Performance Evaluation of Multiband Multi-Sensor Spectrum Sensing Systems

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***Abstract*—Quickest detection theory has previously been applied to the problem of spectrum sensing. By detecting the onset of an idle channel period, quickest detectors minimize the time required to search for an idle period. These methods are based on detection of a single change point, which implies that change in the channel's usage state is assumed to occur only once. Since the channel state may transition continuously between busy and idle states via an ON-OFF process, an alternative formulation based on partially observable Markov decision processes (POMDP) has been recently proposed. In this paper, Page's cumulative sum sequential analysis method (CUSUM) and multiband multi-sensor spectrum sensing (MMSSD) based on POMDP are brought into a similar context and compared. Next, the ON-OFF process model itself is assessed. The POMDP formulation assumes an ON-OFF process model where the busy and idle periods are geometrically distributed. While this model is desirable for its analytical tractability, its applicability to reflect the dynamics of actual spectral usage, which are derived from real data traffic, is assessed.**

I. Introduction and Background

Spectrum sensing has been a widely researched topic as it contributes to better spectrum utilization. It concerns the problem of determining the occupancy of one or multiple spectrum bands. Research related to spectrum sensing is naturally cast as a detection problem, typically through energy or feature detection.

The application of a sequential detection framework to spectrum sensing problems has been a subject of recent research because it optimizes system agility, i.e., the time required to perform a detection. A sequential detection system generates a decision with observations made in each time slot, and processing continues until a prescribed level of reliability is achieved. In [1], the sequential probability ratio test (SPRT) detector is proposed as a spectrum sensing method where it has been shown that detection delay time is reduced compared to detection methods that require a fixed-block of observations. In [2], [3], the use of quickest detection, which is a form of sequential detection that specifically concerns detection of a change in a signal, is explored for application to spectrum sensing. Rather than determining whether a channel is busy or idle, a quickest detection system detects the onset of an idle channel period. The quickest detection formulation considered in [2], [3] assumes that the time where a change in the distribution occurs is unknown and nonrandom, which implies that statistical dynamics that determine when a signal changes are not taken into consideration. On the other hand, Shiryaev's Bayesian quickest detection formulation in [4] assumes the time that the change occurs is random and incorporates a priori knowledge about the probability distribution of the change time.

The quickest detection problem formulations in [2]–[4] assumes that the process experiences only a single change in the time horizon of the detection process. However, spectrum usage is more realistically modelled as an ON-OFF process that consists of multiple busy and idle periods, and has multiple changes of channel state. By assuming a single change point, the formulation in [2]–[4] omits the false alarm situation where the detector declares a channel to be idle after the channel has exited the idle state.

This motivates the problem to be re-cast into a new problem using the framework of partially observable Markov decision processes (POMDP) [5]. Using POMDP, an alternative quickest detector is proposed in [5] where multiple channels are modelled as parallel ON-OFF processes and a single sensor can be deployed to sense unoccupied spectrum in an arbitrary number of channels.

In our previous work [6], the POMDP formulation as presented in [5] is extended to include an arbitrary number of sensors, and a multiband multi-sensor spectrum sensing and reduced-complexity generalization is proposed. By using extra sensors, it has been shown that agility of detection is improved over the detection systems in [5]. However, performance of the POMDP solution in [5], [6] has not been compared to standard quickest detection schemes.

The problem formulations in [5], [6] model the ON-OFF process as a set of time instants where the channel changes its state. Similar to [4], the POMDP formulation assumes that the time intervals between a pair of successive change points are random and geometrically distributed. This is the result of the assumption that users enter into and exit from the channel according to an exponential ON-OFF model. This model is adopted due to its analytical tractability rather than its ability to accurately represent real-world traffic dynamics. In [7]–[10], many traffic models that statistically characterizes spectral usage patterns have been proposed. In particular, the traffic model developed in [10], which is experimentally derived from traffic conditions measured in a WLAN-based network, provides an accurate model for the ON-OFF process.

The main contribution of this paper is to conduct further performance evaluation of the systems developed in [6]. In our comparison, Page's cumulative sum sequential analysis method (CUSUM) is used for the quickest detection system under consideration. In addition, as in [5], the proposed detector is applied to idealized random traffic through the formulation of the POMDP. In this paper, the performance of the systems in [6] are evaluated using a more realistic statistical traffic model, which is based on a WLAN network that has been verified in [10].

