Performance Analysis of Multiuser Uplink Amplify-and-Forward Relay Networks with Fixed-gain Relaying

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***Abstract*—In this paper, a multiuser uplink amplify-and-forward (AF) relay system with a multiple-antenna destination is considered where fixed-gain relaying and opportunistic scheduling are employed. Closed-form expressions for outage probability and average symbol error rate (SER) in Rayleigh fading environments are derived. Also, asymptotic expressions for outage probability and average SER are presented. The coding gain and diversity order are obtained. The analysis demonstrates that the total diversity order of the system equals $\min\{K, M\}$ where K is the multiuser diversity order and M is the spatial diversity order. Finally, comparisons between multiuser relaying systems and multiuser point-to-point systems are made and some insights are provided.**

I. Introduction

Relaying technology has the promise of extending network coverage without increasing transmit power [1]. Existing relaying protocols include amplify-and-forward (AF), in which a relay amplifies and retransmits a noisy signal from a source to a destination. Other protocols include decode-and-forward (DF), where the relay decodes, recodes and forwards the signal from source to destination. The AF protocol can be further classified as adaptive-gain and fixed-gain relaying. For adaptive-gain relaying, the relay needs to continuously monitor the channel from the source to relay. This requires substantial feedback and complexity in relay implementation. Fixed-gain relaying simplifies the relaying operation as it only requires statistical rather than instantaneous channel state information from source to relay as well as retains most of the performance gains as compared to adaptive-gain relaying [2].

Recently, it has come to our attention that performance of multiuser relay networks in a variety of fading environments has been analyzed in [2]–[6] including outage probability, average symbol error rate (SER) and asymptotic results at high SNR. However, the systems in [2]–[6] assume only a single antenna at each node. It is widely known that multiple-antenna technologies substantially improve communication link reliability through spatial diversity.

In this paper, a multiuser uplink AF relay system with fixed-gain relaying method and a multiple-antenna destination is considered. We quantify the combined effect of multiuser diversity and spatial diversity on relay systems. To elaborate, we build on the existing results by making the following contributions: (1) Exact expressions for outage probability and average SER for the system under consideration are presented to aid in practical system design; (2) Asymptotic expressions for outage probability and average SER at high SNR are derived, and applied to determining the coding gain and diversity order. Finally, comparison between the proposed multiuser relaying system and a multiuser point-to-point system is made and some insights are

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provided.

II. System Model

A multiuser AF relay system is considered where the destination is equipped with M antennas, the relay and each of K users are only equipped with a single antenna, which is a scenario applicable to current cellular uplink networks. It is assumed that a direct communication link between source and destination is not available, as is reasonable in cases where deep fades occur frequently and/or large separations exist between source and destination. Also, half-duplex transmission is assumed.

In a signaling interval, the received signal at the relay from the kth user can be written as

$$y_R = h_{SR,k} x_{S,k} + n_R \quad (1)$$

where $x_{S,k}$ denotes the signal sent by the kth user, and $h_{SR,k}$ is a complex Rayleigh fading channel gain from the kth user to the relay, and $n_R \sim \mathcal{CN}(0, \sigma_R^2)$ is Gaussian noise at the relay. In the following signaling interval, the signal received at the destination can be written as

$$\mathbf{y}_D = \mathbf{h}_{RD} x_R + \mathbf{n}_D \quad (2)$$

where $\mathbf{h}_{RD}$ is a complex Rayleigh fading channel gain vector from the relay to the destination, $\mathbf{n}_D \sim \mathcal{CN}(0, \sigma_D^2 \mathbf{I})$ is Gaussian noise vector at the destination, and $x_R = G y_R$ denotes the signal sent by the relay where G is a relay gain amplifying factor. In this paper, fixed-gain amplifying factor $G = \sqrt{P_R/(P_S E[|h_{SR,k}|^2] + \sigma_R^2)}$ is utilized due to its lower complexity and easier deployment where P_S and P_R are average power constraints at the individual user and relay, respectively, and $E[\cdot]$ is expectation operation. Substituting $x_R = G y_R$ into (2) leads to the received signal at the destination given by

$$\mathbf{y}_D = \mathbf{h}_{RD} G h_{SR,k} x_S + \mathbf{h}_{RD} G n_R + \mathbf{n}_D. \quad (3)$$

