Receive Antenna Subset Selection For Time-Varying Channels Using Slepian Subspace Projections

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***Abstract*—Receive antenna selection (AS) preserves the diversity benefits of multiple antennas with considerable reduction of hardware complexity and costs. We propose a receive AS method for time-varying channels which utilizes low-complexity Slepian subspace projection techniques. The proposed method uses Doppler bandwidth knowledge and takes into account practical limitations such as training, packetization and antenna switching. Results show that the proposed AS method outperforms ideal conventional systems with perfect channel state information (CSI) but no AS at the receiver and AS using the conventional Fourier estimation/prediction method. A closed-form expression for the symbol error probability (SEP) of M-ary phase-shift keying (MPSK) with receive AS is derived.**

I. Introduction

Receive antenna selection (AS) reduces hardware complexity by using limited number of radio-frequency (RF) chains at the receiver of a wireless system [1].

Algorithms and performance analysis for AS systems are reported in numerous previous studies [2]–[5]. It is only recently that a limited number of recent studies have investigated practical issues such as pilot-based training and implementation of AS [6]. In the above references, perfect channel knowledge is assumed. However, the wireless channel is time-varying which results in outdated CSI at the receiver. The impact of imperfect channel knowledge on the performance of AS systems is studied in [7]–[10]. The performance of AS systems with CSI feedback delay are studied in [11] and [12]. Weighted AS rules for time-varying channels which use the temporal correlation knowledge are proposed in [13] and [14]. In [13] and [14], only channel gain estimates obtained from the AS training phase are used in the selection and decoding processes. This results in a signal-to-noise ratio (SNR) loss.

Motivated by the above observations, [15], [16] recently proposed a practical training-based receive AS algorithm for time-varying channels. It uses CSI knowledge of the data transmission phase in selection and decoding processes by utilizing low-training overhead Slepian prediction [17] and estimation [18]. The optimal Wiener predictor requires detailed correlation knowledge whereas the Slepian estimator/predictor only requires knowledge of the Doppler bandwidth [17]. However, only the simpler problem of selecting a single antenna at the receiver is considered in [15]. The paper's contributions are summarized as follows:

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- A single receive AS method for time varying channels based on Slepian subspace projections [15], [16] is extended to accommodate the selection of multiple receive antennas.
- A closed-from expression for the symbol-error probability (SEP) of M-ary phase-shift keying (MPSK) with receive AS is provided, and verified with simulations.
- Extensive simulation results are presented to compare the performance of the proposed method to that of ideal conventional SIMO systems with perfect CSI but no AS at the receiver as well as AS based on conventional Fourier basis prediction/estimation.

II. System Model

Consider a system with one transmit and K receive antennas equipped only with $K' \leq K$ RF chains. Depending on the AS switching time, either per-packet or symbol-by-symbol AS can be used. For example, microelectromechanical system (MEMS) switches enable only per-packet switching with negligible attenuations. Solid-state switches can enable switching of antennas between symbols, but with non-negligible attenuations [19].

A. Antenna Selection Training Phase

In total $\lceil \frac{K}{K'} \rceil L$ pilot symbols are transmitted in $L \geq 2$ rounds of transmission, as depicted in Fig. 1. In each round the transmitter sends out $\lceil \frac{K}{K'} \rceil$ pilots, where each pilot is received by at most K' receive AEs. Two consecutive pilots are spaced $T_{\mathrm{p}} \triangleq \alpha T_s$, where T_s is the symbol duration and $\alpha \geq 2$. Therefore, two consecutive pilots transmitted for each of the $\lceil \frac{K}{K'} \rceil$ antenna subsets are separated in time by $T_{\mathrm{r}} \triangleq \lceil \frac{K}{K'} \rceil T_{\mathrm{p}}$. The AS training pilots are received by AE k at times $m \in T_{\mathrm{sp}}^k$, where

$$T_{\mathrm{sp}}^k \triangleq \left\{ \alpha \left[\left(\left\lceil \frac{k}{K'} \right\rceil - 1 \right) + (\ell - 1) \left\lceil \frac{K}{K'} \right\rceil \right] \right\}, \quad 1 \leq \ell \leq L. \tag{1}$$

The observations over the AS training pilots are necessary to perform Slepian channel prediction [17] for each AE over the data transmission phase $\mathcal{I}_{\mathrm{dt}}$. Based on the predicted channel gains $\{\hat{h}_k^{\mathrm{SP}}[m] \mid m \in \mathcal{I}_{\mathrm{dt}}\}$, the receiver then selects its receive AEs subset $\mathfrak{S} \triangleq \{\mathrm{AE}\ \hat{\imath}_1, \mathrm{AE}\ \hat{\imath}_2, \ldots, \mathrm{AE}\ \hat{\imath}_{K'}\}$ and connects them to the RF chains in a duration of $T_{\mathrm{p}} - T_{\mathrm{s}}$. Therefore, the AS training phase spans the discrete time