The rest of the paper is organized as follows: Section II describes the problem formulation and introduces the proposed detectors. Section III presents the spectral sensing system being compared and the comparison results of the simulation.

II. Problem Formulation

A. On-OFF Markov Process Model

Consider a spectrum sensing system with M sensors that tries to opportunistically access unused spectrum within L channels that may be occupied by other wireless systems, such as a primary network. The l^{th} channel, where $1 \leq l \leq L$, is modelled as a discrete-time ON-OFF process which consists of periods of idle and busy states. As suggested in [4], [5], it is assumed that occupants emerge in and exit from the channel according to an exponential ON-OFF model, which means that durations of idle and the busy states are random, and in a discrete-time system, are also geometrically distributed. Hence, given that the l^{th} channel is in a busy state, or in an idle state, the channel can switch to a different state with the probability of p_B^l, or respectively p_I^l. An assumption is made that the traffic dynamics are the same across all L channels, thus, $p_I = p_I^l$ and $p_B = p_B^l$ for all l's. From p_B and p_I, the mean idle time m_I, mean busy time m_B, and the fraction of idle time λ_o are defined as, respectively,

$$m_B = 1/p_B, \tag{1}$$

$$m_I = 1/p_I, \text{ and} \tag{2}$$

$$\lambda_o = \frac{m_I}{m_I + m_B}. \tag{3}$$

The assumption of an exponential ON-OFF model is adopted for its analytical tractability. Later, in the simulation results, this assumption will be tested. The signal observed from the l^{th} channel at time slot t, denoted to be $X_l(t)$, is $X_l(t) = gs(t) + n(t)$ in the busy state, and $X_l(t) = n(t)$ in the idle state, where g is the slow fading channel coefficient, $s(t)$ is a signal transmitted by the primary user, and $n(t)$ is zero-mean additive white Gaussian noise (AWGN) with power σ^2. Together with the channel coefficient, the term $gs(t)$ is assumed to be a zero-mean Gaussian random variable with variance P. It is assumed that both P and σ^2 can be estimated at the receiver and hence are given.

In each time instant, the spectrum sensing system chooses M out of L channels to observe, and based on signal energy observations and parameters mentioned above, decides to either declare a discovery of an idle period, or to continue to observe. Let $T_{declare}$ be the random elapsed time from the beginning of the detection process to when a discovery is declared, and let P_{FA} be the false alarm probability of declaring a channel idle when the state of the channel is busy.

Given false alarm constraint α, the design criterion is:

$$\begin{aligned} &\min E\left[T_{\text{declare}}\right] \\ &\text{subject to } \mathrm{P}_{FA} \leq \alpha. \end{aligned} \tag{4}$$

B. POMDP Formulation

The assumption based on the exponential ON-OFF traffic model allows casting the problem into a POMDP problem. Details of the POMDP formulation can be found in [5], [6], where detection involves a single sensor and L channels. In [6], the formulation extends the results in [5] to incorporate an arbitrary number of sensors when detecting an idle channel across L channels. There are several significant extensions to the original formulation in [5]. The first extension is that the action space, i.e., set of all possible actions at time t, $\{a_t\}$, is expanded to include a new set of *Continue* actions, denoted as $\mathcal{C}$. The subset $\mathcal{C}$ contains actions such as $(C_{k_1}, ..., C_{k_M})$ that represent the action to continue to observe the M channels with indices $k_1, ..., k_M$, where each index k_m, for $1 \leq m \leq M$, is the index of the channel being sensed by mth sensor. Due to the additional number of sensors, the observation model is modified such that the observation at time t contains M elements, which are denoted as $X_{k_1}(t), X_{k_2}(t), ..., X_{k_M}(t)$. Using M observations in each time instant, the extended formulation updates the sufficient statistics, or belief vector, $\mathbf{\Lambda}(t) \equiv [\lambda_1(t), ..., \lambda_L(t)]$, which represent the a posteriori probabilities that the state of the respective channel is idle.

C. Multiband Multi-sensor Spectrum Sensing Detection

The POMDP formulation in [6] leads to the development of two detectors that utilize M sensors to detect an idle period across L channels. The first detector is called *multiband multi-sensor spectrum sensing detector* (MMSSD), which (1) performs detection over multiple channels; (2) simultaneously senses; and (3) tracks the belief values for all channels.