Maximal-ratio combining (MRC) is used at the destination, then the total received SNR from the kth user is given by

$$\gamma_k = \frac{\frac{P_S |h_{SR,k}|^2}{\sigma_R^2} \frac{P_R ||\mathbf{h}_{RD}||^2}{\sigma_D^2}}{\frac{P_R}{G^2 \sigma_R^2} + \frac{P_R ||\mathbf{h}_{RD}||^2}{\sigma_D^2}} = \frac{\gamma_{1,k} \gamma_2}{C + \gamma_2} \quad (4)$$

where $\gamma_{1,k} = P_S |h_{SR,k}|^2 / \sigma_R^2$, $\gamma_2 = P_R ||\mathbf{h}_{RD}||^2 / \sigma_D^2$ are the SNRs of the kth user to relay and relay to destination hops, respectively, and $C = P_R/(G^2 \sigma_R^2)$ is a constant as G is fixed. Using the definition of G given earlier, we obtain $C = E[\gamma_{1,k}] + 1$.

Opportunistic scheduling[1] is used to leverage multiuser diversity [2]. To put complexity at the destination, user selection is made at the destination. To elaborate, a training sequence is first sent from the relay to the destination, which enables the destination to obtain the CSI for R-D hop. Then Orthogonal training sequences from the K sources are forwarded to the destination by the relay. The destination can determine the CSIs of the S-R hops via the obtained CSI for R-D hop and make user selection. Under this method, the instantaneous end-to-end SNR associated with the scheduled user for fixed-gain relaying is defined as

$$\gamma = \max_{1 \le k \le K} \gamma_k. \quad (5)$$

It is readily observed that (5) are equivalent to the following expressions [3]

$$\gamma = \frac{\gamma_1 \gamma_2}{C + \gamma_2} \quad (6)$$

where $\gamma_1 = \max_{1 \le k \le K} \gamma_{1,k}$.

III. Outage Probability and Average SER

In this section, we obtain closed-form expressions for outage probability and average SER for the system under consideration.

Theorem 1: The cumulative distribution function (CDF) of the received SNR γ is given by

$$F_\gamma(\gamma) = 1 + \frac{2}{(M-1)!} \sum_{k=1}^{K} \binom{K}{k} (-1)^k \left(\frac{kC\gamma}{\bar{\gamma}_1 \bar{\gamma}_2} \right)^{M/2} \times e^{-\frac{k\gamma}{\bar{\gamma}_1}} K_M \left(2\sqrt{\frac{kC\gamma}{\bar{\gamma}_1 \bar{\gamma}_2}} \right)$$

where $K_M(\cdot)$ is the M-order modified Bessel function of the second kind defined in [7].

Proof: We assume that all the users are subject to the same power constraint P_S and located in a homogeneous environment, i.e., the signal

[1]Fairness is not guaranteed among users. This may be addressed by the proportional fairness algorithm. However, this topic is beyond the scope of this paper.

from each user to the relay experiences independent identically distributed (i.i.d) Rayleigh fading, but the user-to-relay and relay-to-destination hops may have independent and not identically distributed fading. The CDF of the overall received SNR is

$$P(\frac{\gamma_1\gamma_2}{C+\gamma_2} < \gamma) = \int_0^\infty P(\gamma_1 < \frac{C\gamma + y\gamma}{y}) \quad f_{\gamma_2}(y)\,\mathrm{d}y. \tag{7}$$