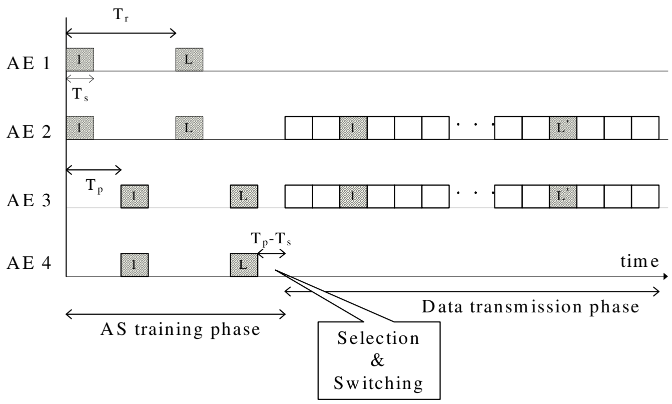

Fig. 1. Antenna selection cycle for the proposed per-packet AS method. (AEs 2 and 3 are selected, $K = 4$, $L = 2$, $L' = 2$, and $T_{\rm p} = 2\,T_{\rm s}$).

interval $\mathcal{I}_{\rm tr} = \{0, 1, \ldots, M-1\}$, where $M = \alpha \left\lceil \frac{K}{K'} \right\rceil L$. In symbol-by-symbol AS, the most suitable receive AEs subset $\mathfrak{S}_m \triangleq \{\text{AE } \hat{\iota}_1^m, \text{AE } \hat{\iota}_2^m, \ldots, \text{AE } \hat{\iota}_{K'}^m\}$ is selected for each symbol at time m.

B. Data Transmission Phase

1) Per-Packet AS: The data transmission phase spans $\mathcal{I}_{\rm dt} = \{M, M+1, \ldots, M+N-1\}$, where the transmitter sends out a length-N packet containing $L' \geq 1$ pilots time-multiplexed with $N - L'$ data symbols as [16]

$$\mathcal{P}_{\rm dp} \triangleq \left\{ \left\lfloor (\ell' - 1)\ \frac{N}{L'} + \frac{N}{2\,L'} \right\rfloor \ \middle|\ 1 \leq \ell' \leq L' \right\} \tag{2}$$

The received signal at AE $\hat{\iota}_k$, for $1 \leq k \leq K'$, is

$$y_{\hat{\iota}_k}[m] = \begin{cases} h_{\hat{\iota}_k}[m]\ d[m] + n_{\hat{\iota}_k}[m], & m \in \mathcal{I}_{\rm dt} \backslash T_{\rm dp} \\ h_{\hat{\iota}_k}[m]\ p[m] + n_{\hat{\iota}_k}[m], & m \in T_{\rm dp} \end{cases} \tag{3}$$

where $d[m]$ and $p[m]$ denote the transmitted data and pilot symbols, respectively. The L' pilot symbols are received by antenna subset $\mathfrak{S}$ at times $m \in T_{\rm dp}$, where

$$T_{\rm dp} \triangleq \left\{ M - 1 + \left\lfloor (\ell' - 1)\ \frac{N}{L'} + \frac{N}{2\,L'} \right\rfloor \right\}. \tag{4}$$

Thus, in total, $L_{\rm tp} \triangleq L + L'$ pilot symbols are received by each selected AE $\hat{\iota}_k$ at times

$$T_{\rm tp}^{\hat{\iota}_k} = T_{\rm sp}^{\hat{\iota}_k} \cup T_{\rm dp}, \qquad 1 \leq k \leq K' \tag{5}$$

with $T_{\rm sp}^{\hat{\iota}_k}$ and $T_{\rm dp}$ given in (1) and (4), respectively. From these $L_{\rm tp}$ pilots, refined channel gain estimates $\{\hat{h}_{\hat{\iota}_k}^{\rm SE}[m] \mid m \in \mathcal{I}_{\rm dt}\}$ are obtained to decode data.

2) Symbol-By-Symbol AS: After selection, the transmitter sends out a length-N packet which consists of $N - \left\lceil \frac{K}{K'} \right\rceil L'$ data symbols and $\left\lceil \frac{K}{K'} \right\rceil L'$ pilot symbols. The $\left\lceil \frac{K}{K'} \right\rceil L'$ pilot symbols are needed to obtain refined channel gain estimates $\{\hat{h}_{\hat{\iota}_k^m}^{\rm SE}[m] \mid m \in \mathcal{I}_{\rm dt}\}$. This is because in symbol-by-symbol AS, for each symbol an antenna subset $\mathfrak{S}_m$ is selected at time m. Since different antenna subsets might be selected during the data transmission phase, $L' \geq 1$ pilots should be sent to each of the $\left\lceil \frac{K}{K'} \right\rceil$ antenna subsets. The symbol locations in the packet that carry the L' pilots for AE k, for $1 \leq k \leq K$, are given by

$$\mathcal{P}_{\rm dp}^k \triangleq \left\{ \left(\left\lceil \frac{k}{K'} \right\rceil - 1 \right) + \left\lfloor (\ell' - 1)\ \frac{N}{L'} + \frac{N}{2\,L'} \right\rfloor \right\}. \tag{6}$$

