$ and $g_j = j$, i.e., user class j requires the decoder to be able to decode layers 1 to j with probability larger than P_j'. For notational simplicity, we omit the $'$ symbols in the remainder of the paper.

Problem P1.1: given the reception quality of each user class $\delta_j, j = 1, 2..., L$, and maximum QoS outage probabilities $1 - P_j, j = 1, 2..., L$, find a set of probabilities $\rho_1, \rho_2, ..., \rho_L$ ($\sum_{i=1}^{L} \rho_i = 1$), to minimize the transmission overhead ε, such that

$$\prod_{l=1}^{j}(1 - P_e(l, j)) \geq P_j \quad j = 1, 2, ..., L \tag{2}$$

Raptor codes used in the 3GPP standard, known as standardized Raptor codes, can be directly applied to the Raptor encoder blocks of our proposed layered design. Standardized Raptor codes are designed such that they are systematic, offer high performance, and can be decoded very quickly for a wide range of code dimensions. For details about the pre-code, the degree distribution and the construction of standardized Raptor codes, readers can refer to [3] (Annex B). When standardized Raptor codes are used, for $k > 200$, the probability that the receiver fails to fully recover k source symbols after m symbols are successfully received can be well modeled by the empirically determined equation [7],

$$P_e(m, k) = \begin{cases} 1 & \text{if } m \leq k \\ a \times b^{m-k} & \text{if } m > k \end{cases} \tag{3}$$

where a and b are constants given by $a = 0.85$, $b = 0.567$. Note that for $k < 200$, the efficiency of a Raptor code will be lower than Eq. (3) suggests, which is as expected. One way to improve the code performance when the number of bits in each layer is very small is to group several source layers with similar optimized values of ρ_j into one larger layer for Raptor encoding. For high-bit-rate media such as video, the condition $k < 200$ is unlikely to occur. In addition, as the results in Section V suggest, the proposed scheme still performs better than existing schemes with the same Raptor code applied.

For a given transmission overhead ε, the total number of encoded symbols transmitted for layer l in each transmission block is $t_l = (1 + \varepsilon) \times K \times \rho_l$, [2] and satisfies $\sum_{l=1}^{L} t_l = (1+\varepsilon) \times K$. The number of encoded symbols of layer l received by users in class j with reception quality δ_j is then $r_j^l = t_l \times \delta_j = (1 + \varepsilon) \times K \times \rho_l \times \delta_j$. When standardized Raptor codes are used, substituting $m = r_j^l = t_l \times \delta_j$ and $k = S_l$ into Eq. (3), the failure probabilities $P_e(l, j)$ can be re-expressed as

$$P_e(l, j) = a \times b^{(t_l \times \delta_j - S_l)}. \tag{4}$$

Therefore, the original problem is transformed to:

Problem P1.2: Given $\delta_i, i = 1, 2, ..., L$, where $0 \leq \delta_1 \leq \delta_2... \leq \delta_L \leq 1$, $P_i, i = 1, 2, ..., L$, where $0 \leq P_i \leq 1$ and $S_i, i = 1, 2, ..., L$, find variables $t_i, i = 1, 2, ..., L$, such that $\sum_{i=1}^{L} t_i$ is minimized, subject to

$$\prod_{l=1}^{j}(1 - a \times b^{(t_l \times \delta_j - S_l)}) \geq P_j \quad j = 1, 2, ..., L. \tag{5}$$

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[1] From a system designer perspective, however, it is best to provide users with QoS guarantees in line with their channel qualities. A highly disparate user with a very poor channel but a high QoS demand should be grouped to a different multicast session

[2] Strictly speaking, for each implementation, t_l is a Binomial distributed random variable with mean $(1 + \varepsilon) \times K \times \rho_l$. However, the randomization of t_l has little effect on the problem of interest when averaged out over a large number of implementations. In addition, one can always monitor the selection of layers to make sure that t_l is proportional to ρ_l in every implementation.