To derive the CDF of γ, we need the CDF of γ_1 and the probability density function (PDF) of γ_2 given by, respectively,

$$F_{\gamma_1}(x) = 1 + \sum_{k=1}^{K} \binom{K}{k} (-1)^k e^{-\frac{kx}{\bar{\gamma}_1}} \tag{8}$$

$$f_{\gamma_2}(y) = \frac{y^{M-1} e^{-\frac{y}{\bar{\gamma}_2}}}{(M-1)!} \left(\frac{1}{\bar{\gamma}_2}\right)^M \tag{9}$$

where $\bar{\gamma}_1$ and $\bar{\gamma}_2$ denote average SNRs from each user to the relay and from the relay to each destination antenna, respectively. Substituting (8) and (9) into (7), after the algebraic manipulation yields

$$F_\gamma(\gamma) = 1 + \tfrac{1}{(M-1)!} \textstyle\sum_{k=1}^{K} \binom{K}{k} (-1)^k \left(\tfrac{1}{\bar{\gamma}_2}\right)^M \times e^{-\frac{k\gamma}{\bar{\gamma}_1}} \textstyle\int_0^\infty y^{M-1} e^{-\frac{kC\gamma}{\bar{\gamma}_1 y} - \frac{y}{\bar{\gamma}_2}}\,\mathrm{d}y. \tag{10}$$

Finally, applying [8, eq. (3.471.9)], (7) is obtained. ■

Remark: Substituting $\gamma = \gamma_{\mathrm{th}}$ into (7) leads to a closed-form expression for system outage probability, where γ_{th} denotes SNR threshold to avoid system outage.

The average SER is one of the most commonly used performance criteria in digital communication systems. When the source uses general modulation method, the conditional SER has the form of $P_e(\gamma) = aQ(\sqrt{b\gamma})$ where $(a, b) = (1, 2)$ for binary phase-shift keying (BPSK), $(a, b) = (1, 1)$ for binary frequency-shift keying (BFSK), and $(a, b) = (2(M-1)/M, 6/(M^2-1))$ for M-ary pulse amplitude modulation (MPAM). The following CDF-based method can be used to compute average SER [9]:

$$\bar{P}_{\mathrm{e}} = -\int_0^\infty F_\gamma(\gamma) P'_{\mathrm{e}}(\gamma)\,\mathrm{d}\gamma \tag{11}$$

where $P'_{\mathrm{e}}(\gamma) = -\frac{a\sqrt{b}}{2\sqrt{2\pi}} e^{-\frac{b\gamma}{2}} \gamma^{-\frac{1}{2}}$ denotes the derivation of $P_e(\gamma)$. Substituting (7) into (11), Using [8, eq.(3.361.2) and eq.(6.643.3)] yields a closed-form expression for average SER as below

$$\begin{aligned} \bar{P}_e = \frac{a}{2} + \frac{a\sqrt{b}\Gamma(M+\frac{1}{2})}{2\sqrt{2}(M-1)!} \sum_{k=1}^{K} \binom{K}{k} (-1)^k \\ \times \left(\frac{kC}{\bar{\gamma}_1\bar{\gamma}_2}\right)^{(M-1)/2} \left(\frac{2k+b\bar{\gamma}_1}{2\bar{\gamma}_1}\right)^{-M/2} \\ \times \exp\left(\frac{kC}{2k\bar{\gamma}_2 + b\bar{\gamma}_1\bar{\gamma}_2}\right) \\ \times W_{-\frac{M}{2},\frac{M}{2}}\left(\frac{2kC}{2k\bar{\gamma}_2 + b\bar{\gamma}_1\bar{\gamma}_2}\right) \end{aligned} \tag{12}$$

where $\Gamma(\cdot)$ denotes gamma function, $W_{\lambda,\mu}(\cdot)$ is Whittaker function [7].

IV. High SNR Analysis

To gain insight as to how parameters influence system performance, we next consider asymptotic behavior of outage probability and average SER at high SNR.