Thus $L_{\rm tp} = L + L'$ pilots are received by each AE k, for $1 \leq k \leq K$, at times

$$T_{\rm tp}^k = T_{\rm sp}^k \cup T_{\rm dp}^k, \tag{7}$$

where

$$T_{\rm dp}^k \triangleq \left\{ M - 1 + \left(\left\lceil \frac{k}{K'} \right\rceil - 1 \right) + \left\lfloor (\ell' - 1)\ \frac{N}{L'} + \frac{N}{2\,L'} \right\rfloor \right\}. \tag{8}$$

The data symbols received by $\mathfrak{S}_m$ are given by

$$y_{\hat{\iota}_k^m}[m] = h_{\hat{\iota}_k^m}[m]\ d[m] + n_{\hat{\iota}_k^m}[m] \tag{9}$$

where $m \in \mathcal{I}_{\rm dt} \backslash \left(T_{\rm dp}^1 \cup \ldots \cup T_{\rm dp}^K \right)$.

III. Slepian Basis Expansion Model

A. Slepian Estimator

The true channel vector $\boldsymbol{h} \triangleq \left[h[0], \ldots, h[M'-1] \right]^T$ of size $M' \times 1$ is estimated as [17]

$$\boldsymbol{h} \approx \hat{\boldsymbol{h}}^{\rm SE} = \boldsymbol{U}\,\hat{\boldsymbol{\gamma}} = \sum_{i=0}^{D-1} \hat{\gamma}_i\, \boldsymbol{u}_i \tag{10}$$

where the Slepian function $\boldsymbol{u}_i \triangleq \left[u_i[0], \ldots, u_i[M'-1] \right]^T$ consists of time-limited discrete prolate spheroidal (DPS) sequences corresponding to eigenvalue λ_i. The DPS sequences $\{u_i[m] \mid m \in \mathbb{Z}\}_{i=0}^{M'-1}$ are defined as [18]

$$\sum_{l=0}^{M'-1} \frac{\sin\left(2\pi\nu_{\rm max}(l-m)\right)}{\pi(l-m)}\, u_i[l] = \lambda_i\, u_i[m], \ \ m \in \mathbb{Z} \tag{11}$$

where $i \in \mathcal{I}_{\rm bl} = \{0, 1, \ldots, M'-1\}$ and $\nu_{\rm max}$ is the normalized Doppler bandwidth. In (10), $\boldsymbol{U} \triangleq \left[\boldsymbol{u}_0, \ldots, \boldsymbol{u}_{D-1} \right]$ is an $M' \times D$ matrix. The coefficient vector $\hat{\boldsymbol{\gamma}} \triangleq \left[\hat{\gamma}_0, \hat{\gamma}_1, \ldots, \hat{\gamma}_{D-1} \right]^T$ is estimated using the J interleaved pilots $\{p[l] \mid l \in \mathcal{J}\}$, received at times $l \in \mathcal{J}$, via [18]

$$\hat{\boldsymbol{\gamma}} = \boldsymbol{G}^{-1} \sum_{l \in \mathcal{J}} y[l]\ p^*[l]\ \boldsymbol{f}^*[l] \tag{12}$$

where $y[l]$ is the observation over the transmitted pilot symbol $p[l]$, $\boldsymbol{f}[l] \triangleq \left[u_0[l], \ldots, u_{D-1}[l] \right]^T$, and $\boldsymbol{G}$ is a $D \times D$ matrix given by

$$\boldsymbol{G} = \sum_{l \in \mathcal{J}} \boldsymbol{f}\ [l]\ \boldsymbol{f}^\dagger[l]. \tag{13}$$

In (10), D is given by [17]

$$D = \operatorname*{argmin}_{d \in \{1, \ldots, J\}} \left(\frac{1}{2\,\nu_{\rm max}\, J} \sum_{i=d}^{J-1} \lambda_i + \frac{d}{J}\, N_0 \right) \tag{14}$$

where N_0 is the noise variance.

B. Slepian Predictor

The Slepian predictor approximates $h[m]$ as [17]

$$\hat{h}^{\mathrm{SP}}[m] = \boldsymbol{f}^{T}[m]\,\hat{\boldsymbol{\gamma}} = \sum_{i=0}^{D-1} \hat{\gamma}_i\, u_i[m], \quad m \in \mathbb{Z} \setminus \mathcal{I}_{\mathrm{bl}} \tag{15}$$

where $\{u_i[m] \mid m \in \mathbb{Z} \setminus \mathcal{I}_{\mathrm{bl}}\}_{i=0}^{M'-1}$ can be calculated from (11).