The constraint that t_l is a non-negative integer is not explicitly shown here. However, the non-negativity is implicitly guaranteed by the constraints. To ensure the solution of t_l is an integer, we can always compute t_l as if it is a continuous variable, then round it to a nearest integer that is larger than the computed solution. When more general Raptor codes with a degree distribution $\Omega(x)$ (for analytical simplicity, no precode is considered) are used, the failure probability $P_e(l, j)$ can be approximated by

$$P_e(l, j) = 1 - (1 - e_{l,j})^{S_l}, \tag{6}$$

where $e_{l,j}$ is bit error probability of a class j user successfully decoding layer l. This approximation is based on the asymptotic assumption that the bit errors of the iterative decoder are independent of each other. The bit error probability $e_{l,j}$ of the iterative decoder can be analytically measured by "and-or" tree analysis as in [4]. Applying the same and-or tree technique to our layered design, the probability that a single bit in layer l has not been successfully recovered by users in class j using an iterative decoder after n iterations, $e_{l,j}^n$, is

$$e_{l,j}^0 = 1 \tag{7}$$

$$e_{l,j}^n = exp(-(1+\varepsilon)K\rho_l\delta_j\Omega'(1 - e_{l,j}^{n-1})) \ \ n \geq 1, \tag{8}$$

where Ω' denotes the derivative of Ω. The asymptotic bit error probability after an infinite number of decoding iterations $e_{l,j} = \lim_{n \to \infty} e_{l,j}^n$. In practice, $e_{l,j}$ can be obtained by choosing a large n in Eq. (8).

IV. Solving the Problem

Problem $P1.2$ can be transformed into a convex optimization problem. For convenience, let $c_l = a \times b^{-S_l}$ and let $\alpha_j = b^{\delta_j}$, and Problem $P1.2$ can be rewritten as:

Problem P1.3:

$$\min_{t_1,...,t_L} \quad \Sigma_{i=1}^L t_i \tag{9}$$

$$s.t. - \Sigma_{l=1}^j \log[1 - c_l\alpha_j^{t_l}] + \log P_j \leq 0, \quad j = 1, 2, ..., L. \tag{10}$$

The convexity of the constraint functions can be proved by using the seconder order convexity condition [8]. For a convex optimization problem, $P1.3$ can be easily solved numerically. However, in many practical cases, the above problem can be further simplified. Since class l users have to decode the corresponding layer l with high probability, it is reasonable to assume users in subsequent classes with better receiving capabilities will likely be able to decode layer l with very high probability since the Raptor code error rate curves are very steep [7]. As a result, it is much simpler to find the solution by assuming that the optimal solution occurs when all the inequality constraints are active, i.e., all inequality constraints achieve equality at the optimal solution point. The optimality of the solution obtained from the above assumption can be verified by checking Karush-Kuhn-Tucker (KKT) optimality conditions of the primal and dual problems of $P1.3$ [8].

In the following example we illustrate the problem formulation and solution. We assume that the original source

file contains three layers with $S_1 = 1000$, $S_2 = 3000$ and $S_3 = 6000$ bits. There are also three classes of users: class 1, 2, and 3 require a minimum playback quality that needs to correctly decode up to layer 1, 2 and 3 with probabilities larger than $P_1 = 0.9$, $P_2 = 0.8$ and $P_3 = 0.7$, respectively. The channel reception qualities for classes 1,2 and 3 are $\delta_1 = 0.4$, $\delta_2 = 0.8$ and $\delta_3 = 1.0$, respectively. For the UEP Raptor code design, we encode layer 1, 2 and 3 using separate standardized Raptor codes with dimension S_1, S_2 and S_3, respectively. Denote the portions of bits in the encoded symbols to be ρ_1 for layer 1, ρ_2 for layer 2 and ρ_3 for layer 3 , respectively.