Let $D_l, 1 \leq l \leq L$, be the action of declaring the l^{th} channel idle, and $(C_{k_1}, ..., C_{k_M})$ be the action to continue the detection in channels with indices $k_1, ..., k_M$. The MMSSD has the following structure for the case of general M and L:

$$\pi_{\text{MMSSD}}(\mathbf{\Lambda}(t)) = \begin{cases} D_l, & \text{if } l = \underset{1\leq j\leq L}{\arg\max}(\mathbf{\Lambda}(t)) \\ & \text{and } \lambda_l \geq \eta_d(\mathbf{\Lambda}^-(t)) \\ (C_{k_1}, ..., C_{k_M}) & \text{Otherwise,} \end{cases} \tag{5}$$

where $k_1, ..., k_M$ denote the indices of the L channels that have the M largest belief values, $\mathbf{\Lambda}(t)$ is updated using the M observations, $\mathbf{\Lambda}^-(t)$ denotes the set of belief values for the $L - M$ unobserved channels at time t, and $\eta_d(\mathbf{\Lambda}^-(t))$ is a threshold function to determine when the detection process terminates. $\mathbf{\Lambda}(t)$ and $\mathbf{\Lambda}^-(t)$ are both functions of observations and the a priori parameters of the channels.

Each element $\lambda_l(t)$, $1 \le l \le L$, in the belief vector $\mathbf{\Lambda}(t)$ is updated using the following equation, and the update only occurs if the previous action has been one of the *Continue* actions:

$$\lambda_l(t) = \begin{cases} \widehat{\mathcal{T}}(\lambda_l(t-1), x) & \text{if observed}, X_l(t) = x \\ \widetilde{\mathcal{T}}(\lambda_l(t-1)) & \text{if not observed.} \end{cases} \quad (6)$$

If the l^{th} channel has been observed, the corresponding $\lambda_l(t)$ is updated using $\widehat{\mathcal{T}}(\lambda, x)$, which is

$$\begin{aligned} &\widehat{\mathcal{T}}(\lambda, x) \\ &= \frac{(\lambda\overline{p}_I + \overline{\lambda}p_B)f_1(x)}{(\lambda\overline{p}_I + \overline{\lambda}p_B)f_1(x) + (\lambda p_I + \overline{\lambda}\,\overline{p}_B)f_0(x)} \end{aligned} \quad (7)$$

where the operator $\overline{(\cdot)}$ is defined as $1-(\cdot)$, p_I and p_B are given a priori, and $f_0(x)$ and $f_1(x)$ are the probability density functions of an observation in either a busy or an idle state, respectively. Otherwise, the following update equation is used:

$$\widetilde{\mathcal{T}}(\lambda) = \lambda\overline{p}_I + \overline{\lambda}p_B. \quad (8)$$

In the structure (5), at each time t, the detector chooses the M channels with highest belief values to observe at time $t+1$. The detector continues to observe and update the belief values until one of the belief values crosses the threshold, $\eta_d(\mathbf{\Lambda}^-(t))$. Unfortunately, the computation of the function $\eta_d(\mathbf{\Lambda}^-(t))$ is intractable. Thus an alternative threshold design method is proposed here, where η_d is constant. For MMSSD to maintain its false alarm rate below a false alarm constraint α, it is sufficient that $\eta_d = 1-\alpha$ [6].

D. Reduced-Complexity Multiband Multi-Sensor Spectrum Sensing

A downside to MMSSD, which tracks the belief vector for all L channels and ranks the top M belief values out of all the L, is that it contains high costs of storage and computation complexity. As the number of available channels, L, grows large, MMSSD becomes less practical. This motivates the development of a variant of MMSSD, which is called reduced complexity MMSSD, or RC-MMSSD.

The complexity of RC-MMSSD is independent of number of available channels. The RC-MMSSD maintains a belief vector of size M, $[\lambda_1(t), ..., \lambda_M(t)]$, where $\lambda_m(t), 1 \le m \le M$, is the belief value that corresponds to the channel y_m, where $y_m \in (1, 2, ..., L)$ is a pointer to the channel to be observed by m^{th} sensor, for $1 \le m \le M$. With every iteration, the detector updates each belief value using the observation from its corresponding observed channels via (7). Based on the updated belief vector, the detector decides on the action at time t based on the policy in Table I.