IV. Receive Antenna Selection Algorithm

We propose the following training-based K' *out of* K receive AS algorithm for time-varying channels:

1) Every antenna subset of the $\lceil \frac{K}{K'} \rceil$ total subsets is trained using $L \geq 2$ pilot symbols. The spacing between consecutive pilot symbols transmitted for each of the $\lceil \frac{K}{K'} \rceil$ antenna subsets is $T_r = \alpha \lceil \frac{K}{K'} \rceil T_{\mathrm{s}}$.
2) The receiver then performs channel prediction over $\mathcal{I}_{\mathrm{dt}}$ via (15)

$$\hat{h}_k^{\mathrm{SP}}[m] = \boldsymbol{f}^{T}[m]\,\hat{\boldsymbol{\gamma}}_k = \sum_{i=0}^{D-1} \hat{\gamma}_{k,i}\, u_i[m] \tag{16}$$

where $\hat{\boldsymbol{\gamma}}_k \triangleq [\hat{\gamma}_{k,0}, \hat{\gamma}_{k,1}, \ldots, \hat{\gamma}_{k,D-1}]^T$ is obtained via (12) (with T_{sp}^{k} replacing $\mathcal{J}$).

   a) *Per-Packet AS:* Selects its receive antenna subset $\mathfrak{S}$ which maximizes the post-processing SNR over $\mathcal{I}_{\mathrm{dt}}$, i.e., the first K' order statistics of $\left\{\sum_{m=M}^{M+N-1} \left|\hat{h}_k^{\mathrm{SP}}[m]\right|^2\right\}$.
   b) *Symbol-By-Symbol AS:* Selects its instantaneous antenna subset $\mathfrak{S}_m$ which consists of the first K' order statistics of $\left\{\left|\hat{h}_k^{\mathrm{SP}}[m]\right|^2\right\}$.
3) a) *Per-Packet AS:* The transmitter sends a length-N data packet in which L' pilots are time multiplexed with $N - L'$ data symbols as (2).
   b) *Symbol-By-Symbol AS:* The transmitter sends out a length-N data packet, which consists of $N - \lceil \frac{K}{K'} \rceil L'$ data symbols and $\lceil \frac{K}{K'} \rceil L'$ pilots interleaved as (6).
4) a) *Per-Packet AS:* Refined channel gain estimates $\{\hat{h}_{\hat{\imath}_k}^{\mathrm{SE}}[m] \mid m \in \mathcal{I}_{\mathrm{dt}}\}$ for $\mathfrak{S}$ are obtained via

$$\hat{\boldsymbol{h}}_{\hat{\imath}_k}^{\mathrm{SE}} = \boldsymbol{U}'\,\hat{\boldsymbol{\gamma}}_{\hat{\imath}_k} = \sum_{i=0}^{D-1} \hat{\gamma}_{\hat{\imath}_k,i}\, \boldsymbol{u}'_i \tag{17}$$

   where $\hat{\boldsymbol{h}}_{\hat{\imath}_k}^{\mathrm{SE}} \triangleq \left[\hat{h}_{\hat{\imath}_k}^{\mathrm{SE}}[M], \ldots, \hat{h}_{\hat{\imath}_k}^{\mathrm{SE}}[M+N-1]\right]^T$, $\boldsymbol{U}' \triangleq [\boldsymbol{u}'_0, \ldots, \boldsymbol{u}'_{D-1}]$ is the $N \times D$ submatrix of the complete $(M+N) \times D$ Slepian sequences matrix $\boldsymbol{U}$, and $\boldsymbol{u}'_i \triangleq [u_i[M], \ldots, u_i[M+N-1]]^T$.
   b) *Symbol-By-Symbol AS:* To decode data the receiver obtains $\{\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m] \mid m \in \mathcal{I}_{\mathrm{dt}}\}$ for $\mathfrak{S}_m$ via

$$\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m] = \sum_{i=0}^{D-1} \hat{\gamma}_{\hat{\imath}_k^m,i}\, u_i[m]. \tag{18}$$

V. Symbol Error Probability Analysis

The soft estimate is obtained by maximum-ratio-combining (MRC) of the received soft symbols from $\mathfrak{S}_m$ as

$$r_{\mathfrak{S}_m}[m] = \sum_{k=1}^{K'} \left(\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m]\right)^* y_{\hat{\imath}_k^m}[m]. \tag{19}$$

Conditioned on $\left\{\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}$, $\left\{\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}$ and $d[m]$, using standard results on moments of conditional Gaussian RVs [20] it can be shown that $r_{\mathfrak{S}_m}[m]$ in (19) is a complex Gaussian RV whose conditional mean $\mu_{r_{\mathfrak{S}_m}}[m]$ depends on both $\left(\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m]\right)^* \hat{h}_{\hat{\imath}_k^m}^{\mathrm{SP}}[m]$ and $\left|\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m]\right|^2$ which makes the analysis analytically intractable. We next derive an SEP expression which provides insights for both symbol-by-symbol and per-packet AS as described in Sec. IV. The first and second conditional moments of (19) are conditioned only on $\left\{\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}$.