Therefore, the problem is to find ρ_1, ρ_2 and ρ_3 to minimize $t_1 + t_2 + t_3$. such that

$$\begin{cases} (1 - a \times b^{(t_1 \times \delta_1 - S_1)}) \geq P_1 \\ (1 - a \times b^{(t_1 \times \delta_2 - S_1)}) \times (1 - a \times b^{(t_2 \times \delta_2 - S_2)}) \geq P_2 \\ (1 - a \times b^{(t_1 \times \delta_3 - S_1)}) \times (1 - a \times b^{(t_2 \times \delta_3 - S_2)}) \\ \times(1 - a \times b^{(t_3 \times \delta_3 - S_3)}) \geq P_3 \end{cases} \tag{11}$$

Assuming all the inequality constraints are active, each inequality can be solved independently, and we easily obtain that the minimum overhead $\varepsilon_{min} = (t_1 + t_2 + t_3)_{min}/K - 1 = 0.2266$, which is achieved when $\rho_1 = 0.20463$, $\rho_2 = 0.30605$ and $\rho_3 = 0.48932$. Upon verification of the solution using KKT conditions, we found that this solution is indeed optimal. Note that an equal error protection (EEP) allocation gives a minimum overhead 1.5100 which is about seven times higher than the optimal UEP solution.

Remark: In the above, the problem is formulated in a static model where the reception capabilities and QoS demands of users do not change over time. However, the above formulation can also be utilized in a dynamic model. If the user reception capabilities δ_j change slowly, the configuration parameters ρ_l and minimum transmission overhead ε can be adjusted periodically according to feedback, which is usually available given the limited number of user classes and the slowly changing user dynamics. In this way, the overall throughput is maximized.

The paper considers the model where the number of received symbols for each user class is equal to the user reception capability times the transmission volume, which is the same as that used in the EWF code in [5]. For some multimedia streaming systems, there is an upper limit ε_0 on the transmission overhead because of end users' delay requirements. In this paper, we generally assume that ε_0 is larger than the optimal solution of the minimum overhead ε. For systems that want to utilize the maximum allowable transmission overhead to provide additional best-effort QoS, the optimal parameters ρ in the above problem can still provide a suboptimal solution that proportionally increases users' QoS according to their demands. For situations where the optimal minimum transmission overhead is found to be larger than the limit ε_0, no feasible solution is available for the given requirements. However, the system designer can choose to refer the most demanding user class to a different multicast server and the optimal configuration parameters can be recalculated without the constraint of this user class.

V. NUMERICAL RESULTS

In order to demonstrate the advantage obtained by optimization, we first compare the optimized UEP Raptor code scheme to an EEP Raptor code scheme. For simplicity, only two layers are considered. In these comparisons (Figs. 2 to 4), the proposed layered UEP design employing standardized Raptor codes is used. The code dimension of the standardized Raptor code used in layer l is S_l. The small inefficiencies of the standardized Raptor codes are characterized by the failure probability P_e in Eq. (3). The optimal configuration parameter ρ_1 and minimum overhead for the UEP scheme are obtained by the simplified method described in Section IV for solving Problem P1.3, i.e., by assuming all the inequality constraints are active. Note that all the results of the optimized UEP scheme shown in Figs. 2 to 4 satisfy the KKT optimality conditions after verifications. The EEP scheme allocates each encoded symbol such that every information symbol has the same priority. Therefore, the ratio ρ_1/ρ_2 is fixed to S_1/S_2. For this predesigned ρ_1/ρ_2, we find the minimum transmission overhead required to satisfy all the users' QoS constraints. For all the results shown in this section, the parameters chosen are described in the caption of each corresponding figure. Fig. 2 shows the minimum transmission overhead requirement for optimized UEP and EEP Raptor codes as the ratio between the number of bits in the two layers S_1/S_2 is varied. Fig. 3 shows the same comparison when the channel reception quality of the first class δ_1 varies. It can be seen that the UEP scheme has a significant advantage over the EEP scheme in almost all cases. When the channel reception qualities of the two classes approach each other, EEP approaches the optimized UEP in performance.

Fig. 4 shows the minimum overhead when different predesigned layer allocation parameter ρ_1 values are used. It can be seen that the minimum required transmission overhead is very sensitive to the choice of ρ_1, and an arbitrary non-optimized allocation scheme can perform much worse than the optimal allocation scheme and even EEP.

Fig. 3. UEP and EEP comparison. $L = 2; K = 9000; S = [1000, 8000]; \delta_2 = 0.8, P = [0.95, 0.8]$.