RC-MMSSD has two thresholds: η_d and η_s, where the former is the stopping threshold and the latter is the switching threshold. The design of η_d is the same as MMSSD and is chosen to meet the false alarm constraint α. The threshold η_s

TABLE I

ALGORITHM FOR RC-MMSSD

Ω denotes the pool of unobserved channels at the current time t

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for m = 1 to M do
    if λ_m ≥ η_d then
        {Declare channel y_m as idle}
    else if λ_m < λ_o then
        {Switch out of channel y_m}
        y_m ← index of a new channel selected from Ω
        λ_m(t) ← λ_o {Resetting λ_m(t)}
        Put the index of the replaced channel back into Ω
    else
        {Continue to observe channel y_m}
    end if
end for
```

determines when a sensor switches into a new channel and is set to value λ_o [6].

In [6], it has been shown that the correct operation of RC-MMSSD requires the condition $L >> M$. If this condition is not met, the false alarm rate rises above the false alarm constraint α and the design rule $\eta_d = 1-\alpha$ becomes invalid. In practical scenarios, the situation with $L >> M$ is often the case in order to scan large numbers of channels. It also has been shown that as L grows much larger than M, the behaviour converges to that of RC-MMSSD and thus the performance of the two detection systems also converge [6].

III. Simulation Results

The performance of the proposed detectors MMSSD and RC-MMSSD are compared to CUSUM using Monte Carlo simulation. The signal model used in the simulation is described in Section II-A and the signal-to-noise ratio, where SNR$\equiv 10\log(P/\sigma^2)$, is chosen to be 10dB.

A. Comparison to Page's CUSUM

The performance of the MMSSD and RC-MMSSD is compared to that of Page's CUSUM [11]. Since CUSUM performs detection on a single channel at a time, in order to perform a comparison in the multiband context, a full sensing CUSUM detection system is developed.

A full sensing CUSUM detection scheme contains L sensors that simultaneously perform CUSUM detection on its assigned channels. Based on this scheme, CUSUM always performs detection in a full sensing configuration. The detection terminates when a sensor first declares a channel to be idle. To conduct a fair comparison between different detection systems, each detector is designed with the same false alarm rate of 0.01. The 95% confidence interval for each false alarm rate is estimated during simulation to ensure that they can be compared statistically. Since there is no closed-form expression to design CUSUM to satisfy a false alarm constraint, a search is used for a threshold that yields the desired false alarm rate of 0.01 is used.

There is an inherent incompatibility issue when applying CUSUM to the ON-OFF model. First, over the time horizon of an ON-OFF process, multiple transitions between idle

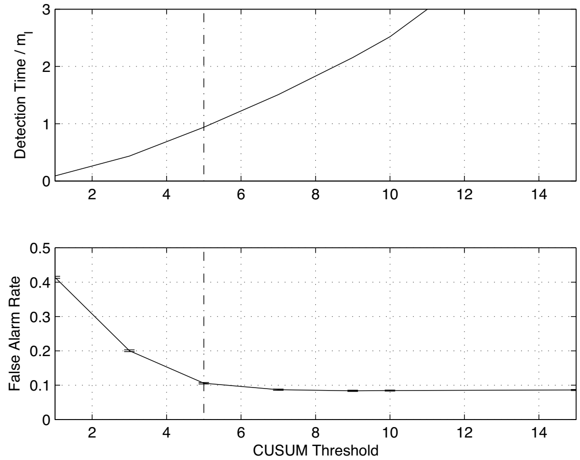

Fig. 1. The false alarm rates reach an error floor in spite of raising the detector threshold. The dashed line, i.e., where the ratio of average detection delay time to average idle time m_I in the top plot, crosses 1 in the top plot, indicates where average detection delay time begins to exceed the average idle time, and where the false alarm rates begin to flatten in the bottom plot.