Theorem 1 With MRC decision variable in (19) conditioned only on the refined channel gain estimates $\left\{\hat{h}_{\hat{\imath}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}$, the MPSK SEP for a symbol received at time m for a system with one transmit and K receive antennas employing the K' *out of* K symbol-by-symbol receive AS in Sec. IV is

$$\begin{aligned} SEP'_m(\eta) = \; & \frac{1}{\pi} \sum_{P_\pi \in \mathfrak{P}} \left(\prod_{k=1}^{K} \left[\sum_{r=1}^{k} \frac{\Gamma_{P_\pi(k)}[m]}{\Gamma_{P_\pi(r)}[m]} \right]^{-1} \right) \\ & \int_0^{\frac{M-1}{M}\pi} \left(\prod_{k=1}^{K'} \frac{\sin^2(\theta)}{\sin^2(\theta) + \sigma^2_{X^{SE}_{\hat{\imath}_k^m}}[m]} \right) \times \\ & \left(\prod_{k=1}^{K} \frac{\sin^2(\theta) + \sigma^2_{X^{SE}_{\hat{\imath}_k^m}}[m]}{\sin^2(\theta) + \sigma^2_{X^{SE}_{\hat{\imath}_k^m}}[m] + \digamma_k} \right) \mathrm{d}\theta \end{aligned} \tag{20}$$

where $\eta \triangleq \frac{E_s}{N_0}$ is SNR, P_π is a permutation of set $\{1, 2, \ldots, K\}$ and $\mathfrak{P}$ the set of all $K!$ permutations. $\sigma^2_{X^{\mathrm{SE}}_{\hat{\imath}_k^m}}[m] = \frac{b^{\mathrm{SE}}_{\hat{\imath}_k^m}[m]\left(1 - \left|\rho_{\hat{h}^{\mathrm{SE}}_{\hat{\imath}_k^m}[m],\, \hat{h}^{\mathrm{SP}}_{\hat{\imath}_k^m}[m]}\right|^2\right)}{\zeta^{\mathrm{SE}}_{\hat{\imath}_k^m}[m]}$ where $\zeta^{\mathrm{SE}}_{\hat{\imath}_k^m}[m] \triangleq \frac{1}{1 + \sigma^2_{e^{\mathrm{SE}}_{\hat{\imath}_k^m}}[m]}$, $b^{\mathrm{SE}}_{\hat{\imath}_k^m}[m] \triangleq \frac{\left(\zeta^{\mathrm{SE}}_{\hat{\imath}_k^m}[m]\right)^2 \sin^2\left(\frac{\pi}{\mathrm{M}}\right)}{1 - \zeta^{\mathrm{SE}}_{\hat{\imath}_k^m}[m] + \eta^{-1}}$, and $\rho_{\hat{h}^{\mathrm{SE}}_{\hat{\imath}_k^m}[m],\, \hat{h}^{\mathrm{SP}}_{\hat{\imath}_k^m}[m]}$ denote the correlation coefficient of $\hat{h}^{\mathrm{SE}}_{\hat{\imath}_k^m}[m]$ and $\hat{h}^{\mathrm{SP}}_{\hat{\imath}_k^m}[m]$. Also $\digamma_k \triangleq k\left[\sum_{r=1}^{k} \frac{1}{\Gamma_{P_\pi(r)}[m]}\right]^{-1}$ for $k \leq K'$, and $\digamma_k \triangleq K'\left[\sum_{r=1}^{k} \frac{1}{\Gamma_{P_\pi(r)}[m]}\right]^{-1}$ for $k > K'$ where $\Gamma_{\hat{\imath}_k^m}[m] = \frac{b^{\mathrm{SE}}_{\hat{\imath}_k^m}[m] \left|\rho_{\hat{h}^{\mathrm{SE}}_{\hat{\imath}_k^m}[m],\, \hat{h}^{\mathrm{SP}}_{\hat{\imath}_k^m}[m]}\right|^2}{\zeta^{\mathrm{SE}}_{\hat{\imath}_k^m}[m]}$.

Proof: (9) can be expressed as

$$y_{\hat{\imath}_k^m}[m] = \hat{h}^{\mathrm{SE}}_{\hat{\imath}_k^m}[m]\, d[m] - e^{\mathrm{SE}}_{\hat{\imath}_k^m}[m]\, d[m] + n_{\hat{\imath}_k^m}[m] \tag{21}$$

where $\hat{h}_k^{\mathrm{SE}}[m] = h_k[m] + e_k^{\mathrm{SE}}[m]$ with $e_k^{\mathrm{SE}}[m]$ the estimation error. Conditioned on $\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]$ and $d[m]$, $y_{\hat{\iota}_k^m}[m]$ in (21) is a complex Gaussian RV whose conditional mean $\mu_{y_{\hat{\iota}_k^m}}[m]$ and variance $\sigma^2_{y_{\hat{\iota}_k^m}}[m]$ are given by

$$\begin{aligned}\mu_{y_{\hat{\iota}_k^m}}[m] &\triangleq \mathbb{E}\left\{y_{\hat{\iota}_k^m}[m] \mid \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m], d[m]\right\} \\ &= \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\, d[m]\, \zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] \qquad (22)\\ \sigma^2_{y_{\hat{\iota}_k^m}}[m] &\triangleq \mathrm{var}\left\{y_{\hat{\iota}_k^m}[m] \mid \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m], d[m]\right\} \\ &= 1 - \zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] + \eta^{-1}. \qquad (23)\end{aligned}$$

The maximum-likelihood (ML) soft estimate is obtained by conducting MRC on the received soft symbols from $\mathfrak{S}_m$ as

$$r'_{\mathfrak{S}_m}[m] = \sum_{k=1}^{K'} \left(\frac{\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] \left(\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right)^* y_{\hat{\iota}_k^m}[m]}{1 - \zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] + \eta^{-1}} \right) \qquad (24)$$

where the scaling factor explicitly takes account of the channel estimation uncertainty, i.e., the variances in (23).