Fig. 4. The effect of layer allocation probability ρ_1. $L = 2; K = 9000; S = [1000, 8000]; \delta = [0.4, 0.8]; P = [0.95, 0.8]$.
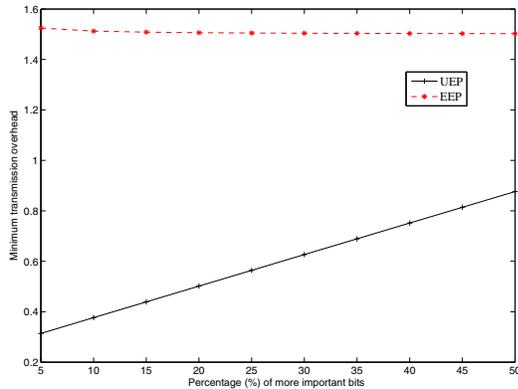

Fig. 2. UEP versus EEP. $L = 2; K = 9000; \delta = [0.4, 0.8]; P = [0.95, 0.8]$.

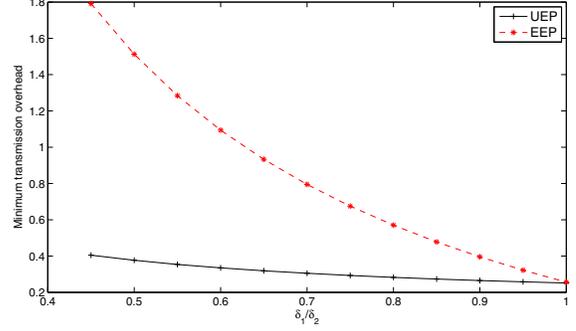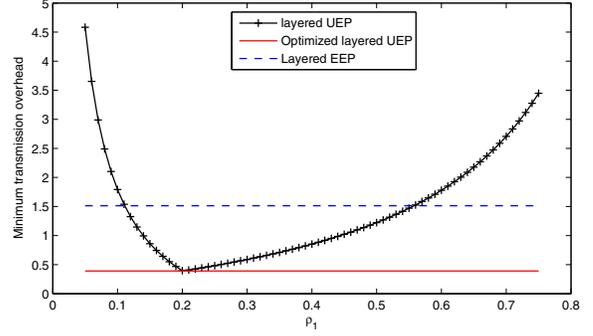Next, to investigate the performance of the proposed layered

UEP Raptor code scheme, we compare its performance to a recent UEP Raptor code design by Rahnavard *et. al.* [4] as well as EWF codes by Vukobratovic *et. al.* [10]. Because of their non-modular designs, Rahnavard's UEP Raptor code and the EWF code cannot deploy existing standardized Raptor codes directly. Therefore, to make fair comparisons, instead of using standardized Raptor codes, we use Raptor codes with degree distribution $\Omega(x)$

$$\Omega(x) = 0.007969x + 0.493570x^2 + 0.166622x^3$$
$$+0.072646x^4 + 0.082558x^5 + 0.056058x^8 + 0.037229x^9$$
$$+0.055590x^{19} + 0.025023x^{65} + 0.003135x^{66}. \quad (12)$$

(Eq. (12)) for all the layers of our proposed layered UEP code in Figs. 5 and 6. It should be stressed that the advantage of being able to utilize high performance standardized Raptor codes is not shown in these comparisons, which would further favor the proposed design. This degree distribution $\Omega(x)$ is originally designed from [2] and has been adopted by Rahnavard's UEP scheme in [4]. The same degree distributions $\Omega(x)$ are applied to Rahnavard's UEP Raptor code and all the windows of the EWF code. For analytical simplicity, no precode is used for all the three schemes in the comparisons in Figs. 5 and 6.