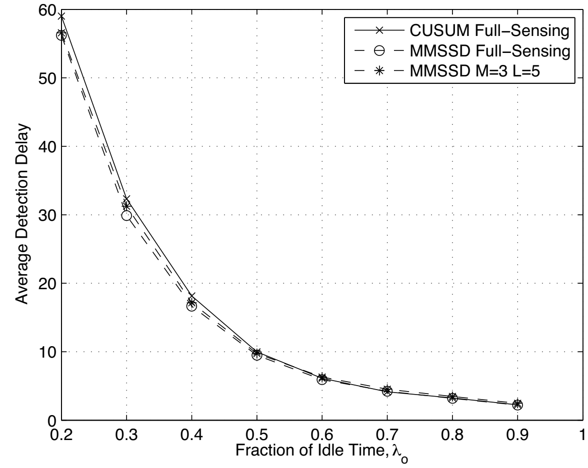

Fig. 2. A comparison of the average detection delay time between a full sensing CUSUM detection and MMSSD detection that uses 3 and 5 sensors. (SNR=10dB, $m_B = 620, L = 5, F_s = 10^6$).

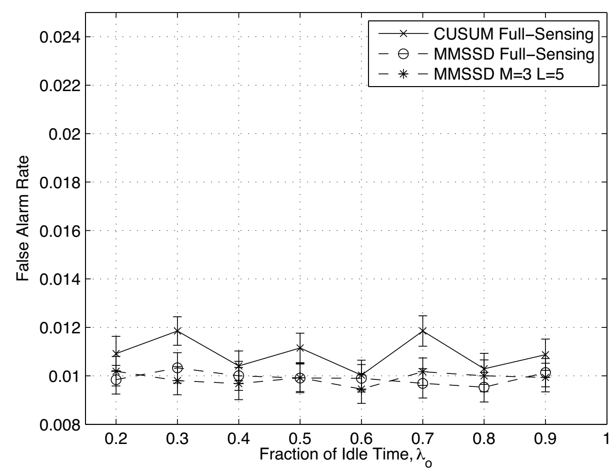

Fig. 3. A comparison of the false alarm rate between a full sensing CUSUM detection and MMSSD detection that uses 3 and 5 sensors. The error bars are the 95% confidence interval for each points. (SNR=10dB, $m_B = 620, L = 5$).

and busy states can occur. Thus, the definition of detection delay time according to the CUSUM formulation becomes inadequate. Second, due to multiple state transitions, once a channel enters an idle state, it may again switch back to the busy state. This introduces a new type of false alarm situation, in which the CUSUM detector declares the discovery of an idle state after the channel has exited the idle state. Since CUSUM does not consider this false alarm, the false alarm rate of the CUSUM detector reaches an error floor when the average detection delay time exceeds the average idle period, even as the threshold of CUSUM is raised to a large value. In Fig. 1, the false alarm rate stops decreasing with respect to increase in the detector threshold after the ratio of average detection delay time to average idle duration starts to exceed 1. Due to the error floor, the CUSUM detector cannot be designed to meet false alarm requirements below the error floor. To avoid the error floor in simulation, SNR levels and average idle durations are chosen to achieve a sufficiently low ratio of average detection delay time to average idle period time for all the comparisons.

Figs. 2 and 3 show the comparison between MMSSD and CUSUM. It is assumed that the time intervals between each sample are $1\mu sec$ in duration. The false alarm rates of MMSSD and CUSUM have been designed to achieve $\alpha = 0.01$ and Fig. 3 shows that the false alarm rates of the CUSUM are equal to or higher than that of the MMSSD detectors by 0.005. Fig. 2 shows that the MMSSD average detection delay time that uses 5 sensors performs similarly to CUSUM detection with 5 sensors. This suggests that MMSSD does not show a performance over CUSUM in a full sensing configuration. It can also be observed that MMSSD with $M = 3$ and $L = 5$ performs similarly to full-sensing CUSUM. The MMSSD's ability for sensors to switch channels dynamically allow for the reduction of one sensor without paying performance penalty. Although the reduction in number of sensors is small, this behaviour is consistent with the observations in [6]. In the next section, the comparison between RC-MMSSD and CUSUM further demonstrates the benefits of the detector's ability to dynamically switch.

Fig. 4 shows the performance comparison between RC-MMSSD and CUSUM. Similar to the comparison between MMSSD and CUSUM, RC-MMSSD detection is designed to have similar false alarm rate to the CUSUM system. The average detection delay times for CUSUM detection with $M = L = 3$ and $M = L = 30$ in Fig. 4 illustrate the potential performance gain if the number of channels L increases from 3 to 30. In order for CUSUM detection to realize such a