Conditioned on $\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}$ and $d[m]$, $r'_{\mathfrak{S}_m}[m]$ in (24) is a complex Gaussian RV whose conditional mean $\mu_{r'_{\mathfrak{S}_m}}[m]$ and variance $\sigma^2_{r'_{\mathfrak{S}_m}}[m]$ are given by

$$\begin{aligned}\mu_{r'_{\mathfrak{S}_m}}[m] &\triangleq \mathbb{E}\left\{r'_{\mathfrak{S}_m}[m] \mid \left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}, d[m]\right\} \\ &= \sum_{k=1}^{K'} \frac{\left|\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right|^2 \left(\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right)^2 d[m]}{1 - \zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] + \eta^{-1}} \qquad (25)\\ \sigma^2_{r'_{\mathfrak{S}_m}}[m] &\triangleq \mathrm{var}\left\{r'_{\mathfrak{S}_m}[m] \mid \left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}, d[m]\right\} \\ &= \sum_{k=1}^{K'} \frac{\left|\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right|^2 \left(\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right)^2}{1 - \zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] + \eta^{-1}}. \qquad (26)\end{aligned}$$

The SEP of an $\mathtt{M}$PSK symbol received at time m conditioned on $\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}$ and $\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}$ $\mathrm{SEP}'_m\left(\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}, \left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}\right)$, denoted for brevity by $\mathrm{SEP}'_m(\Xi)$, is [15], [20], [21]

$$\begin{aligned}\mathrm{SEP}'_m(\Xi) &= \frac{1}{\pi}\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi} \exp\left(\frac{-\left|\mu_{r'_{\mathfrak{S}_m}}[m]\right|^2 \sin^2\left(\frac{\pi}{\mathtt{M}}\right)}{\sigma^2_{r'_{\mathfrak{S}_m}}[m]\sin^2(\theta)} \right) \mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi} \exp\left(\frac{-Y_{\mathfrak{S}_m}^{\mathrm{SE}}[m]}{\sin^2(\theta)} \right) \mathrm{d}\theta \qquad (27)\end{aligned}$$

where $\frac{\left|\mu_{r'_{\mathfrak{S}_m}}[m]\right|^2}{\sigma^2_{r'_{\mathfrak{S}_m}}[m]} = \sum_{k=1}^{K'} \frac{\left|\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right|^2 \left(\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right)^2}{1-\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]+\eta^{-1}}$ and the last equality follows from $Y_{\mathfrak{S}_m}^{\mathrm{SE}}[m] \triangleq \sum_{k=1}^{K'} \left|X_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right|^2$ where $X_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] \triangleq \sqrt{b_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]}\, \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]$.

The SEP averaged over $\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\right\}_{k=1}^{K'}$ $\mathrm{SEP}'_m\left(\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}\right)$, denoted for brevity by $\mathrm{SEP}'_m(\varpi)$, is

$$\mathrm{SEP}'_m(\varpi) = \frac{1}{\pi}\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi} \mathcal{M}_{\varkappa}\left(\frac{-1}{\sin^2(\theta)}\right) \mathrm{d}\theta \qquad (28)$$

where $\mathcal{M}_{\varkappa}(\cdot)$ is the moment generating function (MGF) of $Y_{\mathfrak{S}_m}^{\mathrm{SE}}[m]$ conditioned on $\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}$, i.e., $\mathcal{M}_{Y_{\mathfrak{S}_m}^{\mathrm{SE}}[m] \mid \left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}}(\cdot)$.

Now conditioned on $\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}$, $X_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]$ is a complex Gaussian RV with conditional mean $\mu_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m]$ and variance $\sigma^2_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m]$ are given by

$$\begin{aligned}\mu_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m] &\triangleq \mathbb{E}\left\{X_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] \mid \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\} \\ &= \sqrt{\frac{b_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\, \zeta_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]}{\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]}}\, \rho_{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m],\, \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]}\, \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m] \\ \sigma^2_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m] &\triangleq \mathrm{var}\left\{X_{\hat{\iota}_k^m}^{\mathrm{SE}}[m] \mid \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\} \\ &= \frac{b_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\left(1 - \left|\rho_{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m],\, \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]}\right|^2\right)}{\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]}. \qquad (29)\end{aligned}$$

Therefore $Y_{\mathfrak{S}_m}^{\mathrm{SE}}[m]$ follows a non-central Chi-squared distribution with conditional MGF given by [22]

$$\begin{aligned}\mathcal{M}_{\varkappa}(x) &= \left(\prod_{k=1}^{K'} \frac{1}{1 - \sigma^2_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m]\, x}\right) \\ &\quad \times \exp\left(\sum_{k=1}^{K'} \frac{\mu^2_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m]\, x}{1 - \sigma^2_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m]\, x}\right). \qquad (30)\end{aligned}$$