First, we determine the value of constant C. As the channel from each user to the relay has a Rayleigh fading distribution, $C = E[\gamma_{1,k}] + 1$ can be denoted as

$$C = \bar{\gamma}_1 + 1. \tag{13}$$

Proposition 1: In the high SNR regime, system outage probability can be approximated by the expression (14) shown at the top of the next page.

Proof: To obtain asymptotic behavior of outage probability at high SNR, set $\bar{\gamma}_2 = \mu\bar{\gamma}_1$ where μ is a constant. Using power series expansions of $K_M(z)$ from [8, eq. (8.446)] and the exponential function from [8, eq. (1.211.1)], substituting (13) into (7) yields

$$\begin{aligned} F_\gamma(\gamma) = 1 + \tfrac{2}{(M-1)!} \textstyle\sum_{k=1}^{K} \binom{K}{k} (-1)^k \left(\tfrac{k\gamma(\bar{\gamma}_1+1)}{\mu\bar{\gamma}_1^2}\right)^{\frac{M}{2}} \\ \times \textstyle\sum_{l=0}^{\infty} \frac{(-\frac{k\gamma}{\bar{\gamma}_1})^l}{l!} \left(\tfrac{1}{2} \textstyle\sum_{j=0}^{M-1} (-1)^j \frac{(M-j-1)!}{j!\left(\frac{k\gamma(\bar{\gamma}_1+1)}{\mu\bar{\gamma}_1^2}\right)^{(M-2j)/2}} \right. \\ + (-1)^{M+1} \tfrac{1}{2} \textstyle\sum_{j=0}^{\infty} \frac{\left(\frac{k\gamma(\bar{\gamma}_1+1)}{\mu\bar{\gamma}_1^2}\right)^{(M+2j)/2}}{j!(M+j)!} \\ \left. \times \left(\ln \tfrac{k\gamma(\bar{\gamma}_1+1)}{\mu\bar{\gamma}_1^2} - \varphi(j+1) - \varphi(M+j+1) \right) \right). \end{aligned}$$

Through some mathematical manipulations, we

$$F_{\gamma}(\gamma_{th}) = \begin{cases} \sum_{k=0}^{K} \binom{K}{k} \frac{(M-k-1)!}{(M-1)!} (\frac{1}{\mu})^k (\frac{\gamma_{th}}{\bar{\gamma}_1})^K + o(\bar{\gamma}_1^{-K}) & K < M \\ \Big(\sum_{k=0}^{K-1} \binom{K-1}{k} \frac{K!}{(K-k)!} (\frac{1}{\mu})^k + \frac{(-1)^{K+1}}{K!(K-1)!} (\frac{1}{\mu})^K \sum_{k=1}^{K} \binom{K}{k} (-1)^k k^K \ln k & \\ \quad - \frac{1}{(K-1)!} \frac{1}{\mu^K} \big(\ln \frac{\gamma_{\text{th}}}{\mu \bar{\gamma}_1} - \varphi(1) - \varphi(K+1) \big) \Big) (\frac{\gamma_{th}}{\bar{\gamma}_1})^K + o(\bar{\gamma}_1^{-K}) & K = M \\ \left(\frac{(-1)^{M+1}}{M!(M-1)!} \sum_{k=1}^{K} \binom{K}{k} (-1)^k k^M \ln k \right) \frac{1}{\mu^M} (\frac{\gamma_{th}}{\bar{\gamma}_1})^M + o(\bar{\gamma}_1^{-M}) & K > M. \end{cases} \tag{14}$$