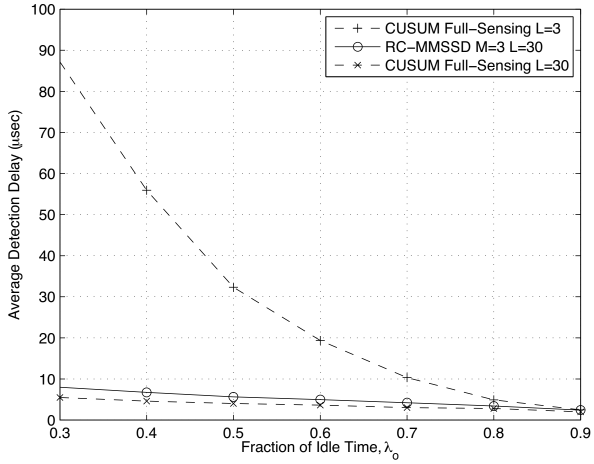

Fig. 4. A comparison of the average detection of RC-MMSSD system that uses 3 sensors to CUSUM with 3 sensors and 30 sensors. (SNR=10dB, $m_B = 620, F_s = 10^6$).

gain, the number of sensors needs to be increased by tenfold. Without increasing the number of sensors, Fig. 4 shows that the detection delay time of RC-MMSSD is significantly lower than that of a CUSUM detection with $M = 3$. In addition, compared to CUSUM with $M = 30$ sensors, RC-MMSSD that uses one-tenth the number of sensors is able to realize much of the performance gain from the increase in L. Therefore, in scenarios where L is large and M is relatively small, the RC-MMSSD system equipped with a small number of sensors can achieve similar performance to that of CUSUM equipped with a large number of sensors.

B. Performance in WLAN Traffic

An important assumption in the POMDP formulation is that the distribution of the durations of the idle and busy times are both geometrically distributed, which is a consequence of the exponential ON-OFF model. This assumption is tested on a traffic model derived from WLAN network characteristics with stationary UDP traffic.

An ON-OFF model is developed in [10] to model the traffic dynamic of UDP in a WLAN network. Channel measurements of over-air UDP traffic generated by three 802.11 wireless terminals are performed. Using this empirical data, the measured traffic pattern is fit to mixture-distributions according to the Kolmogorov-Smirnov criterion.

The ON-OFF process developed in [10] has the following structure: an idle period can arise when the channel is either in the contention phase, i.e., the period in which different terminals determine who gains access, or in the period when the channel is not used by any of the terminals and is truly free. Thus, the cdf of the duration of an idle period is given by [10]

$$F(t; k, \sigma) = p_c F_c(t) + p_f F_f(t; k, \sigma), \tag{9}$$

where p_c and $p_f \equiv (1-p_c)$ are fractions of contention time and free time, respectively, $F_c(t)$ is the cdf of the contention time that is assumed to be uniformly distributed on [0, 0.0007s], and $F_f(t; k, \sigma)$ denotes the generalized Pareto cdf of the free channel time, parameterized by k and σ:

$$F_f(t; k, \sigma) = 1 - (1 + k\frac{t}{\sigma})^{1/k}. \tag{10}$$

The values of the parameters k and σ for different measured traffic conditions tabulated in [10] with respect to different packet rates are adopted in our evaluation. The value of p_c is assumed to grow linearly with packet rate of transmission. The busy period is modelled as a period with a deterministic duration (0.0062s) because 802.11 packets are of fixed length. Since according to the 802.11 standard, the unlicensed 2.4GHz band contains three non-overlapping channels, $L = 3$ is assumed here.

The fraction of the idle time for the WLAN network is measured from simulation of the ON-OFF process, and in order to identify performance deviation in the WLAN network, the traffic model with geometrically distributed idle and busy periods is used as a nominal, baseline model to compare against. The nominal traffic model and the WLAN-based traffic model are given the same first-order statistics, e.g., the mean duration of the idle period. In addition, a sampling rate of 1MHz is considered for the discrete-time system.