Substituting (30) in (28), yields

$$\begin{aligned}\mathrm{SEP}'_m(\varpi) &= \frac{1}{\pi}\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi} \left(\prod_{k=1}^{K'} \frac{\sin^2(\theta)}{\sin^2(\theta) + \sigma^2_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m]}\right) \times \\ &\quad \exp\left(-\sum_{k=1}^{K'} \frac{\xi_{\hat{\iota}_k^m}[m]\left|\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right|^2}{\sin^2(\theta) + \sigma^2_{X_{\hat{\iota}_k^m}^{\mathrm{SE}}}[m]}\right) \mathrm{d}\theta \qquad (31)\end{aligned}$$

where $\xi_{\hat{\iota}_k^m}[m] \triangleq \frac{b_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\, \zeta_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\left|\rho_{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m],\, \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]}\right|^2}{\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]}$. Denoting by $\Upsilon_{\hat{\iota}_k^m}[m] \triangleq \xi_{\hat{\iota}_k^m}[m]\left|\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right|^2$ the exponential RV with mean $\Gamma_{\hat{\iota}_k^m}[m] \triangleq \mathbb{E}\left\{\Upsilon_{\hat{\iota}_k^m}[m]\right\} = \frac{b_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]\left|\rho_{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SE}}[m],\, \hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]}\right|^2}{\zeta_{\hat{\iota}_k^m}^{\mathrm{SE}}[m]}$.

Averaging over $\left\{\hat{h}_{\hat{\iota}_k^m}^{\mathrm{SP}}[m]\right\}_{k=1}^{K'}$ and using the virtual branch combining (VBC) technique [23], yield the desired result. ■

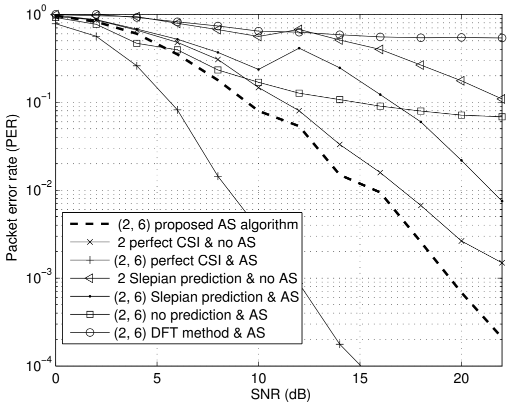

Fig. 2. PER performance of the proposed AS method for a $(2, 6)$ system. (QPSK, data packet length $N = 40$, training pilots $L = 2$, post-selection pilots $L' = 2$, and $T_p = 3\,T_s$).

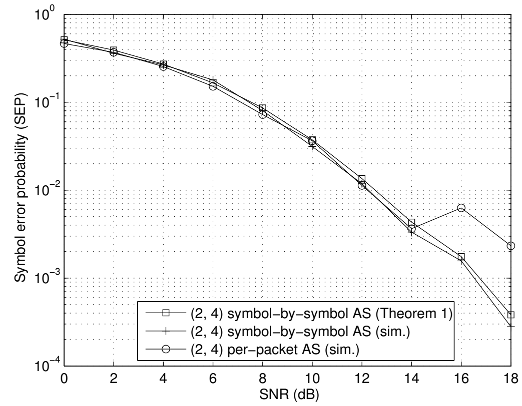

Fig. 4. SEP for the first 8PSK data symbol for a $(2, 4)$ system. (Data packet length $N = 40$, AS training pilots $L = 2$, $L' = 2$, and $T_p = 3T_s$).

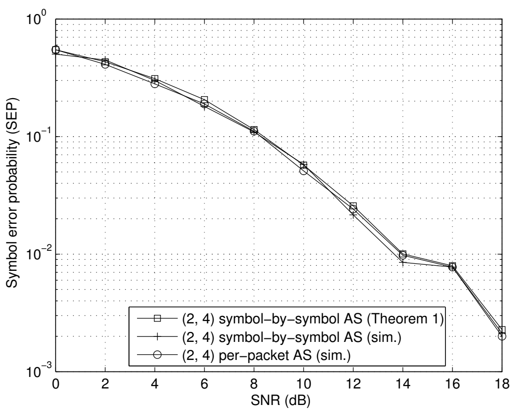

Fig. 3. SEP for the 20-*th* 8PSK data symbol for a $(2, 4)$ system. (Data packet length $N = 40$, AS training pilots $L = 2$, $L' = 2$, and $T_p = 3T_s$).

VI. SIMULATIONS

A system with one transmit and K receive antennas out of which K' is selected, denoted by (K', K), is simulated. The carrier frequency $f_c = 2\,\text{GHz}$ and the user moves with radial velocity $v_{\max} = 100\,\text{km/h}$. The packet consists of $N = 40$ $\mathtt{M}$PSK symbols each of duration $T_s = 20.57\ \mu$s [16]. These parameters give a Doppler bandwidth $\nu_{\max} = 3.8 \times 10^{-3}$. The realizations of the time-varying channel are generated as

$$h\,[m] = \sum_{p=0}^{P-1} \frac{1}{\sqrt{P}} \exp\left(j\left(2\pi\nu_{\max}\,\cos\alpha_p m + \psi_p\right)\right) \qquad (32)$$

where the number of propagation paths $P = 30$, and the path angles α_p and ψ_p are independent and uniformly distributed over $[-\pi\ \pi)$.