obtain

$$\begin{aligned} &F_{\gamma}(\gamma) \\ &\approx \sum_{k=1}^{K} \binom{K}{k} (-1)^k \sum_{j=1}^{M-1} \binom{M-1}{j} \left(-\frac{k\gamma}{\mu \bar{\gamma}_1} \right)^j \\ &\quad \times \sum_{l=0}^{\infty} \frac{(-\frac{k\gamma}{\bar{\gamma}_1})^l}{l!} + \sum_{k=1}^{K} \binom{K}{k} (-1)^k \sum_{l=1}^{\infty} \frac{(-\frac{k\gamma}{\bar{\gamma}_1})^l}{l!} \\ &\quad + \sum_{k=1}^{K} \binom{K}{k} (-1)^k \sum_{j=0}^{\infty} \frac{(-1)^{M+1}}{j!(M+j)!(M-1)!} \\ &\quad \times \left(-\frac{k\gamma_{\text{th}}}{\mu \bar{\gamma}_1} \right)^{M+j} \sum_{l=0}^{\infty} \frac{(-\frac{k\gamma}{\bar{\gamma}_1})^l}{l!} \\ &\times \left(\ln \frac{k\gamma(\bar{\gamma}_1 + 1)}{\mu \bar{\gamma}_1^2} - \varphi(j+1) - \varphi(M+j+1) \right). \end{aligned} \tag{15}$$

Using the identity $\sum_{i=1}^{M} \binom{M}{i} (-1)^i i^L = 0$, for $L = 1, \cdots, M-1$ and $\lim_{z \to \infty} z \ln(1 + \frac{1}{z}) = 1$, and substituting $\gamma = \gamma_{\text{th}}$ to (15) yields (14). ■

At high SNR, asymptotic outage probability (14) can also be denoted by [10, Proposition 5]

$$P_{out} = \rho (\frac{\gamma_{th}}{\bar{\gamma}})^{t+1} + o(\bar{\gamma}^{-(t+1)}). \tag{16}$$

Comparing (14) with (16), we obtain

$$t = \min\{K, M\} - 1 \tag{17}$$

and ρ is given at the top of this page. The asymptotic SER expression is given by [10, Proposition 1]

$$\bar{P}_e = \frac{2^t \rho \Gamma(t + \frac{3}{2})}{\sqrt{\pi}} (\beta \bar{\gamma})^{-(t+1)} + o(\bar{\gamma}^{-(t+1)}) \tag{19}$$

where β is a constant that depends on the specific modulation scheme used.

Substituting (17) and (18) into (19), we obtain an asymptotic average SER expression for the system under consideration at high SNR: $\bar{P}_e \approx (G_c \bar{\gamma}_1)^{-G_d}$ where the diversity order is $G_d = \min(K, M)$ and coding gain is given by

$$G_c = \beta (\frac{(2G_d)! \rho}{2^{G_d+1} G_d!})^{-\frac{1}{G_d}}. \tag{20}$$

V. Numerical Examples and Observations

In the following simulations, the instantaneous SNR threshold is set to $\gamma_{\text{th}} = 0$dB. Figs. 1 and 2 show outage probability and average SER for BPSK modulation method for the system under consideration, respectively. It is observed that computer simulations agree closely with the analytical results. At high SNR, asymptotic results are able to accurately predict the analytical results. It is clear that the diversity order of the system, observable by the slope of the outage probability and average SER curves, cannot be improved through simply adding users only or destination antennas only for the case that the number of destination antennas equal the number of users ($M = K$). This is as expected from the high SNR analysis derived earlier, where the total diversity order of the system is shown to be equal to the minimum of multiuser diversity order and spatial diversity order, i.e., $G_d = \min(K, M)$. These interesting observations will help system design.