Fig. 5 shows that the average detection delay time of MMSSD is larger in a WLAN-based traffic model than that of the nominal exponential ON-OFF traffic model, and the observed deviation is relatively small in the region of low spectrum utilization (i.e. large fraction of idle time) and grows to be more significant as the idle periods becomes more scarce (i.e. low fraction of idle time). In Fig. 6, it is observed that the false alarm rate generally rises in the WLAN-based traffic model compared to the nominal exponential ON-OFF traffic model. In regions of low spectrum utilization, false alarm performance does not deteriorate significantly; on the other hand, as the spectrum utilization increases and idle periods become rare, the false alarm rate deviates further from the performance observed in the nominal exponential ON-OFF traffic model. In conclusion, the assumption of the geometrically-distributed idle and busy times has little impact on the performance only when the spectrum bands experience low utilization, but becomes inadequate as channel utilization rises.

IV. Conclusions

The performance of MMSSD and a reduced-complexity version is compared to that of a CUSUM detector via Monte Carlo simulation. In the full sensing configuration, i.e., $M = L$, MMSSD performs similarly but no worse than the CUSUM detector. Interestingly, it is observed that the performance achieved by the full sensing configuration can be mostly obtained by deploying fewer than L sensors in a MMSSD detector. Similarly, it is shown that RC-MMSSD with M sensors performs comparably to a CUSUM detector with L sensors, when L is the number of channels available and $M << L$. These results suggest that MMSSD and RC-MMSSD take advantage of extra available channels more

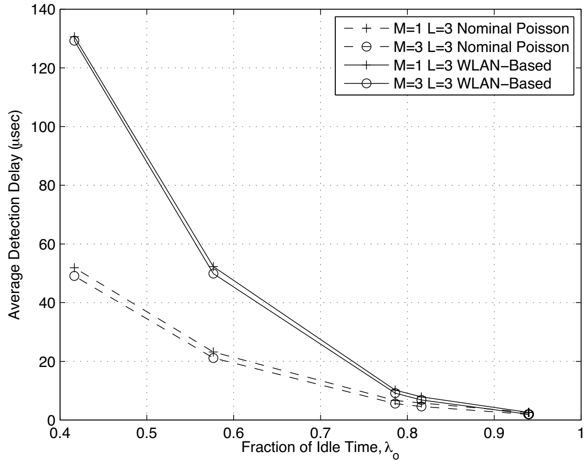

Fig. 5. The average detection delay time of MMSSD in a WLAN-based traffic model is compared to the nominal traffic model (SNR=10dB, $m_B = 620, L = 3$).

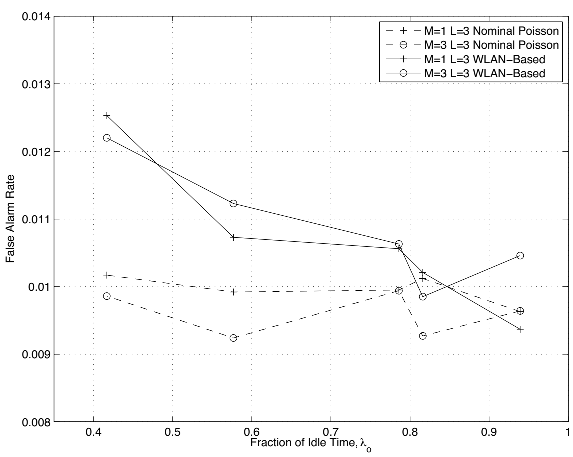

Fig. 6. The false alarm rate of MMSSD in a WLAN-based traffic model is compared to the nominal traffic model. The 95% confidence interval is $\pm$ 0.001 and is not plotted to maintain legibility. (SNR=10dB, $m_B = 620, L = 3$).

efficiently in terms of the use of multiple sensors compared to CUSUM. In other words, the utility per sensor of MMSSD and RC-MMSSD is much higher. The second phase of the study examined the assumption of a nominal exponential ON-OFF traffic model in the formulation of POMDP in a WLAN-based traffic model that was verified in an experimental-based study [10]. In light of applying MMSSD to the more realistic traffic model, it is seen that MMSSD performance results using the nominal exponential ON-OFF model are overly optimistic: significant performance degradation occurs for the WLAN-based traffic model. This suggests that future work in the development of detection methods based on more realistic traffic models are warranted.

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