The packet error rate (PER) of the proposed AS approach for a $(2, 6)$ system is illustrated in Fig. 2. For comparison, we also show the PER performance of (i) a 2 receive antenna system with perfect CSI using MRC and no AS, (ii) a $(2, 6)$ system employing AS without channel prediction, and (iii) a $(2, 6)$ system employing discrete Fourier transform (DFT) basis expansion method [18]. Inspection of Fig. 2 reveals that the $(2, 6)$ system employing the proposed per-packet AS algorithm achieves an SNR performance gain in excess of 2 dB over the 2 receive antenna system with perfect CSI and no AS at a PER $= 10^{-3}$. The performance of the same proposed $(2, 6)$ system is about 7 dB worse than $(2, 6)$ system employing AS with perfect CSI at PER of 10^{-3}. The complexity of the proposed AS method is higher than that of a system employing AS with DFT expansion model. The generation of length-M' Slepian sequences requires the use of singular value decomposition (SVD) to calculate the eigenvectors of the $M' \times M'$ matrix $\mathbf{C}$ with (l, m) entry defined as $C\,[l, m] = \frac{\sin(2\pi\nu_{\max}(l-m))}{\pi(l-m)}$. This requires $\mathcal{O}\big(\,(M')^3\,\big)$ complex multiplications [24]. However, the Fourier basis functions, which do not require the knowledge of the Doppler spread and SVD, can be stored in memory using pre-calculated look-up tables.

The SEP of the 20-*th* and first 8PSK symbols as a function of average SNR for a $(2, 4)$ system employing the proposed receive AS algorithm are depicted in Figs. 3 and 4, respectively. A gap can be observed in Fig. 4 between the per-packet and symbol-by-symbol AS curves at moderate to high SNRs. This is because the channel prediction for the first symbol is better than that for the 20-*th* symbol. Also, there are upward transitions in the curves which are the result of an increase of the subspace dimension D in (14) to avoid error-floors. The SEP of the 20-*th* and first 4PSK symbols as a function of average SNR for a $(2, 6)$ system employing the proposed receive AS algorithm are also depicted in Figs. 5 and 6, respectively. From Figs. 3–6 and from other simulations (not included), we also observe that *Theorem 1* reasonably approximates the SEP performance of systems employing the symbol-by-symbol AS algorithm in Sec. IV.

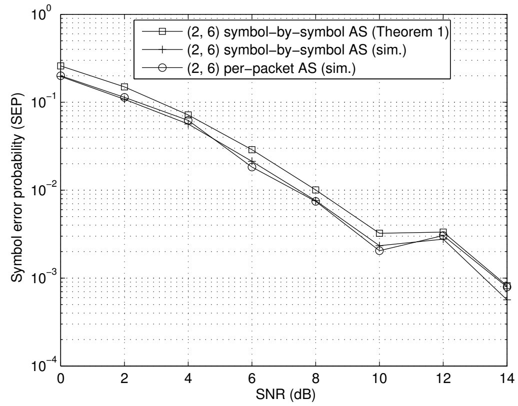

Fig. 5. SEP for the 20-*th* 4PSK data symbol for a $(2, 6)$ system. (Data packet length $N = 40$, AS training pilots $L = 2$, $L' = 2$, and $T_p = 3T_s$).

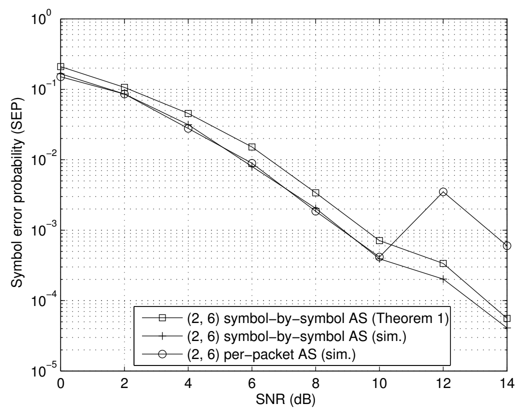

Fig. 6. SEP for the first 4PSK data symbol for a $(2, 6)$ system. (Data packet length $N = 40$, AS training pilots $L = 2$, $L' = 2$, and $T_p = 3T_s$).

VII. Conclusions

We have proposed a receive antenna subset selection approach which uses Slepian basis expansion for prediction and estimation. It takes into account practical constraints imposed by next-generation wireless standards such as training and packet reception for antenna selection (AS). We have derived a closed-form expression for the MPSK SEP with the receive AS method. It is shown that the proposed AS scheme outperforms ideal conventional systems with perfect channel knowledge but no AS at the receiver and conventional complex basis based estimation.

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