For all the three schemes, in order to find the minimum

transmission overhead, we increase the value of the transmission overhead (starting from 0) in very small increments until the constraints in Eq. (2) are all satisfied. The failure probability P_e on the left side of the constraint functions in Eq. (2) is evaluated as follows: the error probability of each layer in Rahnavard's scheme and the EWF code can both be estimated by the "and-or" Tree technique [4] [9]. The bit error probability e_l of layer l for Rahnavard's scheme, the EWF code, and the proposed layered UEP scheme are obtained using Eqs. (6) and (7) in [4], Eqs. (6) and (7) in [9], and Eqs. (7) and (8) in this paper, respectively. The failure probability of decoding each layer can then be estimated as $P_e(l) = 1 - (1 - e_l)^{S_l}$.

Fig. 5 shows the minimum transmission overhead required to satisfy all the user constraints of the proposed layered UEP scheme and Rahnavard's UEP scheme with different values for parameter k_M, which governs the degree of non-uniformity of input symbol selections (see [4]). It can be seen that the optimized layered UEP scheme outperforms Rahnavard's UEP scheme even if k_M is optimized. This is reasonable under the given QoS constraints as layered coding provides better guaranteed performance for the more important layers. Fig. 6 shows a similar comparison between the proposed layered scheme and the EWF code. The size of the first window in the EWF code is fixed to the number of symbols in layer 1 (S_1). Parameter Γ_1 is the probability of choosing the first window (the more important layer) during encoding (see [9]). It can be observed that the proposed layered scheme, when optimized, also performs better than the EWF code with optimized Γ_1. Note that there are two local minima in Fig. 6 because the evaluated bit error rates of the more important bits are not monotonically decreasing as Γ_1 increases (see Fig. 1 in [9]).

Fig. 6. Proposed layered scheme versus the EWF code. $L = 2; K = 9000; S = [1000, 8000]; \delta = [0.4, 0.8]; P = [0.95, 0.8]$.

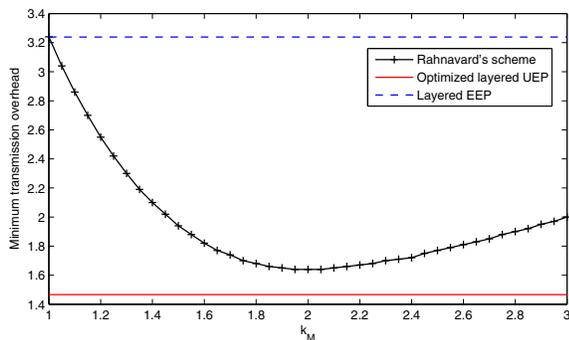
Fig. 5. Proposed layered scheme versus Rahnavard's scheme. $L = 2; K = 9000; S = [1000, 8000]; \delta = [0.4, 0.8]; P = [0.95, 0.8]$.

VI. CONCLUSIONS

In this paper, we formulated a general problem of optimizing UEP Raptor codes for scalable multimedia multicasting systems. The design objective is to minimize the transmission overhead given users with different receiving capabilities and different corresponding levels of QoS guarantees. A random interleaved UEP Raptor c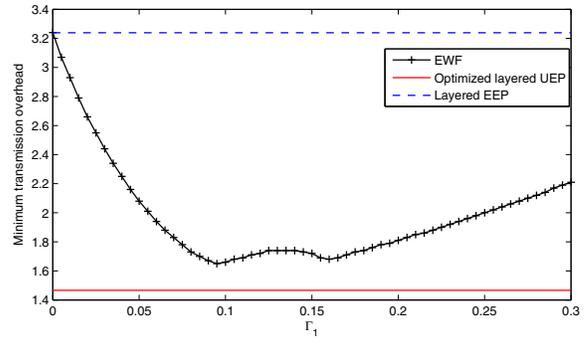ode design is proposed that can take advantage of the high performance of existing standardized Raptor codes. The formulated problem is converted into a convex optimization problem which can be solved numerically. Based on the properties of Raptor codes, the original problem is further simplified and the optimal configuration parameters can be found in closed-form. We demonstrated the advantage of the optimization by showing that the optimized layered UEP Raptor code design performs much better than EEP and non-optimized UEP Raptor codes. Furthermore, the optimized layered UEP Raptor codes requires less transmission overhead compared to existing UEP Raptor code designs for the same QoS guarantees.

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