To assess the impact of relaying, comparison between a multiuser relaying system and multiuser point-to-point system is made. Specifically, we set $K = 2$ and $M = 2$ and the relay is located between the source and destination. The normalized distance between the source and destination is assumed to be unity. The path loss exponent is set to 4, and it is assumed that shadowing effects are the same for the user-relay and relay-destination links, with a standard deviation $\delta = 8dB$, a value typically assumed in urban

$$
\rho = \begin{cases} \sum_{k=0}^{K} \binom{K}{k} \frac{(M-k-1)!}{(M-1)!} \left(\frac{1}{\mu}\right)^k & K < M \\ \Bigg(\sum_{k=0}^{K-1} \binom{K-1}{k} \frac{K!}{(K-k)!} \left(\frac{1}{\mu}\right)^k & \\ \quad + \frac{(-1)^{K+1}}{K!(K-1)!} \left(\frac{1}{\mu}\right)^K \sum_{k=1}^{K} \binom{K}{k} (-1)^k k^K \ln k - \frac{1}{(K-1)!} \frac{1}{\mu^K} \left(\ln \frac{\gamma_{\mathrm{th}}}{\mu \bar{\gamma}_1} - \varphi(1) - \varphi(K+1) \right) \Bigg) & K = M \\ \left(\frac{(-1)^{M+1}}{M!(M-1)!} \sum_{k=1}^{K} \binom{K}{k} (-1)^k k^M \ln k \right) \frac{1}{\mu^M} & K > M. \end{cases} \tag{18}
$$

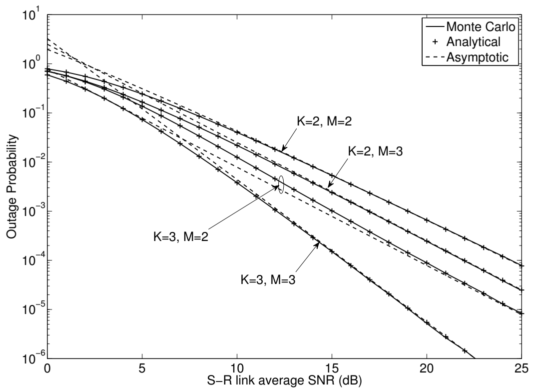

Fig. 1: Outage probability for different antenna configurations.

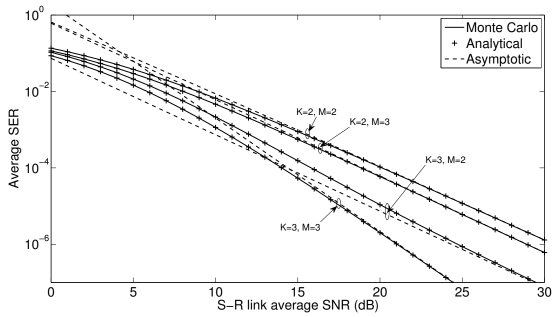

Fig. 2: Average SER for different antenna configurations.

cellular environments. For a fair comparison, the total transmit SNR as well as the receiver strategy are assumed to be identical for the above systems. Fig. 3 compares multiuser relaying systems and multiuser point-to-point systems. It is clear that from Fig. 3 that the multiuser relaying system outperform the multiuser point-to-point system in the low SNR regime. As SNR increases, the multiuser point-to-point system gains an advantage over the relaying system as its total diversity order is $K * M$ [11].

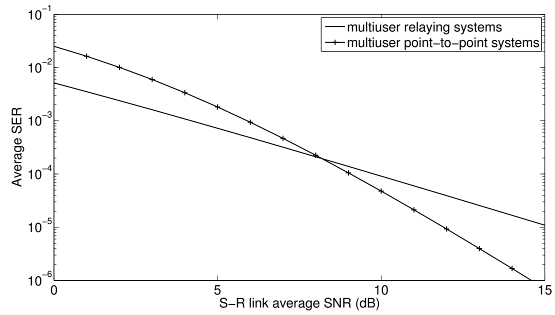

Fig. 3: Average SER Comparison between multiuser relaying systems and multiuser point-to-point systems.

VI. Conclusion

This paper focuses on the performance analysis of a multiuser uplink AF relay network with fixed-gain relaying method and a multiple-antenna destination. Also, the combined effect of the multiuser diversity and spatial diversity on the system are quantified. It is illustrated that the multiuser relaying system has an advantage over the multiuser point-to-point system in the low SNR regime through numerical analysis.

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